SIGHT REDUCTION TABLES FOR MARINE NAVIGATION

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Abstract

The purpose of this article is to describe the new tables, with the above title, which are shortly to be published by the U. S. Naval Oceanographic Office as H.O. Pub. No. 229, and as a Hydrographic Publication of the United Kingdom.

Historical survey

The most comprehensive tables designed for the application of the intercept method of plotting position lines in the practice of astronomical navigation at sea are the well-known *Tables of Computed Altitude and Azimuth,* originally published in the years 1936-1945 as H.O. Pub. No. 214 by the U. S. Navy Hydrographic Office (as it was then known). These are by no means the earliest of such tables in concept and objective, for Lord KELVIN, generally considered the father of modern navigation methods, expressed interest in tables from which solutions could be extracted direct; he recognized, however, that the tabulation of $90^3 \times 60^3$ or 157 464 000 000 solutions, which would be required to avoid interpolation, was quite impracticable. It was left to Frederick BALL, chaplain and naval instructor in the Royal Navy, to compute and publish his position line tables early in the twentieth century; these are the first inspection tables employing the altitude method. But H.O. Pub. No. 214 so far surpassed all others in range and completeness that thereafter they became identified with the "tabular method" of sight reduction at sea. The measure of their success is not merely the very large numbers of copies sold, and in use, but more importantly the effect of their availability in influencing methods of sight reduction.

The marine navigator, whether in the military or merchant navies, is by training and by nature extremely conservative in the methods he uses for sight reduction; and it takes many years for a new method to become generally accepted through the training schools and instruction manuals. Until H.O. Pub. No. 214 was available it was not praticable to teach sight reduction by direct tabular solution of the navigational spherical triangle; this has now become the standard method, with immense benefit in simplifying the basic problem, facilitating teaching, and—almost incidentally—saving actual calculation.

However, direct calculation using logarithmic-trigonometrical tables, or short tabular methods or " mixed " methods, will doubtless long continue to be used — with efficiency — by those who were so trained in their use. During the early part of the twentieth century most prominent maritime nations published such abbreviated navigational tables to facilitate the rapid reduction of observations and the plotting of the resulting position lines. Tables of this type are still used in considerable quantities as evidenced by the U.S. Naval Oceanographic Office's distribution last year, through official issues and sales, of some 400 copies of H.O. Pub. No. 208 (Dreisonstok), and 2600 copies of H.O. Pub. No. 211 (Ageton).

With the full permission of the U.S. Naval Oceanographic Office — and with every assistance in the supply of reproducible material — H.O. Pub. No. 214 has been reproduced, generally by photo-lithography, in several other countries, notably as Hydrographic Department publication H.D. 486 in the United Kingdom, Istituto Idrografico della Marina publication I.I. 3137 in Italy, and Instituto Hidrografico de la Marina, Special Publication No. 4 in Spain. Together with *The Nautical Almanac*, the basic principles of which are now used in almost all countries, it can be said that H.O. Pub. No. 214 has unified to a very large extent the practice of astronomical navigation at sea on an almost world-wide basis.

Although H.O. Pub. No. 214 is far from perfect, its imperfections do not significantly affect its use for navigation. There are about 1 ¼ million entries in these tables, and they were calculated at a time before even punched-card computers were available; essentially each value had to be calculated individually with separate entry into the appropriate trigonometrical tables. It is not surprising that there are minor end-figure and rounding-off errors; the larger errors, mainly introduced in the process of the printing, have already been found and corrected. Moreover, the design of the tables, particularly in respect of the provision for interpolation, is not the most suitable for obtaining the full tabular precision of 0ʹ.1; the limitation to altitudes greater than 5° has also been found restrictive in some applications.

These essentially minor imperfections in H.O. Pub. No. 214, which have little practical effect on its use, have been long appreciated by the compilers, the U.S. Naval Oceanographic Office, and the question has often been raised of the desirability of recomputing and reprinting the whole tables with all the effort and expense it would entail. It is interesting to record that the British Hydrographic Department decided, in 1950, that the advantages of a complete redesign did not warrant the recomputation, reprinting and, perhaps above all else, the necessity for complete proof-reading of new matter as compared with the direct photo-lithographic reproduction of H.O. Pub. No. 214. However, the present availability of high-speed electronic computers, of automatic photo-setting from the computer output, and of automatic methods of checking the printed figures has changed the degree of the effort required; for example, the whole of the 1 ½ million entries in the present tables can be computed, and recorded on magnetic tape, on a fast computer in two or three hours.

The U.S. Naval Oceanographic Office accordingly decided to undertake a design-study for a possible replacement of H.O. Pub. No. 214 by a newly-designed, newly-computed and newly-presented set of tables, and drew up specifications. In accord with the earlier successful collaborative work on H.O. Pub. No. 249, *Sight Reduction Tables for Air Navigation,* these specifications were discussed with the Nautical Almanac Office of the U.S. Naval Observatory, and with H.M. Nautical Almanac Office at the Royal Greenwich Observatory. The resulting design, in all its detail, is the result of collaborative work at, and discussion between, the three establishments.

The whole production of the *Sight Reduction Tables for Marine Navigation* is, in fact, a cooperative effort. However, for obvious practical considerations, the actual computations have been centralised on the IBM 1410 electronic computer at the U.S. Naval Observatory, which also produced the magnetic-tape output from which the plates for photo-lithographic printing were automatically produced by the Linofilm process by the U.S. Government Printing Office. Calculations for certain randomly-selected pages, and independent checks on the main tabulations and on the auxiliary tables, were carried out at H.M. Nautical Almanac Office. All three establishments shared in various stages of the examination, and checking, of the printing process.

Purpose and scope of the tables

The main purpose of the tables is to facilitate the practice of astronomical navigation at sea, to the highest precision possible by conventional methods of observation and altitude correction. Experience with H.O. Pub. No. 214, and with other tables and methods, has shown that interpolation of the tabulated altitude and azimuth for hour angle and/or latitude, as well as for declination, either introduces significant errors or involves unacceptably complicated procedures. Such difficulties can be avoided by plotting the position line from a position chosen so that interpolation for hour angle and latitude unnecessary; the resulting errors of plotting can in turn be avoided by using an appropriate projection, and by small corrections for curvature of the position lines. Accordingly, the tables have been designed primarily to provide the calculated altitude and azimuth, as would have been observed (after application of observational corrections) from a chosen position at the same time as the actual observation, and so to allow the corresponding position line to be plotted by the intercept method.

A secondary purpose is to provide, within the limits of the navigational precision required, a fundamental table of the solutions of a spherical triangle in which two sides and the included angle are given.

It was thus decided to provide tabulated altitudes and azimuths for all combinations of latitude, hour angle and declination at a uniform interval of 1° in each of the three arguments; interpolation, to the highest precision, is provided only for the altitude in the direction of declination. Within this scope, the aim has been to achieve absolute accuracy with a high standard of design and presentation.

Description of the main tables

Precision. The altitude is tabulated to 0'.1 and the azimuth angle to 0°.1, the highest precision required for conventional astronomical navigation at sea. It is recognized that such precision is not generally realised in the observations, and that in some cases the lower precision of the *Sight Reduction Tables for Air Navigation* (H.O. Pub. No. 249 in U.S.A. and A.P. 3270 in U.K.) may suffice.

Arrangement. Of the three arguments, that which determines the division into volumes must be latitude; the tables are in fact divided into six volumes, each of which contains two zones of latitude as follows :

Volume No.	First zone of latitude			Second zone of latitude		
1	0°	(1°)	7°	8°	(1°)	15°
2	15	(1)	22	23	(1)	30
3	30	(1)	37	38	(1)	45
4	45	(1)	52	53	(1)	60
5	60	(1)	67	68	(1)	75
6	75	(1)	82	83	(1)	90

There is an overlap of one degree (15°, 30°, 45°, 60°, 75°) between volumes.

Within each of the twelve zones of latitude, for which the 182 pages of tabulations constitute a self-contained entity, the main argument is local hour angle (measured westwards from the local meridian through 360°); to each value of the local hour angle (L.H.A. = P°) in the range 0° (1°) 90° there corresponds an opening of two facing pages, which contain the tabulations for :

(i) local hour angles, L.H.A. = P° and 360° — P°, for declinations of both the same and contrary names;

(ii) local hour angles, L.H.A. = 180° — P° and 180° + P°, for declinations of the same name; for these hour angles bodies with declinations of contrary name are below the horizon.

Within each opening, the left-hand page is always restricted to the tabulations for local hour angle L.H.A. = P° and 360° — P°, and for

29°, 331° L.H.A. LATITUDE SAME NAME AS DECLINATION

N. Lat. {L.H.A. greater than 180°...Zn = Z; L.H.A. less than 180°........Zn = 360° − Z}

Dec.	75° Hc	75° d	75° Z	76° Hc	76° d	76° Z	77° Hc	77° d	77° Z	78° Hc	78° d	78° Z	79° Hc	79° d	79° Z	80° Hc	80° d	80° Z	81° Hc	81° d	81° Z	82° Hc	82° d	82° Z	Dec.
°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	°
0	13 05.0	+59.5	150.2	12 12.9	+59.6	150.3	11 20.8	+59.6	150.4	10 28.6	+59.7	150.5	9 36.4	+59.7	150.5	8 44.1	+59.8	150.6	7 51.8	+59.9	150.7	6 59.5	+59.9	150.8	0
1	14 04.5	59.5	150.0	13 12.5	59.6	150.1	12 20.4	59.7	150.3	11 28.3	59.7	150.4	10 36.1	59.8	150.5	9 43.9	59.8	150.5	8 51.7	59.8	150.6	7 59.4	59.8	150.7	1
65	76 11.4	+50.4	120.9	75 39.1	+52.4	124.2	75 04.0	+54.0	127.3	74 26.4	+55.4	130.2	73 46.7	+56.4	132.8	73 04.9	+57.3	135.2	72 21.5	+58.0	137.5	71 36.6	+58.5	139.5	65
66	77 01.8	49.0	118.5	76 31.5	51.2	122.2	75 58.0	53.1	125.6	75 21.8	54.6	128.7	74 43.1	55.9	131.6	74 02.2	57.0	134.2	73 19.5	57.7	136.6	72 35.1	58.4	138.8	66
67	77 50.8	47.3	115.9	77 22.7	49.9	119.9	76 51.1	52.1	123.6	76 16.4	53.9	127.0	75 39.0	55.4	130.2	74 59.2	56.5	133.0	74 17.2	57.5	135.6	73 33.5	58.2	138.0	67
68	78 38.1	45.1	112.8	78 12.6	48.3	117.3	77 43.2	50.9	121.4	77 10.3	53.0	125.1	76 34.4	54.6	128.5	75 55.7	56.0	131.7	75 14.7	57.1	134.5	74 31.7	57.9	137.1	68
69	79 23.2	42.5	109.4	79 00.9	46.2	114.3	78 34.1	49.2	118.8	78 03.3	51.8	122.9	77 29.0	53.8	126.7	76 51.7	55.4	130.1	76 11.8	56.6	133.3	75 29.6	57.6	136.1	69
70	80 05.7	+39.4	105.4	79 47.1	+43.6	110.8	79 23.3	+47.3	115.8	78 55.1	+50.3	120.4	78 22.8	+52.7	124.6	77 47.1	+54.6	128.4	77 08.4	+56.2	131.8	76 27.2	+57.3	134.9	70
71	80 45.1	35.4	100.9	80 30.7	40.5	106.8	80 10.6	44.8	112.3	79 45.4	48.4	117.4	79 15.5	51.4	122.1	78 41.7	53.7	126.4	78 04.6	55.4	130.2	77 24.5	56.8	133.6	71
72	81 20.5	30.6	95.7	81 11.2	36.4	102.1	80 55.4	41.7	108.3	80 33.8	46.0	114.0	80 06.9	49.6	119.2	79 35.4	52.4	124.0	79 00.0	54.6	128.3	78 21.3	56.2	132.1	72
73	81 51.1	24.8	89.8	81 47.6	31.6	96.7	81 37.1	37.6	103.5	81 19.8	42.9	109.9	80 56.5	47.3	115.8	80 27.8	50.8	121.2	79 54.6	53.5	126.0	79 17.5	55.5	130.3	73
74	82 15.9	18.2	83.2	82 19.2	25.6	90.6	82 14.7	32.6	98.0	82 02.7	39.0	105.1	81 43.8	44.3	111.7	81 18.6	48.7	117.8	80 48.1	52.0	123.3	80 13.0	54.6	128.1	74
89	75 52.0	52.0	2.0	76 52.0	52.0	2.1	77 51.9	51.9	2.3	78 51.9	51.9	2.5	79 51.8	51.8	2.8	80 51.7	51.7	3.1	81 51.6	51.6	3.4	82 51.5	51.5	3.9	89
90	75 00.0	−52.9	0.0	76 00.0	−52.9	0.0	77 00.0	−53.0	0.0	78 00.0	−53.0	0.0	79 00.0	−53.1	0.0	80 00.0	−53.1	0.0	81 00.0	−53.2	0.0	82 00.0	−53.3	0.0	90
	75°			76°			77°			78°			79°			80°			81°			82°			

29°, 331° L.H.A. LATITUDE SAME NAME AS DECLINATION

LATITUDE CONTRARY NAME TO DECLINATION L.H.A. 29°, 331°

Dec.	75° Hc	75° d	75° Z	76° Hc	76° d	76° Z	77° Hc	77° d	77° Z	78° Hc	78° d	78° Z	79° Hc	79° d	79° Z	80° Hc	80° d	80° Z	81° Hc	81° d	81° Z	82° Hc	82° d	82° Z	Dec.
°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	°
0	13 05.0	−59.5	150.2	12 12.9	−59.5	150.3	11 20.8	−59.6	150.4	10 28.6	−59.7	150.5	9 36.4	−59.7	150.5	8 44.1	−59.7	150.6	7 51.8	−59.8	150.7	6 59.5	−59.9	150.8	0
1	12 05.5	59.5	150.3	11 13.4	59.6	150.4	10 21.2	59.7	150.5	9 28.9	59.6	150.6	8 36.7	59.8	150.6	7 44.4	59.8	150.7	6 52.0	59.8	150.8	5 59.6	59.8	150.8	1
2	11 06.0	59.5	150.4	10 13.8	59.6	150.5	9 21.5	59.6	150.6	8 29.3	59.7	150.7	7 36.9	59.7	150.7	6 44.6	59.8	150.8	5 52.2	59.8	150.9	4 59.8	59.9	150.9	2
3	10 06.5	59.5	150.5	9 14.2	59.6	150.6	8 21.9	59.6	150.7	7 29.6	59.7	150.8	6 37.2	59.7	150.8	5 44.8	59.8	150.9	4 52.4	59.9	150.9	3 59.9	59.9	151.0	3
4	9 07.0	59.5	150.7	8 14.6	59.5	150.7	7 22.3	59.7	150.8	6 29.9	59.7	150.9	5 37.5	59.8	150.9	4 45.0	59.8	151.0	3 52.5	59.8	151.0	3 00.0	59.8	151.0	4
5	8 07.5	−59.6	150.8	7 15.1	−59.6	150.9	6 22.6	−59.6	150.9	5 30.2	−59.7	151.0	4 37.7	−59.7	151.0	3 45.2	−59.8	151.1	2 52.7	−59.8	151.1	2 00.2	−59.9	151.1	5
6	7 07.9	59.5	150.9	6 15.5	59.6	151.0	5 23.0	59.6	151.0	4 30.5	59.7	151.1	3 38.0	59.8	151.1	2 45.4	59.8	151.1	1 52.9	59.8	151.2	1 00.3	59.8	151.2	6
7	6 08.4	59.5	151.1	5 15.9	59.6	151.1	4 23.4	59.7	151.1	3 30.8	59.7	151.2	2 38.2	59.7	151.2	1 45.6	59.7	151.2	0 53.1	−59.9	151.2	0 00.5	−59.9	151.2	7
8	5 08.9	59.5	151.2	4 16.3	59.6	151.2	3 23.7	59.6	151.3	2 31.1	59.7	151.3	1 38.5	59.8	151.3	0 45.9	−59.8	151.3	0 06.8	+59.8	28.7	0 59.4	+59.9	28.7	8
9	4 09.4	59.5	151.3	3 16.7	59.5	151.3	2 24.1	59.7	151.4	1 31.4	59.7	151.4	0 38.7	−59.7	151.4	0 13.9	+59.8	28.6	1 06.6	59.8	28.6	1 59.3	59.8	28.6	9
10	3 09.9	−59.6	151.4	2 17.2	−59.6	151.5	1 24.4	−59.6	151.5	0 31.7	−59.7	151.5	0 21.0	+59.7	28.5	1 13.7	+59.8	28.5	2 06.4	+59.9	28.5	2 59.1	+59.9	28.6	10
11	2 10.3	59.5	151.6	1 17.6	59.6	151.6	0 24.8	−59.6	151.6	0 28.0	+59.7	28.4	1 20.7	59.8	28.4	2 13.5	59.8	28.4	3 06.3	59.8	28.5	3 59.0	59.9	28.5	11
12	1 10.8	59.5	151.7	0 18.0	−59.6	151.7	0 34.8	+59.7	28.3	1 27.7	59.7	28.3	2 20.5	59.7	28.3	3 13.3	59.8	28.4	4 06.1	59.8	28.4	4 58.9	59.8	28.4	12
13	0 11.3	−59.5	151.8	0 41.6	+59.6	28.2	1 34.5	59.6	28.2	2 27.4	59.7	28.2	3 20.2	59.8	28.2	4 13.1	59.8	28.3	5 05.9	59.8	28.3	5 58.7	59.9	28.4	13
14	0 48.2	+59.6	28.1	1 41.2	59.6	28.1	2 34.1	59.7	28.1	3 27.1	59.6	28.1	4 20.0	59.7	28.1	5 12.9	59.7	28.2	6 05.7	59.9	28.2	6 58.6	59.8	28.3	14
90	75 00.0	+52.0	0.0	76 00.0	+52.0	0.0	77 00.0	+51.9	0.0	78 00.0	+51.9	0.0	79 00.0	+51.8	0.0	80 00.0	+51.7	0.0	81 00.0	+51.6	0.0	82 00.0	+51.5	0.0	90
	75°			76°			77°			78°			79°			80°			81°			82°			

S. Lat. {L.H.A. greater than 180°...Zn = 180° − Z; L.H.A. less than 180°........Zn = 180° + Z}

LATITUDE SAME NAME AS DECLINATION L.H.A. 151°, 209°

FIGURE 1B

FIG. 1

declinations of the same name. The right-hand page contains : on the upper portion, the tabulations for local hour angle L.H.A. = P° and 360° — P°, and for declinations of the contrary name; and, on the lower portion, the tabulations for the supplementary local hour angle L.H.A. = 180° — P° and 180° + P°, and for declinations of the same name, which dovetail in exactly with those on the upper portion. (See fig. 1).

The local hour angle, which is the primary entering argument, is prominently displayed at the top and bottom of each page; the horizontal argument heading each column is latitude; and the vertical argument is declination.

With this arrangement, the opening required for the reduction of a sight taken within a particular zone of latitude (either north or south) is determined uniquely by the value of the local hour angle. Whatever the local hour angle, tabulations for all declinations and for all latitudes within the zone are immediately available on the one opening. The value of the L.H.A., prominently displayed, is thus the most significant element for entering the tables — far more important than the page number as an index and reference.

Tabulated quantities. For each combination of arguments there is tabulated : the altitude, Hc, to 0'.1; in smaller type, the actual difference, d, with sign, to the tabulated altitude for the next higher degree of declination (referred to later as the " altitude difference "); the azimuth angle, Z, to 0°.1.

Rules are given, on each opening, for converting the tabulated azimuth angle into true azimuth, measured from the north through east; the rules differ according to the hemisphere (northern or southern) and the range of local hour angle.

Figs 1A and 1B illustrate the appearance of a portion of (a) a left-hand page and (b) a right-hand page from the same opening.

Interpolation

Provision is made for interpolation of the altitude in the direction of the declination by the tabulation of the actual first difference, d, referred to as the " altitude difference ", and by special interpolation tables, included in each volume. The latter are designed to enable interpolation to be made to the exact declination of the body observed, to full precision including (where necessary) the effect of second differences.

The main, vertical, argument of the interpolation table (see fig. 2) is the excess of the actual declination over that used (an integral degree) for the tabular entry; since it is recommended that the tabular entry used should *always* be for the integral degree of declination numerically *less* than (or equal to) the actual declination, this excess should always be the actual minutes of the declination itself, referred to as the " declination increment ". It is abbreviated to " Dec. Inc. ". The other argument is the tabulated altitude difference, d, which for convenience is divided into two

INTERPOLATION TABLE

Dec. Inc.	10′	20′	30′	40′	50′	Decimals	0′	1′	2′	3′	4′	5′	6′	7′	8′	9′
28.0	4.6	9.3	14.0	18.6	23.3	.0	0.0	0.5	0.9	1.4	1.9	2.4	2.8	3.3	3.8	4.3
28.1	4.7	9.3	14.0	18.7	23.4	.1	0.0	0.5	1.0	1.5	1.9	2.4	2.9	3.4	3.8	4.3
28.2	4.7	9.4	14.1	18.8	23.5	.2	0.1	0.6	1.0	1.5	2.0	2.5	2.9	3.4	3.9	4.4
28.3	4.7	9.4	14.1	18.9	23.6	.3	0.1	0.6	1.1	1.6	2.0	2.5	3.0	3.5	3.9	4.4
28.4	4.7	9.5	14.2	18.9	23.7	.4	0.2	0.7	1.1	1.6	2.1	2.6	3.0	3.5	4.0	4.5
28.5	4.8	9.5	14.3	19.0	23.8	.5	0.2	0.7	1.2	1.7	2.1	2.6	3.1	3.6	4.0	4.5
28.6	4.8	9.5	14.3	19.1	23.8	.6	0.3	0.8	1.2	1.7	2.2	2.7	3.1	3.6	4.1	4.6
28.7	4.8	9.6	14.4	19.2	23.9	.7	0.3	0.8	1.3	1.8	2.2	2.7	3.2	3.7	4.1	4.6
28.8	4.8	9.6	14.4	19.2	24.0	.8	0.4	0.9	1.3	1.8	2.3	2.8	3.2	3.7	4.2	4.7
28.9	4.9	9.7	14.5	19.3	24.1	.9	0.4	0.9	1.4	1.9	2.3	2.8	3.3	3.8	4.2	4.7

Double Second Diff.	Corr.
0.8	
	0.1
2.4	
	0.2
4.0	
	0.3
5.6	
	0.4
7.2	
	0.5
8.8	
	0.6
10.4	
	0.7
12.0	
	0.8
13.6	
	0.9
15.2	
	1.0
16.8	

Dec. Inc.	10′	20′	30′	40′	50′	Decimals	0′	1′	2′	3′	4′	5′	6′	7′	8′	9′
34.0	5.6	11.3	17.0	22.6	28.3	.0	0.0	0.6	1.1	1.7	2.3	2.9	3.4	4.0	4.6	5.2
34.1	5.7	11.3	17.0	22.7	28.4	.1	0.1	0.6	1.2	1.8	2.4	2.9	3.5	4.1	4.7	5.2
34.2	5.7	11.4	17.1	22.8	28.5	.2	0.1	0.7	1.3	1.8	2.4	3.0	3.6	4.1	4.7	5.3
34.3	5.7	11.4	17.1	22.9	28.6	.3	0.2	0.7	1.3	1.9	2.5	3.0	3.6	4.2	4.8	5.3
34.4	5.7	11.5	17.2	22.9	28.7	.4	0.2	0.8	1.4	2.0	2.5	3.1	3.7	4.3	4.8	5.4
34.5	5.8	11.5	17.3	23.0	28.8	.5	0.3	0.9	1.4	2.0	2.6	3.2	3.7	4.3	4.9	5.5
34.6	5.8	11.5	17.3	23.1	28.8	.6	0.3	0.9	1.5	2.1	2.6	3.2	3.8	4.4	4.9	5.5
34.7	5.8	11.6	17.4	23.2	28.9	.7	0.4	1.0	1.6	2.1	2.7	3.3	3.9	4.4	5.0	5.6
34.8	5.8	11.6	17.4	23.2	29.0	.8	0.5	1.0	1.6	2.2	2.8	3.3	3.9	4.5	5.1	5.6
34.9	5.9	11.7	17.5	23.3	29.1	.9	0.5	1.1	1.7	2.2	2.8	3.4	4.0	4.5	5.1	5.7
35.0	5.8	11.6	17.5	23.3	29.1	.0	0.0	0.6	1.2	1.8	2.4	3.0	3.5	4.1	4.7	5.3
35.1	5.8	11.7	17.5	23.4	29.2	.1	0.1	0.7	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4
35.2	5.8	11.7	17.6	23.4	29.3	.2	0.1	0.7	1.3	1.9	2.5	3.1	3.7	4.3	4.9	5.4
35.3	5.9	11.8	17.6	23.5	29.4	.3	0.2	0.8	1.4	2.0	2.5	3.1	3.7	4.3	4.9	5.5
35.4	5.9	11.8	17.7	23.6	29.5	.4	0.2	0.8	1.4	2.0	2.6	3.2	3.8	4.4	5.0	5.6
35.5	5.9	11.8	17.8	23.7	29.6	.5	0.3	0.9	1.5	2.1	2.7	3.3	3.8	4.4	5.0	5.6
35.6	5.9	11.9	17.8	23.7	29.7	.6	0.4	0.9	1.5	2.1	2.7	3.3	3.9	4.5	5.1	5.7
35.7	6.0	11.9	17.9	23.8	29.8	.7	0.4	1.0	1.6	2.2	2.8	3.4	4.0	4.6	5.1	5.7
35.8	6.0	12.0	17.9	23.9	29.9	.8	0.5	1.1	1.7	2.2	2.8	3.4	4.0	4.6	5.2	5.8
35.9	6.0	12.0	18.0	24.0	30.0	.9	0.5	1.1	1.7	2.3	2.9	3.5	4.1	4.7	5.3	5.9
	10′	20′	30′	40′	50′		0′	1′	2′	3′	4′	5′	6′	7′	8′	9′

Double Second Diff.	Corr.
0.8	
	0.1
2.5	
	0.2
4.1	
	0.3
5.8	
	0.4
7.4	
	0.5
9.1	
	0.6
10.7	
	0.7
12.3	
	0.8
14.0	
	0.9
15.6	
	1.0
17.3	
	1.1
18.9	
	1.2
20.6	
	1.3
22.2	
	1.4
23.9	
	1.5
25.5	
	1.6
27.2	
	1.7
28.8	
	1.8
30.4	
	1.9
32.1	
	2.0
33.7	
	2.1
35.4	

Dec. Inc.	10′	20′	30′	40′	50′	Decimals	0′	1′	2′	3′	4′	5′	6′	7′	8′	9′
36.0	6.0	12.0	18.0	24.0	30.0	.0	0.0	0.6	1.2	1.8	2.4	3.0	3.6	4.3	4.9	5.5
36.1	6.0	12.0	18.0	24.0	30.1	.1	0.1	0.7	1.3	1.9	2.5	3.1	3.7	4.3	4.9	5.5
36.2	6.0	12.0	18.1	24.1	30.1	.2	0.1	0.7	1.3	1.9	2.6	3.2	3.8	4.4	5.0	5.6
36.3	6.0	12.1	18.1	24.2	30.2	.3	0.2	0.8	1.4	2.0	2.6	3.2	3.8	4.4	5.0	5.7
36.4	6.1	12.1	18.2	24.3	30.3	.4	0.2	0.9	1.5	2.1	2.7	3.3	3.9	4.5	5.1	5.7
36.5	6.1	12.2	18.3	24.3	30.4	.5	0.3	0.9	1.5	2.1	2.7	3.3	4.0	4.6	5.2	5.8
36.6	6.1	12.2	18.3	24.4	30.5	.6	0.4	1.0	1.6	2.2	2.8	3.4	4.0	4.6	5.2	5.8
36.7	6.1	12.3	18.4	24.5	30.6	.7	0.4	1.0	1.6	2.3	2.9	3.5	4.1	4.7	5.3	5.9
36.8	6.2	12.3	18.4	24.6	30.7	.8	0.5	1.1	1.7	2.3	2.9	3.5	4.1	4.7	5.4	6.0
36.9	6.2	12.3	18.5	24.6	30.8	.9	0.5	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4	6.0

Double Second Diff.	Corr.
0.8	
	0.1
2.5	
	0.2
4.2	
	0.3
5.9	
	0.4
7.6	
	0.5
9.3	
	0.6
11.0	
	0.7
12.7	
	0.8
14.4	
	0.9
16.1	
	1.0
17.8	

Dec. Inc.	10′	20′	30′	40′	50′	Decimals	0′	1′	2′	3′	4′	5′	6′	7′	8′	9′
42.0	7.0	14.0	21.0	28.0	35.0	.0	0.0	0.7	1.4	2.1	2.8	3.5	4.2	5.0	5.7	6.4
42.1	7.0	14.0	21.0	28.0	35.1	.1	0.1	0.8	1.5	2.2	2.9	3.6	4.3	5.0	5.7	6.4
42.2	7.0	14.0	21.1	28.1	35.1	.2	0.1	0.8	1.6	2.3	3.0	3.7	4.4	5.1	5.8	6.5
42.3	7.0	14.1	21.1	28.2	35.2	.3	0.2	0.9	1.6	2.3	3.0	3.8	4.5	5.2	5.9	6.6
42.4	7.1	14.1	21.2	28.3	35.3	.4	0.3	1.0	1.7	2.4	3.1	3.8	4.5	5.2	5.9	6.7
42.5	7.1	14.2	21.3	28.3	35.4	.5	0.4	1.1	1.8	2.5	3.2	3.9	4.6	5.3	6.0	6.7
42.6	7.1	14.2	21.3	28.4	35.5	.6	0.4	1.1	1.8	2.5	3.3	4.0	4.7	5.4	6.1	6.8
42.7	7.1	14.3	21.4	28.5	35.6	.7	0.5	1.2	1.9	2.6	3.3	4.0	4.7	5.5	6.2	6.9
42.8	7.2	14.3	21.4	28.6	35.7	.8	0.6	1.3	2.0	2.7	3.4	4.1	4.8	5.5	6.2	6.9
42.9	7.2	14.3	21.5	28.6	35.8	.9	0.6	1.3	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7.0
43.0	7.1	14.3	21.5	28.6	35.8	.0	0.0	0.7	1.4	2.2	2.9	3.6	4.3	5.1	5.8	6.5
43.1	7.2	14.3	21.5	28.7	35.9	.1	0.1	0.8	1.5	2.2	3.0	3.7	4.4	5.1	5.9	6.6
43.2	7.2	14.4	21.6	28.8	36.0	.2	0.1	0.9	1.6	2.3	3.0	3.8	4.5	5.2	5.9	6.7
43.3	7.2	14.4	21.6	28.9	36.1	.3	0.2	0.9	1.7	2.4	3.1	3.8	4.6	5.3	6.0	6.7
43.4	7.2	14.5	21.7	28.9	36.2	.4	0.3	1.0	1.7	2.5	3.2	3.9	4.6	5.4	6.1	6.8
43.5	7.3	14.5	21.8	29.0	36.3	.5	0.4	1.1	1.8	2.5	3.3	4.0	4.7	5.4	6.2	6.9
43.6	7.3	14.5	21.8	29.1	36.3	.6	0.4	1.2	1.9	2.6	3.3	4.1	4.8	5.5	6.2	7.0
43.7	7.3	14.6	21.9	29.2	36.4	.7	0.5	1.2	2.0	2.7	3.4	4.1	4.9	5.6	6.3	7.0
43.8	7.3	14.6	21.9	29.2	36.5	.8	0.6	1.3	2.0	2.8	3.5	4.2	4.9	5.7	6.4	7.1
43.9	7.4	14.7	22.0	29.3	36.6	.9	0.7	1.4	2.1	2.8	3.6	4.3	5.0	5.7	6.5	7.2
	10′	20′	30′	40′	50′		0′	1′	2′	3′	4′	5′	6′	7′	8′	9′

Double Second Diff.	Corr.
1.0	
	0.1
3.0	
	0.2
4.9	
	0.3
6.9	
	0.4
8.9	
	0.5
10.8	
	0.6
12.8	
	0.7
14.8	
	0.8
16.7	
	0.9
18.7	
	1.0
20.7	
	1.1
22.7	
	1.2
24.6	
	1.3
26.6	
	1.4
28.6	
	1.5
30.5	
	1.6
32.5	
	1.7
34.5	
	1.8
36.4	

The Double-Second-Difference correction (Corr.) is always to be added to the tabulated altitude.

FIG. 2. — A portion of the interpolation table. The reduction factor is about 0.9. The actual type area on the printed page is 8.3 × 11.0 inches.

parts, the first being a multiple of 10′ (10′, 20′, 30′, 40′ or 50′) and the second the remainder in the range 0′.0 (0′.1) 9′.9.

The main part of the interpolation correction (i.e. the quantity to be applied to the tabular entry to convert it to the calculated altitude for the desired declination) is obtained as the simple sum of two quantities :

(i) *the first contribution* : the tabular value corresponding to the Dec. Inc. and the tens of minutes of the altitude difference, d;

(ii) *the second contribution* : the tabular value corresponding to the Dec. Inc. and the remainder of d; the units of this remainder form the horizontal, and the decimals the vertical, arguments for entering the section of the table opposite (on the right-hand side of) the pertinent unit of the Dec. Inc. employed in obtaining the first contribution.

There is no mental interpolation, and the additional error introduced by the division into two parts (as compared with an exact multiplication) has a maximum of 0′.07. If, as recommended, interpolation is made in a forward direction the correction is to be applied to the altitude with the sign of the altitude difference.

In the same table, provision is also made for the application of the second-difference correction; critical tables, each corresponding to a small range of the main argument Dec. Inc., give the second-difference correction with the double-second difference as argument. Since the second difference of the altitude is always negative, the second-difference correction is always to be added to the altitude. This correction is generally small; it can be neglected for normal sight reduction unless the altitude difference, d, is printed in *italic* type, or unless full precision is required. The altitude difference, d, in the main tables is printed in *italic* type (followed by a dot for easier recognition) when the double-second difference is 4′.0 or greater, which can occur only when the altitude exceeds 60°; the maximum error that can arise through the neglect of second differences, when d is not in *italics*, is thus 0′.25.

The interpolation table has been designed so that it can be printed on four pages only; the inside front cover and facing page provide for the range 0′.0 to 31′.9 of the Dec. Inc., while the inside back cover and facing page provide for the range 28′.0 to 59′.9. Part of it is reproduced in Figure 2.

As an illustration of the use of the interpolation table, the altitude is determined for latitude N. 76°, local hour angle 29° (see Fig. 1A) and declination N. 69° 34′.8.

For declination 69°, the same name, the altitude is 79° 00′.9 and the altitude difference, d, is + 46′.2; it is printed in *italic type* and followed by a dot (so that second differences are significant), and the double-second difference, obtained as the difference between the tabulated altitude differences on the preceding (+ 48′.3) and following (+ 43′.6) lines, is — 4′.7. The Dec. Inc. is 34′.8, which is used as the main argument for the first contribution from the interpolation table and then indicates the small tables from which the second contribution and the second-difference correction are taken.

			Correction
First contribution for Dec. Inc. 34′.8	altitude diff.	= 40′	23′.2
Second contribution for Dec. Inc. 34′.8	altitude diff.	= 6′.2	3′.6
First-difference correction : Dec. Inc. 34′.8	altitude diff.	= 46′.2	26′.8
Second-difference correction : Dec. Inc. 34′.8	double-second diff.	= 4′.7	0′.3

The interpolated altitude, using first differences only, is thus 79° 27′.7 = 79° 00′.9 + 26′.8 since d is positive; the correction of second differences makes it 79° 28′.0, since the latter must always be added.

No special table is provided for interpolation of the azimuth angle; and the differences are not tabulated. The successive azimuth differences are less than 10°.0 for altitudes less than 84°, and can easily be formed mentally; to this point second differences can be neglected. If formal interpolation is necessary, the second contribution from the interpolation table is alone required (since the azimuth difference cannot exceed 10°.0); but for most practical applications interpolation by inspection will probably suffice.

Interpolation becomes difficult close to the zenith, but no special tables are provided. Instead, there is described in the Introduction to the tables, a method of interpolation (based on keeping constant the difference between latitude and declination) that overcomes the difficulty in all except extreme cases in very high latitudes.

Although the tables are not designed for interpolation for latitude and local hour angle, interpolation is possible using differences and the interpolation table. But great care, and considerable work, is required if a precision of about 0′.2 or 0′.3 is to be obtained. For this reason there is also included in each volume two printed diagrams, and a simple loose transparency (which will also be available in separate blocks), which will provide a graphical method for interpolating the altitude to the D.R. position to full precision. The same diagrams can also be used for plotting a series of position lines using intercepts from chosen positions; the resulting fix can then be transferred to the chart.

Auxiliary tables

Because of the provision of altitude-reduction tables in *The Nautical Almanac*, there is little requirement for auxiliary tables; the only one included is a Speed-Time-Distance table.

Production

Method of Computation. For latitude (lat.), declination (dec.) and local hour angle (L.H.A.), the azimuth (Az.) and altitude (Alt.) were calculated from the following formulae :

$$\tan(\text{Az.}) = \frac{\cos(\text{dec.})\ \sin(\text{L.H.A.})}{\cos(\text{lat.})\ \sin(\text{dec.}) - \sin(\text{lat.})\ \cos(\text{dec.})\ \cos(\text{L.H.A.})}$$

$$\sin(\text{Alt.}) = \sin(\text{lat.})\ \sin(\text{dec.}) + \cos(\text{lat.})\ \cos(\text{dec.})\ \cos(\text{L.H.A.})$$

The computations were performed on an IBM 1410 electronic calculator, using nine significant figures, to assure the desired accuracy of a tenth of a minute of arc in altitude and a tenth of a degree in azimuth.

The results of the computations for any one volume are transcribed on to two magnetic tapes; one contains the altitude and azimuth calculated for declinations and latitudes of the same name (left-hand pages) and the other contains the altitude and azimuth calculated for declinations and latitudes of contrary names (right-hand pages).

The calculations for any one page proceed according to the following algorithm. The local hour angle of the page is first established. Then, for the smallest value of the latitude contained within the volume, all azimuths and altitudes are calculated for the range of declinations 0° (1°) 90°. The latitude is then augmented by one degree and, again, all altitudes and azimuths are calculated. The process of augmenting the latitude and calculating the altitudes and azimuths proceeds until the largest value of the latitude in the volume is reached. The succeeding pages are computed in the same fashion, augmenting the local hour angle by one degree for each set of four pages.

Checks before copy production. Checks on the final values of altitude and azimuth were applied by two methods :

(i) manual recalculation of several quantities on each page;

(ii) automatic differencing of consecutive values of altitude and azimuth in the direction of declination.

In addition, all values of the altitude within 1°30′ of the zenith (for which values the limitation to nine figures may introduce errors of order 0′.0005 and so affect the rounding-off to 0′.1) were re-calculated by direct methods, not subject to the limitations of the standard formulae. The formula used is equivalent to

$$\sin^2 1/2(\text{z.d.}) = \cos^2 1/2(\text{L.H.A.})\ \sin^2 1/2(\text{lat.} - \text{dec.}) + \sin^2 1/2(\text{L.H.A.})\ \cos^2 1/2(\text{lat.} + \text{dec.})$$

where z.d. is the zenith distance.

The re-calculated values were automatically compared with the previous ones during the editing program; as a result 3 end-figures were changed in Volume 6.

Automatic Photo-Composition. An editing and page-composition program on the IBM 1410 was used to assemble automatically the previously calculated values of the altitude and azimuth in the form of the final printed page. In this program the numerical data are combined with editing instructions, and the combined data are transcribed on to magnetic tape. At the beginning of the program the headings which appear on every page are assembled, coded in binary notation, and stored. The records for a single page are read in from the magnetic tapes produced in the computing run on the IBM 1410. The L.H.A. values for that page are placed in their proper heading area and the headings and spaces between lines at the top of the page are transferred to the output tape. Each line of the tabular data is then converted and stored on the output tape at two lines per record. Whenever the declination value ends in 4 or 9 an extra space between lines is provided. At the end of the pages the terminal headings are put on the tape record and the page number is calculated from the value of the L.H.A. This process requires approximately 40 seconds of computer time for each page; or about four hours time for one volume of H.O. Pub. No. 229. This binary-coded magnetic tape is then automatically translated to the fifteen-channel paper tape necessary to operate the Linofilm photo-composing equipment of the U.S. Government Printing Office. The data are photo-set, character by character, to form a photographic positive of the final page. The columnar rulings and the minor headings which explain the rules for deriving azimuth from azimuth angle are added to the Linofilm prints using overlays, and the completed pages are prepared photographically using standard procedures. The stair-step rulings (on the right-hand pages to separate the tabulations for same and contrary names) are scribed by hand on the final negative.

Proof reading. Ozalid proofs of the photo prints were examined for completeness and imperfections. The overlays were then placed on the photoprints to make offset negatives with an enlargement to final page size (approximately 127 %). Van Dyke proofs of these negatives were used for further checks. All data on these page proofs were punched on cards, one line to a card; and these cards were finally compared automatically on the electronic computer with the originally computed data.

Commentary

For some time it has been thought that automatic astronomical, electronic, and now satellite, navigation developments would outmode and replace the older stereotyped systems including astronomical navigation. Regardless of the impact of these influences and developments, this forecast has not come to pass, or at least has been impeded to the extent that, in spite of tremendous developments in navigational systems of all kinds, navigators are today still vitally interested in improved facilities for sight reduction, and in more rapid and accurate techniques for establishing the line of position. Astronomical navigation is still a fundamental procedure on most navigable craft. Greater accuracy and efficiency in positioning by all acceptable methods become economically feasible, and even practical, as speeds increase, cargoes become larger and more valuable, and the ship's environment becomes better known. Military demands and accelerated means of transportation and communication require that approximate solutions and estimating procedures be abandoned in favour of greater accuracies, improved techniques, and more rigorous solutions. There is no doubt as to the need for better and faster methods for sight reduction.

At the time when the navigational journals are full of descriptions of completely automatic navigational systems using artificial satellites, it may seem reactionary even to consider the production of tables for the solution of the navigational spherical triangle. Such solutions are a tiny fraction of the computing required with these systems, and take milliseconds only to complete. But a very large percentage of surface navigators still use logarithms, and it will be many years before even the majority of ships are fully equipped with automatic position-fixing systems. Moreover, such systems will not, without considerable effort, solve the occasional great circle, or other problems for which the tables will provide solutions at once. We are thus convinced that there is a need for these tables; and we have made the utmost use of experience, and of modern computing and printing methods, to try to make them accurate and convenient for the user.