GEODETIC CONNEXION BETWEEN FRANCE AND NORTH AFRICA BY SIMULTANEOUS SIGHTING ON THE ECHO I ARTIFICIAL SATELLITE

by THE FRENCH NATIONAL GEOGRAPHICAL INSTITUTE

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I. – GENERAL PRINCIPLES OF SATELLITE TRIANGULATION

The basic principle is very simple : a luminous object is seen against a different star background according to the observer's position on the earth's surface. It can be seen that a photograph of this luminous object and the surrounding stars will give information which permits fixing the position of points on the earth's surface, on the assumption that the positions of other observation points are known.

Let us clarify this idea : the solid earth, assumed to be indéformable and to which we shall attach a trihedral TXYZ, has a known directional movement relative to the stars in function of time, this movement being given by the astronomy of star positions. Fixing the time means immobilizing the position of the trihedral in the stellar field; each star then constitutes an absolute direction in relation to the trihedral TXYZ, such direction being given in the star almanac.

In the same way we must " immobilize " the luminous object, which in practice requires simultaneous or nearly simultaneous observations from the different points on the earth's surface which are to be connected.

If the luminous object emits flashes then simultaneity is automatically ensured. If it is continuously luminous, then " artificial flashes " have to be arranged by using, at the various observation points, rotating shutters which can be synchronized.

" Anna " is a flashing satellite which has been used for geodetic operations. It is more expensive than a continuously luminous satellite. It operates only to order. There have been some failures in its operation.

Echo I and Echo II are continuously lum inous satellites : Echo I was the one chosen for the France-North Africa link.

In practice, each photograph includes :

- $-$ a certain number of images of stars from which it is possible to calculate the orientation of the camera;
- $-$ a certain number of images of flashes. In the case of the French observations, the shutter rotating at 1 r.p.s., gives a series of 60 dots of light or ' flashes ' per minute.

In essence, the problem is to interpolate these dots among the known positions of the stars. First of all it should be pointed out that the 60 dots are reduced to one ' central flash' and that the direction of this ' central f lash' in the trihedral $TXYZ$ is then obtained.

Each photo therefore gives a known direction SF . S being the station and F the fixed point. We shall see, for the case of the connexion between France and North Africa, how the geodetic problem is tackled on the basis of such fundamental data.

Here are some values essential for accuracy.

If an angular precision of $1''$ (or $1/200000$ radian) is desired it will be necessary to have

- an accuracy of the rotating shutter to within $1/10^3$ s;
- a siderial time to within $1/20$ s;
- $-$ a determination of the position of the stars and the dots on the photographic plate to within 2 to 3μ (for a focal length of 30 cm).

II. - INSTRUMENTS

The apparatus consists essentially of

- $-$ a 30 cm focal length camera, fitted with a rotating shutter and a chopping shutter;
- $-$ a quartz chronometer regulating the rotating shutter at the speed of 1 r.p.s. for photographing the satellite, and actuating the chopping shutter for photographing the stars.

The whole apparatus is extremely simple and is easily transportable. The time readings are made ;

- for the satellite : direct on the graduation of the rotating shutter. The dial of the shutter is illuminated at exactly the same point at every pip giving the second of the time signa], so that the observer can make a reading correct to within 10^{-3} s;
- $-$ for the stars : the same principle applies, using a revolving graduated disc incorporated in the quartz chronometer.

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III. - THE GEODETIC CONNEXION BETWEEN FRANCE AND NORTH AFRICA

The Institut Géographique National, in agreement with the Algerian authorities, carried out observations for the geodetic connexion between France and North Africa by synchronous photographic observations of the Echo I satellite.

Known points :

- Lacanau (near Bordeaux on the A tlantic coast)
- Agde (near Sète on the Mediterranean)
- Oletta (Corsica)

Points to be determined :

- Hammaguir (Western Sahara)
- O uargla (Eastern Sahara)

In fact, all five points are known within the European Datum (the so-called 1950 European Datum), and the work is therefore designed for comparing the results of conventional geodesy with those of satellite geodesy.

Satellites used

The only observations selected are those made on Echo I as this culminates at approxim ately 47?5, w hich is a very suitable altitude for observations over the Mediterranean.

The observations were spread over the period 4-26 May. It was sometimes possible to record four usable transits per night.

Echo II was also observed from 20 May onwards, but none of the results were worth retaining. Echo II has at the most only one usable transit.

Geodetic configuration

In the final computation all observations of absolute directions will provide observation relations, making it possible to fix the unknown points (together with a table of errors). However for the observations themselves it is necessary to define a general outline for the work to ensure from the outset a satisfactory overall configuration.

The following reasoning may be applied :

First point of view

The central meridian of the zone of observation is approximately $2 \text{ }^{\circ}E$ and the central parallel is 38 °N. The most im portant points for the link are the points of the satellite's path along the mean parallel, respectively to the east and west of the central meridian, the eastern point being vertically over the south of Italy, and the western point vertically over Spain. These points are determined by intersection from the three French base points, and these points themselves by means of intersection determine the points in Africa. If we look at the figure it will be realized that the latter intersection is satisfactory, but that the first is not excellent as the French points constitute rather a short base.

Second point of view

The set of directions defined by the stars determines a certain number of planes : these planes in association make it possible to define straight lines joining the points on the ground.

An examination of the figure shows that all the straight lines in the figure are well defined except that between Ouargla and Hammaguir, and this is because it was difficult to find points in Africa situated outside the shadow cone. Here again it will be realized that the African points are fixed by means of intersection from a French base that was a little too short.

Forecasting the satellite transit

In practice, the point of the satellite's path to be sighted on each transit is determined in advance. The coordinator of the work chooses this point on the basis of various criteria :

- in the first place, the need for a general geometrical figure of good shape, as mentioned above ;
- hours of observation suitable for photography of the stars. From this point of view, stations at high altitudes are very unfavoured in summer;
- -- also, the satellite must come out of the shadow cone sufficiently soon for the observers not to be caught off guard, for the time of the satellite's arrival cannot be forecast to within less than five minutes.

In theory, it would also be possible to take into account the successful observations of the previous nights, but in practice this is hardly possible; in fact, the general coordinator draws up a plan and follows it almost strictly.

For the forecasts we use the ephemeris of Echo I provided by the Smithsonian Astrophysical Observatory of Massachusetts.

As soon as an ephemeris is received, the transits of the satellite are num bered; the ephemeris is extrapolated for about a fortnight and the transit points are chosen, that is to say, the points in the sky towards which all the ballistic cameras are to be directed. An electronic programme then gives a list indicating all the observation data for each station.

In practice deviations of the order of 10 minutes in time and 2° -3° longitude are regularly noted between the extrapolated data of the ephemeris and the values of the next ephemeris. Such deviations although undesirable are within acceptable limits. They result from the fact that as Echo I is very light it is extremly sensitive to radiation pressure.

Field work

Only two operators are required for the photography, but in practice it is desirable to have a team of three. Throughout the effective period photographs are taken nightly every two hours, and in these circumstances it is desirable to allow each operator to rest for one night in three.

The photographic work includes the following steps :

- $-$ (Setting up the camera two hours before the observation);
- $-$ (Starting the chronometer);
- $-$ Loading the camera with a plate;
- Checking the chronometer against the time signals;
- Star photography (first exposure);
- Starting the disc shutter, adjustment by means of time signals;
- Photography of the satellite at the rate of 1 exposure per second, except at the 60th second of each minute, this exposure being eliminated by a disc manipulated by an operator;
- Stopping the disc : star photography (second exposure);
- $-$ Photography of the fiducial marks;
- Reloading of the camera;
- Setting up the camera for the next observation.

The work also comprises :

- development of the plate ;
- making one annotated paper print showing the direction of the satellite transit, the numbering of the minutes and the direction of the diurnal motion.

The most important factors in field work are :

- steadiness of the camera between the two exposures of the stars. At present this period is 15 minutes, and will be reduced to less than 10 minutes ;
- $-$ time recording on the rotating shutter, which must be correct to within $10⁻³$ s.

Successful night observations

Approxim ately 60 positions of the satellite were to be photographed, 60 exposures being made of each. The following table shows the num ber of successful photographs taken :

Naturally when any point is observed simultaneously from at least two stations the results can be processed. In theory, the probability of determining one position with five successful plates is

> 15 40 40 35 25 1 $60 \t 60 \t 60 \t 60 \t 60 \t 37$

In fact, there were only two entirely successful positions out of 60 plates.

$IV. - SCALING OF PLATES - COMPUTATIONS$

The field observations were worked out by the IGN using the following stages of computation :

- $A -$ Calculation of the theoretical positions of the stars on the plates;
- $B -$ Scaling of the stars and the satellite images with a comparator;
- C Smoothing of the positions of the satellite images on the plate;
- $D -$ Calculation, from the plate coordinates of the stars, of the direction cosines in the three-dimensional earth system. Application of the transformation to the plate coordinates of the satellite;
- E Study of the dispersion of intersections on Echo from the five stations, considered as determined by conventional geodesy;
- $F -$ Adjustment of the overall geodetic network from a fixed point considered as origin, and determination of the coordinates of the other four stations. Comparison with conventional triangulation.

These six points will be examined below :

In view of the small effective aperture of the lenses of the ballistic cameras (7.0 cm), and the disadvantages arising from unsatisfactory information on stellar positions, the IGN has so far preferred to confine itself to perfectly known stars, and to process only those appearing in F.K.4. During the observations a double exposure is made (one just before and one immediately after the satellite's transit).

The observer determines the azimuth and the zenith distance of the camera axis correct to 1 or $\frac{1}{2}$ centesimal grade, and determines the instant of exposure of the stars correct to within at least 0.05 s.

As the approximate position (zenith distance, azimuth of the axis and exact time of observation) is given for each photographic plate, it is possible by computation to select the stars appearing in $F.K.4$ that are within a radius of 6 cm from the centre of the plate and to compute the theoretical (x_c, y_c) position.

Refraction is taken into account in this computation.

B. *Com parator measurement*

The Institut Géographique National has two Zeiss-Jena comparators for the measurement on plates.

They are installed in an air-conditioned basement in which a constant temperature to within two degrees is maintained. The processing of a plate takes about one day. The measurements are made by two operators. The difference in the readings recorded is generally not significant for stellar images; for the satellite it is at most 5 microns. This difference seems to be due to the lack of sharpness of the satellite image at its edges in the direction of the path. A study is being carried out to detect systematic errors in measurements on the plates. No conclusions have yet been reached.

C. *Sm oothing of the images of the satellite*

It might seem attractive to consider all the images of the satellite and to introduce them simultaneously into the overall adjustment. In has not been possible to use this procedure so far, owing to the lack of sufficiently powerful computers. An attempt has therefore been made to select a small num ber of such observations that may be considered representative from the 60 images of the satellite measured on the plate. The satellite's coordinates are then " smoothed " for this purpose.

The satellite's path can be considered as representable in space for the duration of one minute by means of a third degree polynomial. There appears in perspective on the photographic plate a significant fourth degree term, due partly to the inclination of the path in relation to the plane of the plate, and partly to its curvature. It is therefore necessary to find an accurate interpolation formula.

Accordingly two polynomials are formulated :

 $X = A_1 + B_1 t + C_1 t^2 + D_1 t^3 + E_1 t^4$ $Y = A_2 + B_2 t + C_2 t^2 + D_2 t^3 + E_2 t^4$

whose coefficients are determined by the least square method. From the residuals it is possible to estimate the accuracy of the sightings on the satellite. As a general rule, the following mean square errors are found :

> \pm 3.3 μ in the direction of the path; ± 2 in the perpendicular direction.

Only the smoothed X and Y coordinates for a central value t_0 will be used in computing the directions of the satellite and their intersections.

D. Establishment of the formula for conversion of the plate coordi*nates to the terrestrial Cartesian system.*

The coordinates $(x_0 y_0)$ suitably corrected for distortion, and the (x_c, y_c) coordinates of the stars correspond to the same set of stellar directions intersected respectively by the theoretical plane corresponding to the approximate azimuthal and zenithal data.

There is a homographic relation between $(x_0 y_0)$ and $(x_c y_c)$

$$
x_c = \frac{a_1 + b_1 x_0 + c_1 y_0}{1 + b_1 x_0 + c_2 y_0}
$$

$$
y_c = \frac{a_2 + b_2 x_0 + c_2 y_0}{1 + b_1 x_0 + c_2 y_0}
$$

Hence the observation relations :

$$
a_1 + b_1 x_0 + c_1 y_0 - b x_0 x_c - c y_0 x_c - x_c = v
$$

$$
a_2 + b_2 x_0 + c_2 y_0 - b x_0 y_c - c y_0 y_c - y_c = v
$$

These contain eight parameters : it was considered necessary to use more parameters in the formulae for the transformation of the coordinates because it is probably better to apply the correct geometric transform ation with a small number of parameters than to use a quadratic transformation formula with many arbitrary parameters, since although the latter method may give a much smaller apparent mean square error it may also lead to miscalculation in the actual interpolation.

All these relations, adjusted by the least square method, provide the coefficients for the above homographic transformation which when applied to the smoothed coordinates of the images give the coordinates $x_{\mathbf{r}}$ and *yr* of the satellite.

By changing the known axes, we can obtain the direction cosines in the terrestrial Cartesian system from x_F and y_F .

The accuracy of the star sighting can be estimated from the residuals of these equations.

Generally speaking the mean square errors for a plate showing equatorial stars are :

 \pm 3 μ in the direction of diurnal motion;

 \pm 2 μ in the perpendicular direction.

On the whole the mean square error in the smoothed direction of the central flash results from three factors :

- $-$ lack of precision in pin pointing the images of the satellites;
	- $-$ inaccuracy of the formula for transformation of the coordinates (x_0, y_0) to (x_c, y_c) ;
	- error in the time of observation (of the order of $1/1000 s$).

These three errors are of the same order of magnitude $(1/200 000)$.

E. *Intersection in space*

On the basis of known geodetic coordinates, the accuracy of intersections at the satellite's position in space gives an idea of the accuracy of the method, insofar as it can be admitted that the consistency of the conventional geodetic coordinates is greater than that provided by satellite $connection - a$ fact which is not true.

Table 1 summarizes the results obtained.

It should be noted that no phase correction has yet been made to these measurements. In fact it is difficult to ascertain the exact behaviour of the satellite as a source of light. In particular it would be necessary to find out whether it acts as a perfect reflector or as a diffuser, or whether the light coming from it is both reflected and diffused, and if so in what proportion. It is certain that the application of a phase correction would further improve the results.

This table indicates for each station the statistical distribution of the metric deviations of sightings observed from the various stations. For example, it shows that at Lacanau Station (No. 3) 30 sightings out of the 50 studied passed at a distance of between 5 and 10 metres from the theoretical position of the satellite. These figures are obviously vitiated by systematic errors resulting from errors in the position of each of the triangulation stations.

To arrive at an idea of the true distribution of deviations in sightings the following value resulting from the work described in paragraph F (see Table 2) can be taken as a basis : the mean square error in an observed direction, after free adjustment ⁽¹⁾, is \pm 7.5 \times 10⁻⁶.

At the time of the observations the mean altitude of the satellite in the zone of work was 1 750 km. If it is admitted that the mean sighting distance was of the order of 2 100 km — the observed distance varies from 1800 to 2500 km $-$ this angular error corresponds to a standard deviation of ± 16 m, or a probable deviation of the order of $10 - 11$ m. This mean square error is the result of multiple errors of observation, measurements, and the error in synchronization, as mentioned above.

F. All the verifications under E having been made, general adjustment of the network was carried out, adopting a point $-$ in this specific case Agde — as the general origin of the network and adopting for the scale the distance Agde-Ouargla already known by triangulation.

Table 2 gives the results of the comparison and some general indications concerning the comparisons that can be made between the various data.

(1) That is to say, independently of errors of position of the known stations.

	1	OLETTA	$\bf{2}$ AGDE	3 LACANAU	4 OUARGLA	5 HAMMAGUIR	
		\overline{s}	312 3707.10 02.38	323 3657.20 58.19	219 6721.93 23.67	249 2169.11 68.86	Azimuths
OLETTA		$\frac{1}{s}$	107 9308.40 03.70	115 2334.08 34.29	17 0024.70 26.06	40 9096.04 95.71	Azimuths
$\mathbf{1}$		\pm	484 184.1 182.6	888 604.5 610.6	1232645.8 650.5	1699842.0 839.3	Distances
			$_{\rm s}^+$	331 5492.67 96.95	190 8647.20 50.20	227 6843.86 44.59	Azimuths
AGDE			$_{\rm s}^+$	127 9146.67 50.16	392 1680.24 82.82	23 2734.09 34.76	Azimuths
$\mathbf{2}$			$^{+}$	420 407. 7 417.4	1270247.4	1491208.9 204.7	Distances
				$\frac{1}{s}$	173 5505.65 05.65	207 2140.75 37.06	Azimuths
LACANAU				$\frac{+}{s}$	378 1501.61 01.89	59437.27 34.19	Azimuths
$\bf{3}$				\overline{s}	1555721.3 728.0	1571977.8 571977.9	Distances
					$\frac{1}{s}$	293 2618.82 21.82	Azimuths
OUARGLA					t^+	88 3611.65 14.96	Azimuths
4					\overline{s}	811 352.5 345.2	Distances

Table 2

+ value arrived at by conventional triangulation Mean square error

S value arrived at by space triangulation.

Azimuths : centesimal system MSE = $\pm 7.5 \times 10^{-6}$

Distances in metres.

Table 3 is devoted to the study of shutter synchronization. It will be remembered that the shutters are synchronized by time signals received directly by means of a flashing unit *on* the rotating shutter itself.

This table shows that it is reasonable to expect synchronization to be accurate to 1/1 000 s, which is approximately the object envisaged at the outset. This result agrees fairly well with the value for the deviation obtained when examining the scatter in the sightings. If we take into account the fact that, as has been seen, the mean square error of the sightings is of the order of ± 16 m, and since the error due to measurement on the plates, etc. is known to be of the order of \pm 2-3 microns (or about $1/150\,000$ — which roughly corresponds to about 12 metres — it can be seen that this agrees well with the preceding result since the mean square error of $\pm 7.5 \times 10^{-6}$ from the angular point of view can be said to be the resultant of the sighting error on the plate (i.e. approximately \pm 1/150 000 $\approx \pm 6.6 \times 10^{-6}$ plus the synchronization error. Since the

Table 2 (continued) *Trirectangular coordinates*

N.B. — The existence of these systematic negative discrepancies seems to prove that the choice of origin was bad as far as height was concerned. It will be noticed that a translation of $x + 10$ m, $y + 3$ m and $z + 5$ m

more satisfactory results.
The result seems in itself interesting enough from this viewpoint for this small
artifice to be dispensed with. It is moreover by no means out of the question that the results of the calculation of the geoid figure w ill account for part of the discrepancy. In the area of the point of origin (Agde) it has not yet been possible to make this calculation because the astronomical survey has not yet been carried out there.

satellite, which is 2 100 km distant, travels at approximately 7.5 per 10^{-3} s, and taking 1/1 000 s as the unit of time, this error would be :

$$
(7.5)^2 = (6.6)^2 + \epsilon^2 \left(\frac{7.5}{2.1}\right)^2
$$

hence $\varepsilon = \pm 10^{-3}$ s.

Conclusions

All these results are encouraging. They show that triangulation by means of luminous satellites of the Echo type, as studied by the IGN with light portable equipment, is a fairly delicate operation as far as organization is concerned but on the whole is simple, rational and accurate.

The accuracy of the France/Algeria operation can be said to be at least $1/100000 - 1/150000$.

These results were obtained with experimental equipment that was far from being perfect, in particular the prototypes of the ballistic cameras used were fitted with rather mediocre lenses.

Numerous conclusions were drawn from this experiment. In particular it was decided :

Date	Time	MSE	Date	Time	MSE	Date	Time	MSE
19/20/6	0h23	士 1.10	28/29/6	21h45 23h48	0.96 1.05	13/14/7	0h57	1.25
	2h20 4h22	0.92 1.48		3h53	0.90			
20/21/6	0 _{h57} 2h57	1.55 1.05	29/30/6	22h21 0h27 2h26	0.46 0.78 1.20	14/15/7	23h32 3h22	1.91 0.77
21/22/6	23h32 1h32	1.26 2.21 1.32	1/2/7	1h39 3h47 4h48	1.29 1.25 1.18	15/16/7	0h09 2h50	0.87 0.69
22/23/6	0 _h 0 ₇	0.88	2/3/7	2h15 4h19	1.09 0.71	16/17/7	2h17	0.91
23/24/6	22h45	1.03	3/4/7	22h52 0h51 2h54 3h46	0.48 0.82 0.79 1.66	17/18/7	21h21 1h18	0.91 0.95
24/25/6	1h22 3h27	1.03 1.09	4/5/7	23h28 3h33	1.05 1.42	21/22/7	23h47 3h15	1.30 1.29
25/26/6	0h02 4h03	0.89 0.32	8/9/7	23h53 3h56 4h43	0.62 0.78 0.72	22/23/7	22h24	0.48
26/27/6	22h34 0h36	1.33 0.32	9/10/7	22h33 0h28	0.93 0.67	23/24/7	23h00	1.05
27/28/6	1h13 3h17	0.58 0.51	12/13/7	22h21 2h37	0.90 0.67			

TABLE 3

Table 3 gives in $1/1000$ s the mean square synchronization error of the rotating shutter in relation to the time signals.

shutter in relation to the time signals.
The first column shows the date of reception, the second column gives the time,
and the third the mean square error in the operation of the shutter. This shutter in
the IGN procedur

The result is a time that is compared with the individual values obtained for each

transmitter.

Comparison shows the accidental error corresponding to each time signal received.
From this is deduced the mean square error of the time received.
The values reproduced in this table correspond to a station of the Continen

The resultant mean square error is :

 $\epsilon = \pm 0.99 \times 10^{-3}$ second.

- to increase the speed of action of the photographic emulsions;
- to check the photographic plates in order to select and sensitize those that are most plane and the most suitable optically;
- $-$ to fit the cameras with very good lenses. The new lenses give every satisfaction in this respect.

As far as observations are concerned, it was agreed that each station would in future be provided with two ballistic cameras on parallel axes,

GEODETIC LINK FRANCE - NORTH AFRICA Echo I May 1964

Points in the sky photographed, with indication of successful stations Successful photograph

- Photograph perhaps usable.
The station may be identified from the direction of the arrow
B: Besançon Observatory.
P: Paris Observatory.
	-

working simultaneously so as to provide reciprocal checking, and to take time signals from various transmitters so as to avoid propagation anomalies, etc.

Thanks to the implementation of these measures it can be stated that stellar triangulation is capable of a very high degree of accuracy : the order of 1/200 000 should easily be obtained. Even higher degrees of accuracy are by no means out of the question; the whole matter is a question of care and of tracing systematic errors. Examination of the results gives the clear impression that satellite triangulation is more accurate than conventional triangulation. The scale can only be fixed by very highly accurate conventional measurements over very long distances \rightharpoonup by traverses by geodimeter \rightharpoonup or by direct measurement to the satellite

using lasers, for example. In any case the tool which is simple in design and easy to operate seems to be one of the most attractive ever placed at the disposal of the geodesist. It should be used to the utmost of its potential, and it can be confidently asserted that it has a great future. World-wide triangulation, various projects of which are at an advanced stage of preparation, is certainly for the near future.