# CAMPAIGN FOR DETERMINING THE LONGITUDE OF THE FUNDAMENTAL POINT OF THE ASTRONOMIC OBSERVATORY IN NAPLES

## I. — THE CHAUVENET PROBLEM AND THE DETERMINATION OF LONGITUDE BY MERIDIAN OBSERVATIONS

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### INTRODUCTION

The Astronomic Observatory in Naples has scheduled a series of observations using the most modern methods and equipment for the purpose of revising the longitude value of its fundamental point.

The successive steps in this programme have been established as follows :

*a*) Preliminary study of methods suited to modern techniques;
*b*) Observations and first approximation reductions;
*c*) Final computation and results.

The author, on behalf of himself and his fellow workers, wishes to thank the Directing Committee of the International Hydrographic Bureau for having agreed to publish all the details of our research programme in the International Hydrographic Review.

### 1. — THE CHAUVENET PROBLEM AND THE PURPOSE OF OUR WORK

The Chauvenet Problem is the name generally given to all the practical operations for determining the inclination $i$ of a transit instrument's axis of rotation as a function of the observed zenithal distance, taking the irregularity and the inequalities of the pivots into account.

From the historical angle, we do not know whether it is allowable to attribute the problem defined above to W. CHAUVENET. Our research into the literature has not led us to establish this as a fact, but the work at present being undertaken is not of historical interest, and in accordance with the usual terminology we also shall term this problem the Chauvenet Problem.

Before proceeding further we think it useful to state the exact meaning of the following appellations. The term *irregularities of the pivots* is given to the discrepancies of the cross section of each pivot in relation to the value $r_m$ of its mean radius, whereas the term *inequality of the pivots* indicates the difference $(r_2 - r_1)$ between the two radii of the two opposite pivots of the same axis of rotation.

The Chauvenet Problem is of fundamental importance for meridian observations which necessitate very highly accurate results. However at the present time this problem is almost completely forgotten in actual practice and the reason is that in the observations of astronomical time carried out systematically at observatories for determining the rate of quartz clocks in the reductions we may quite well replace the axis of rotation's inclination $i$ by the inclination $y$ of the instrument's axis of support, in order to ensure the necessary degree of accuracy.

However when it is desired to carry out a high accuracy longitude determination, the determination of values of inclination $i$ for the axis of rotation in both positions of the instrument must be carried out with the greatest accuracy, rigorously applying the practical methods involved in the solution of the Chauvenet Problem. In fact, for the required degree of accuracy, any simplification in the measurement of inclination $i$ would falsify the final values of the instrument's $\Delta$Ts and observed azimuths and, consequently, the definitive longitude value.

Within the framework of preparing the astronomic observatory at Capodimonte-Naples for participation in this campaign for the revision of Italian longitudes the author carried out a series of measurements in order to determine provisional values (for it is not yet possible to know the temperature correction factor) for the graduation of the striding level of the 90 mm Bamberg instrument, fabrication number 11370, the instrument which will be used in this campaign.

Dr. A. PUGLIANO assisted the author in the Repsold-Heurtaux level-check measurements and undertook the reduction of the data.

In the first part of this work we set out the principles of the Chauvenet theory along the lines of the author's lectures to students at the university. In the second part we give the results of the level-check measurements as well as a study of the mistakes made when the practical conclusions arising from the Chauvenet Problem theory are neglected when making meridian observations of longitude.

The author also thanks the technicians Vincenzo CASELLA and Luigi IEVOLELLA for their work with the check-level.

The draughtsmanship was undertaken by the surveyor Pier Giacomo COLOGNA.

## 2. — DETERMINATION OF THE INSTRUMENT'S INCLINATION AND OF ITS AXIS OF SUPPORT

The transit instrument is made up as follows. It has a solid cast iron base from which two legs topped by two bearings in the form of a V rise at right angles to the stand. These Vs hold the pivots of the axis of rotation, and on these pivots rests the spirit level in its support. In the instruments manufactured by the Askania Company the legs of the level terminate in the form of a V. In figure 1 the support is shown simply resting on the pivots, but this does not modify our reasoning.

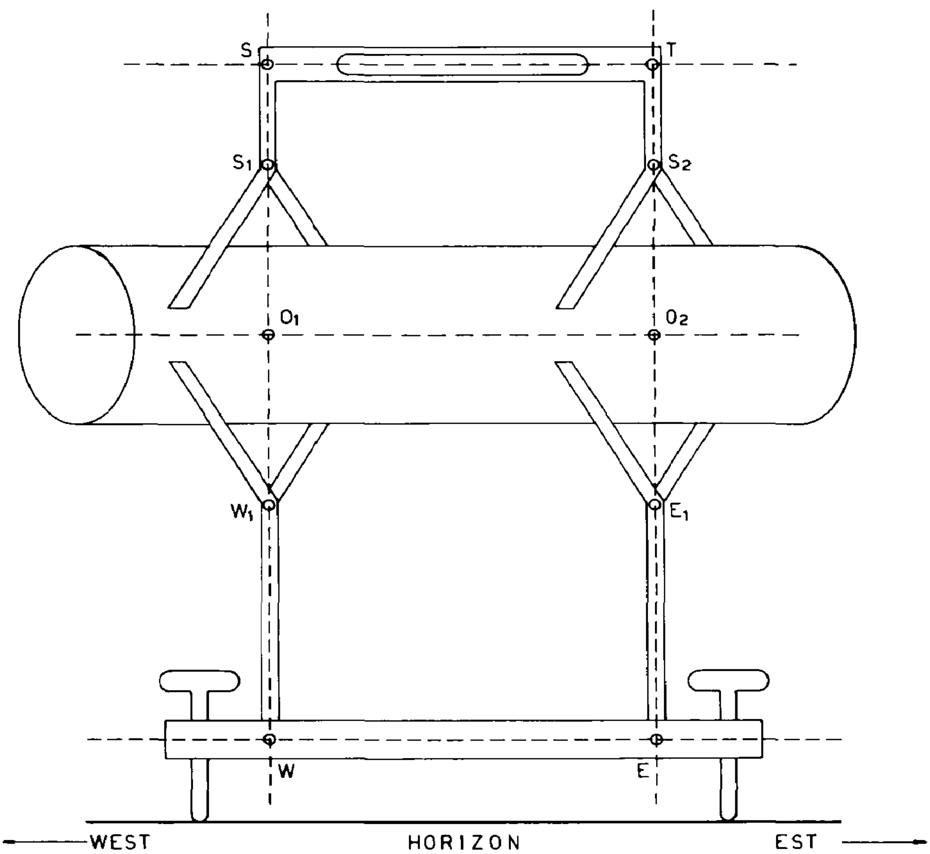


Fig. 1

What should be noticed in this figure is that there are *three axes* in the transit instrument, each independent of one another. These axes are :

*a*) the E W axis which we will call the instrument's *axis of support*;

*b*) axis $O_1O_2$ which is the instrument's *axis of rotation*;

*c*) the axis ST which we shall call the *axis of the level.*

It should be pointed out straight away that it is obviously easier from the technical point of view [1] to maintain the two segments $\overline{WS}$ and $\overline{ET}$ perfectly perpendicular in relation to the segment $\overline{WE}$ (and this is the reason why in the Askania instruments the two Vs of the support containing the level each have a small level) than to make the two segments $\overline{WS}$ and $\overline{ET}$ equal. In fact, in order to satisfy the condition $\overline{WS} = \overline{ET}$ the following three conditions must be fulfilled :

$$v = e$$
$$s = t$$
$$r_1 = r_2$$

The first of these conditions is inherent in the manufacture of highly accurate instruments; if the second condition is not fulfilled, this will not in any way affect the measurement, as we shall later see. As regards the third condition — i.e. to construct cylindrical axes with exactly the same radius — from a technical point of view this is impossible to realize.

We should note [2] that if the two pivots are placed 0.60 m apart (as was the case for the instruments used in our campaign) it will only need a difference of

$$r_2 - r_1 = 3\,\mu$$

to have an error of 1″ in the inclination measurement, i.e. that for spirit levels of the same sensitivity as those in our transit instruments the bubble moves by one division of the scale engraved on the phial.

Then, in order to be able to establish a rigorous theory for the measurement of the inclination, let us consider three different definitions for the inclination of a transit instrument. These are :

a) *the inclination $y$ of the instrument's axis of support* in relation to the horizon;

b) *the inclination $i$ of the axis of rotation* $O_1O_2$ in relation to the horizon;
(It is this value which must be known for the highly accurate observations such as those carried out for determining longitude);

c) *the inclination $x$ of the level's axis* ST in relation to the horizon.

It should be remarked that the first of these inclinations is always fixed, whereas the other two vary : the inclination $i$ according to the instrument's position only; the inclination $x$ in function of the relative position of the level's support in relation to the two positions of the instrument.

Let us call :

$2\,\alpha_S$, the angle of the Vs for $S_1$ of the spirit level;
$2\,\alpha_T$, the angle of the Vs for $T_1$ of the spirit level;
$2\,\alpha_E$ and $2\,\alpha_W$ respectively the angles of the Vs E and W of the EW supporting base;
$O_1$ and $O_2$, the centres of the two pivots of radius $r_1$ and $r_2$ respectively ($r_1 \neq r_2$).

Let us take the case where the inclination $x$ of the level's axis in relation to the horizon (figure 2) is higher than the inclination $y$ of the

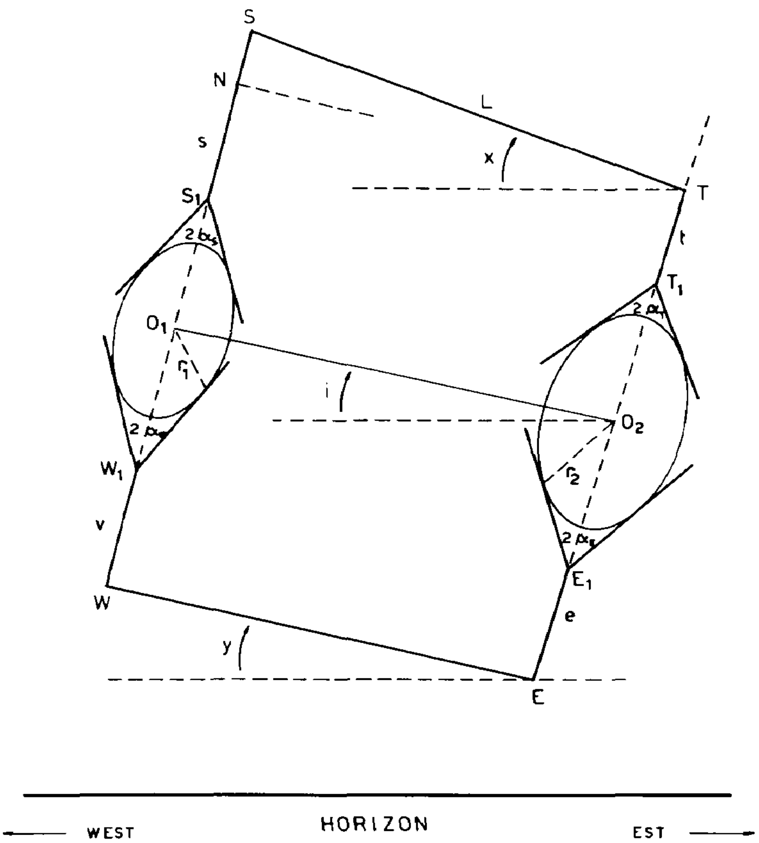


Fig. 2

WE arm of the axis of the instrument's support (always related to the horizontal plane), and that the zero point on the level's scale is to the East in the direction of T. We must take into account the fact that these particular conditions are not in the least restrictive. The demonstration which we shall make holds good for any $y$ and $x$ value, even if the zero of the level's phial were originally to the West. Obviously in drawing a perpendicular line from T to $\overline{WS}$ we obtain :

$$\widehat{STN} = x - y$$

At present, in order to clarify matters, let us consider in succession the effects of the $y$ and the $x$ inclinations. Firstly let us note that if the level's axis were perfectly parallel to the horizontal plane ($x = y = 0$) the ends of the bubble would be on the following scale marks :

$$\lambda - l, \quad \lambda + l$$

where $\lambda$ represents the scale reading of the centre of the bubble, the level being horizontal, and $2\,l$ being the length of the bubble.

If we then make the level deviate from this horizontal position by an angle $y$ by means of raising the W extremity of the instrument's WE axis of support (assuming that $x - y = 0$, i.e. that $\overline{WS}$ is equal to $\overline{ET}$) we shall obtain the following readings on the level scale at the ends of the bubble in its new position, the zero of the scale being to the East :

$$A_0 = \lambda - l + \frac{y}{\sigma};$$

$$B_0 = \lambda + l + \frac{y}{\sigma}$$

in which $\sigma$ is the *sensitivity*, or rate, of the level.

Finally, in this last position of the level if we move the extremity S by a displacement $(x - y)$ the conditions $\overline{WS} \neq \overline{ET}$ will be met, and the readings on the level scale at the ends of the bubble will be :

$$\left.\begin{aligned} A &= \lambda - l + \frac{y}{\sigma} + \frac{x-y}{\sigma} \\ B &= \lambda + l + \frac{y}{\sigma} + \frac{x-y}{\sigma} \end{aligned}\right\} \quad (1)$$

In the rectangular triangle $\widehat{STN}$, designating the length $\overline{ST}$ by L, we obtain :

$$\sin(x - y) = \frac{\overline{WS} - \overline{ET}}{L}$$

and in highly accurate instruments such as ours, by considering

$$x,\ y,\ \frac{\overline{WS} - \overline{ET}}{L}$$

as small quantities of the first order, and for this reason we may neglect the terms of the third order, and the foregoing relation may be written :

$$x = y + \frac{\overline{WS} - \overline{ET}}{L}$$

However :

$$\overline{WS} = \overline{WW_1} + \overline{W_1O_1} + \overline{O_1S_1} + \overline{S_1S} = v + s + r_1\left(\frac{1}{\sin\alpha_W} + \frac{1}{\sin\alpha_S}\right)$$

$$\overline{ET} = \overline{EE_1} + \overline{E_1O_2} + \overline{O_2T_1} + \overline{T_1T} = e + t + r_2\left(\frac{1}{\sin\alpha_E} + \frac{1}{\sin\alpha_T}\right)$$

and consequently :

$$x = y + \frac{(v-e) + (s-t)}{L} + \frac{1}{L}\left(\frac{r_1}{\sin\alpha_W} + \frac{r_1}{\sin\alpha_S} - \frac{r_2}{\sin\alpha_E} - \frac{r_2}{\sin\alpha_T}\right)$$

By substituting this relation in (1) we shall obtain :

$$\left.\begin{aligned} A &= \lambda - l + \frac{y}{\sigma} + \frac{(v-e)+(s-t)}{L\sigma} + \\ &+ \frac{1}{L\sigma}\cdot\left(\frac{r_1}{\sin\alpha_W} + \frac{r_1}{\sin\alpha_S} - \frac{r_2}{\sin\alpha_E} - \frac{r_2}{\sin\alpha_T}\right) \\ B &= \lambda + l + \frac{y}{\sigma} + \frac{(v-e)+(s-t)}{L\sigma} + \\ &+ \frac{1}{L\sigma}\cdot\left(\frac{r_1}{\sin\alpha_W} + \frac{r_1}{\sin\alpha_S} - \frac{r_2}{\sin\alpha_E} - \frac{r_2}{\sin\alpha_T}\right) \end{aligned}\right\} \quad (2)$$

Without touching the level on its supports, and *solely* by reversing the instrument's position (figure 3), the zero of the level scale this time will be to the West. The same reasoning as before is used, but this time the results will be :

$$\widehat{STN} = y - x' = \frac{\overline{ES} - \overline{WT}}{L} =$$

$$= \frac{(e-v)+(s-t)}{L} + \frac{1}{L}\left(\frac{r_1}{\sin\alpha_E} + \frac{r_1}{\sin\alpha_S} - \frac{r_2}{\sin\alpha_W} - \frac{r_2}{\sin\alpha_T}\right)$$

so that finally we shall obtain :

$$\left.\begin{aligned} A_1 &= \lambda - l - \frac{y}{\sigma} + \frac{y-x'}{\sigma} = \lambda - l - \frac{y}{\sigma} + \frac{(e-v)+(s-t)}{L\sigma} + \\ &+ \frac{1}{L\sigma}\left(\frac{r_1}{\sin\alpha_E} + \frac{r_1}{\sin\alpha_S} - \frac{r_2}{\sin\alpha_W} - \frac{r_2}{\sin\alpha_T}\right) \\ B_1 &= \lambda + l - \frac{y}{\sigma} + \frac{y-x'}{\sigma} = \lambda + l - \frac{y}{\sigma} + \frac{(e-v)+(s-t)}{L\sigma} + \\ &+ \frac{1}{L\sigma}\left(\frac{r_1}{\sin\alpha_E} + \frac{r_1}{\sin\alpha_S} - \frac{r_2}{\sin\alpha_W} - \frac{r_2}{\sin\alpha_T}\right) \end{aligned}\right\} \quad (3)$$

By adding up the four previous readings member by member, taking the conventional sign for the ends of a transit instrument bubble into account, we obtain :

$$\frac{1}{4}(A+B-A_1-B_1) = \frac{y}{\sigma} + \frac{v-e}{L\sigma} + \frac{r_1+r_2}{2\,L\sigma}\left(\frac{1}{\sin\alpha_W} - \frac{1}{\sin\alpha_E}\right) \quad (4)$$

Let us now speak of techniques. The instrument's base, i.e. $W_1W\,E\,E_1$, is constructed in a single block, and specialized firms, using special processes, are able to reduce the difference $(v - e)$ to the minimum, so that even for the degree of accuracy we require we may assume :

$$\overline{W\,W_1} = \overline{E\,E_1}$$

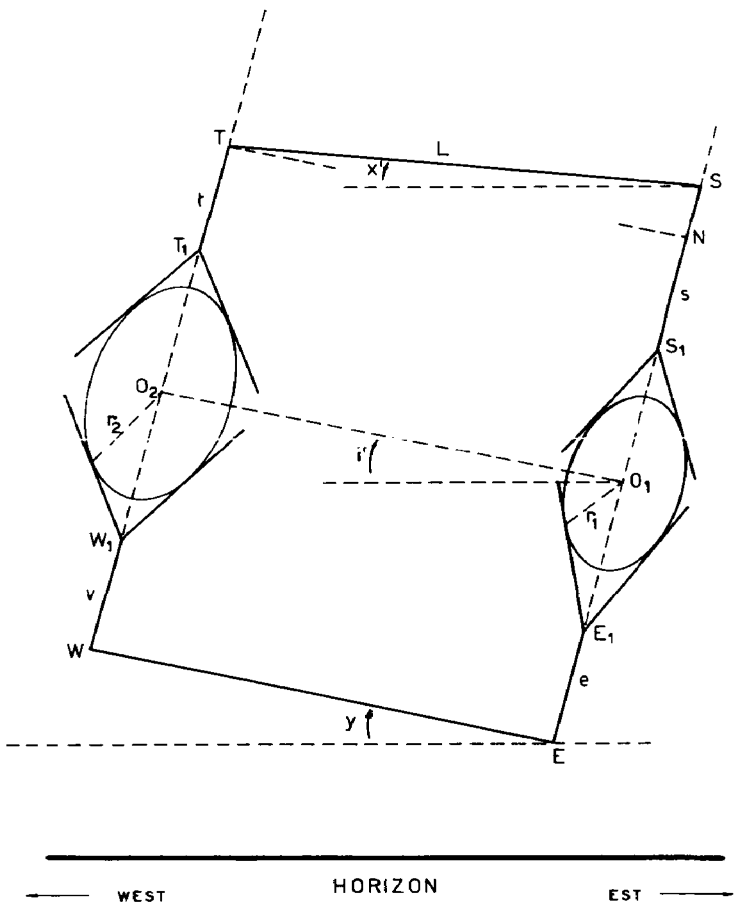


FIG. 3

However it should be remarked that the same is not true for segments $\overline{TT_1}$ and $\overline{SS_1}$; in this case indeed the arms of the support containing the level are screwed to the support (figure 4a and b) after having been housed in. Furthermore, in the support the level fixed at one end is adjusted to the desired position with two pairs of screws (so that this end may act at this point as a pivot), two of the screws being fixed vertically and two horizontally (figure 4a). Luckily, as we have seen, the fact that $s$ differs from $t$ has no influence on the $y$ inclination measurement provided that these two lengths are constant. Obviously this articulated system will be subject to the thermic variations to which it is exposed. This is one of the reasons why the striding level must *never* be continuously illuminated. The other, and still more important, reason is that uninterrupted illumination causes a thermic agitation of the liquid contained in the phial which affects the whole length of the bubble, and in certain cases causes serious errors in the readings. Finally, we should not overlook the fact that the value of the sensitivity $\sigma$ of the level is a function of temperature $t$.

Another of the manufacturers' concerns is to make the angles we have designated by $2\,\alpha_S$, $2\,\alpha_T$, $2\,\alpha_W$, $2\,\alpha_E$ all equal insofar as possible. Particular

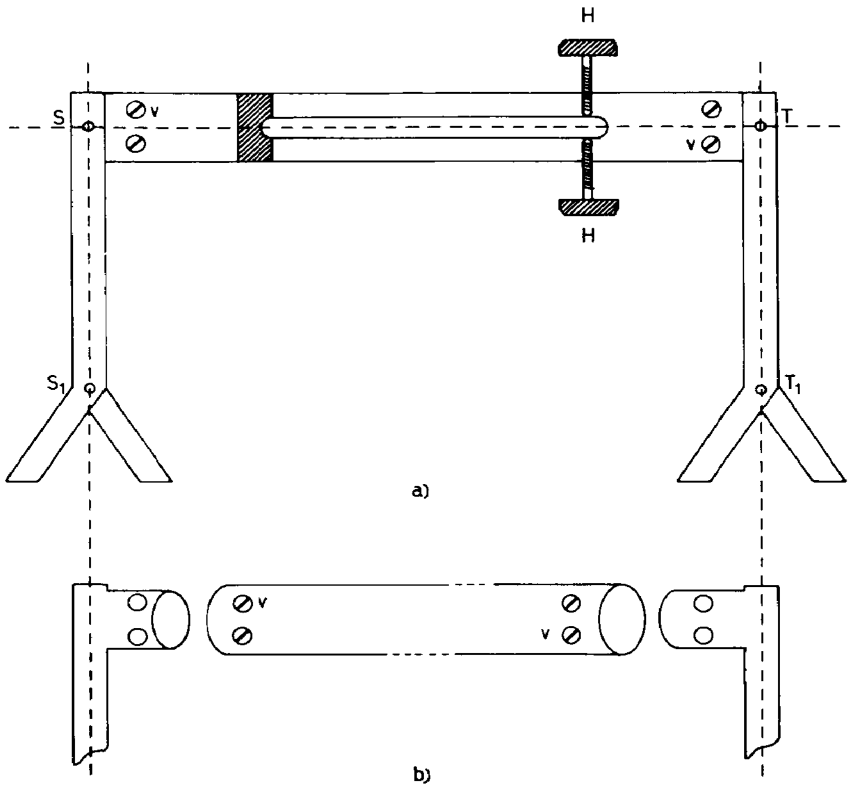


FIG. 4

attention has been paid to the very important point of the construction of the 2 $\alpha_W$ and the 2 $\alpha_E$ angles. In fact, if we wish to determine the shape of the pivots (i.e. their irregularities) slightly different shapes will be found, according to the different angles of the Vs on which the axis of rotation rests; that is to say there is a set pivot irregularity for each pair of values (2 $\alpha_W$, 2 $\alpha_E$), (2$\alpha'_W$, 2 $\alpha'_E$).

This fact, explicable in the light of the mechanics of points of support, was shown up for the first time by BOERDIJK and DE MUNCK who employed very modern instruments [3] for the measurements, for instance the electronic hammer (Sensor G.M. 5537, bridge G.M. 5536 of Philips make). For this reason the makers of instruments supplying data about the irregularities of pivots deduce these data from measurements made on these same Vs (2 $\alpha_E$ and 2 $\alpha_W$) in the manufactured instrument.

Consequently, returning to formula (4) for the following conditions :

$$a) \quad \overline{WW'} = v = \overline{EE'} = e$$
$$b) \quad 2\,\alpha_W = 2\,\alpha_E$$

we obtain :

$$y = \frac{\sigma}{4}(A+B-A_1-B_1) \qquad (5)$$

It follows that *in order to determine the inclination y of the instrument's axis of support, it will only be necessary to take level readings at the* *two opposite positions of the instrument without, however, touching the level, leaving this simply resting on the axis.*

However, for highly accurate determinations we need to know the value of the inclination of the instrument's axis of rotation $O_1O_2$ in relation to the horizontal. As a result of the inequality of the pivots, this inclination $i$ is different from the inclination $y$ of the axis of support, and it will have two values $i$ and $i'$ according to the two diametrically opposite positions of the instrument.

It should be noted that this inclination $i$ is a function of the zenithal distance of the observed star, so that it varies according to the star, and this is why the positions of the bubble ends must be read both before and after each observation in a way different from that indicated in formula (5).

Given that inclination $y$ is fixed — at least during a certain period of time — many observers, believing that this inclination indicates the inclination $i$ of the axis of rotation, have thought it useful, even when carrying out highly accurate observations, to fix the instrument's level by means of a sprung knob; and next to read the ends of the bubble occasionally (and not for each star observed), then to compute the mean of the values obtained for the inclination $y$ and to use this mean value in the reduction. In this way it can be assumed that the discrepancies between the observed values of star transits which are obtained later during the observational period arise either from variations of the instrument's azimuth or from the variation of the residual collimation in relation to the zenithal distance, or else from the pulse error of the impersonal micrometer. All this, together with other factors, is in fact reflected in the discrepancies among the individual star transit observations. However we must bear in mind that the equation allowing us to determine a longitude has only two unknowns ($\Delta T$ of the quartz and Az of the instrument), provided that it is possible to determine inclination $i$ by another means, e.g. with a striding level. If any error is introduced into this last determination the equation's known term will be wholly affected and the values of the two unknowns, $\Delta T$ and Az will obviously be suspect.

However, by blocking the level of the instrument we try to avoid movements of the bubble which are caused when the level's axis and the axis of rotation are not in the same plane, these two axes therefore forming an angle.

In this case if the support of the level revolves around the instrument's axis of rotation the bubble will undergo movements — which have nothing to do with the inclination $i$ — that are in reality due to the fact that the axis ST is deflected in relation to the axis of rotation $O_1O_2$. In order to correct this defect each level has four screws set two by two at right angles, so that by gently swinging the level's support that is fixed to the axis of rotation the position may be adjusted by means of the horizontal screws until the bubble remains motionless for any position in the swing movement. The manual operation that this adjustment entails is so simple that it is usually made by the staff of the astronomic observatory. The reader may refer to a work written by G. SILVA [4] where this subject is dealt with in more detail.

Furthermore, in order to minimize this effect (for it is difficult to eliminate it entirely) it is not absolutely necessary to block the level in order to avoid errors between the two readings of the bubble ends. In fact, in transit instruments the two Vs supporting the level on the pivots are themselves equipped with two small levels. It therefore suffices to take the precaution to read the position at the ends of the bubble of the level when *the bubbles of these two small levels are centered in order to find* conditions that are identical to those of the blocked level.

This being granted, let us now try to determine the value of the inclination of the axis of rotation which, as has been said, is in fact the value used for the reduction of meridian observations for determining longitude at a given position.

## 3. — DETERMINATION OF THE INCLINATION OF THE INSTRUMENT'S AXIS OF ROTATION

From figures 5a and b the following relation between the inclination $i$ of the axis of rotation $O_1O_2$ and the inclination $y$ of the instrument's axis of support is deduced :

$$i = y + \frac{1}{\overline{O_1O_2}}\left(\frac{r_2}{\sin \alpha_E} + e - \frac{r_1}{\sin \alpha_W} - v\right) = y + \frac{1}{\overline{O_1O_2} \sin \alpha}(r_2 - r_1) \qquad (6)$$

whereas by reversing the axis of rotation on the Vs, $W_1$ and $E_1$, of the instrument (figure 5c and d) we obtain :

$$i' = y - \frac{1}{\overline{O_1O_2}}\left(\frac{r_2}{\sin \alpha_W} + v - \frac{r_1}{\sin \alpha_E} - e\right) = y - \frac{1}{\overline{O_1O_2} \sin \alpha}(r_2 - r_1) \qquad (6')$$

In order to simplify to the maximum, let us generalize the conditions necessary for equation (5) by putting :

$$2\,\alpha_S = 2\,\alpha_T = 2\,\alpha_E = 2\,\alpha_W = 2\alpha$$

$$\frac{1}{\overline{ST}} = \frac{1}{L} = \frac{1}{\overline{O_1O_2}}$$

The first condition is technically possible, being inherent in the manufacture of meridian telescopes. As to the second condition, it should be pointed out that if, *by the reductio ad absurdum reasoning we find that in a well adjusted precision instrument* $\overline{SO_1}$ is longer than $\overline{TO_1}$ by at least 3 millimetres (remembering that ST is the axis of the level and not that of the support) then the difference between the length of $\overline{O_1O_2}$ (= 0.60 m) and the length of $\overline{ST}$ will be about 10 $\mu$, so that the numerical value of $1/\overline{O_1O_2}$ will differ from the value of $1/L$ from the fifth decimal figure onwards.

Consequently, even for the high degree of accuracy that we have set ourselves we may, without entailing any error, write (6) and (6′) as follows :

$$i = y + \frac{r_2 - r_1}{\mathrm{L}\ \sin\ \alpha}$$
$$i' = y - \frac{r_2 - r_1}{\mathrm{L}\ \sin\ \alpha} \qquad (7)$$

Next, given that we are able to determine the inclination $y$ of the instrument's axis of support, we must find, by means of readings taken at the ends of the striding level bubble, the right method to determine the quantities :

$$\Delta y = \pm \frac{r_2 - r_1}{\mathrm{L}\ \sin\ \alpha} \qquad (8)$$

If we succeed in determining these values of $\Delta y$ we shall also be able during the observations to take the effect produced by the pivot inequalities

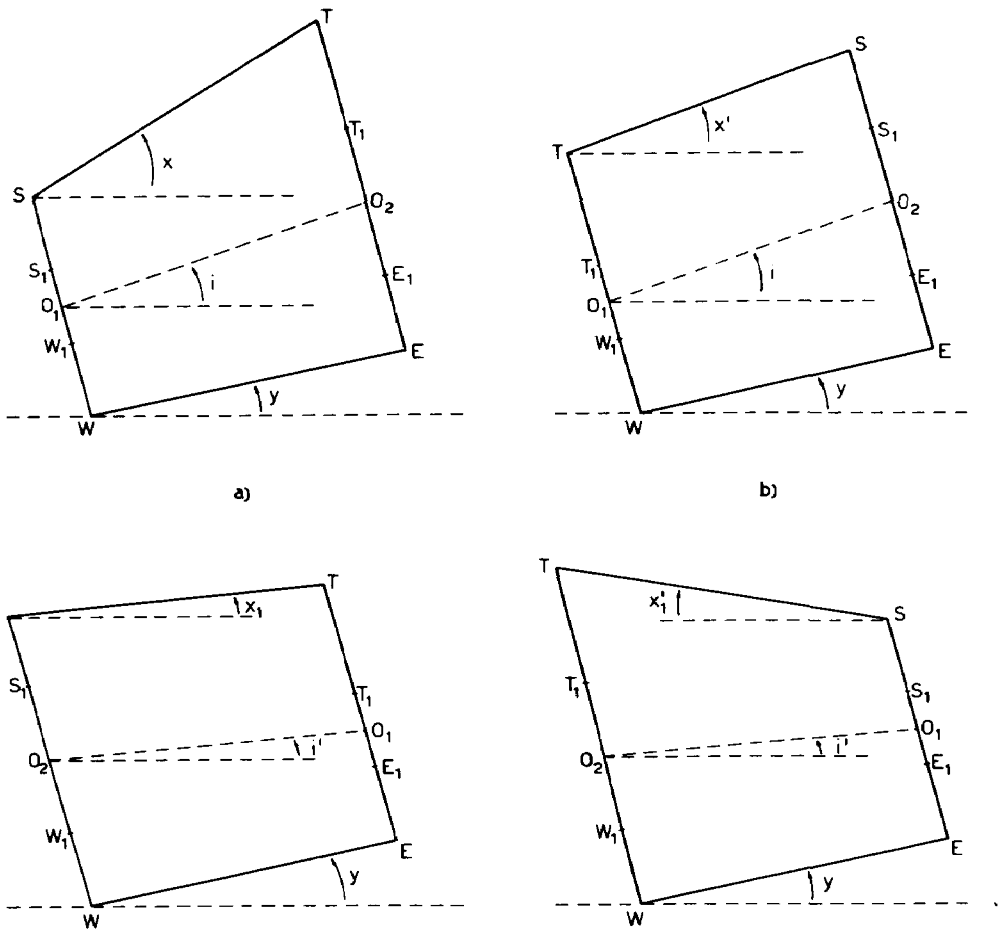


FIG. 5

into account at the moment of observation. (Irregularities are also included in this effect).

We should note that the term " at the moment of observation " has been employed because if we take what has just been said into account — namely that a difference of 3 $\mu$ between the two pivots, which in our Askania instruments are placed 0.60 m apart, produces a variation of one second of arc in the inclination (for levels like ours having a sensitivity of $\sigma = 1''$ the centre of the bubble will move by one division on the scale) then a coating of vaseline applied to the pivots, spreading in different ways for the various positions of the telescope, will be enough to produce a significant error in our $i$ measurements. Then if we realise that atmospheric dust has approximately the same thickness as three microns, we can see that the irregularities and inequalities of the pivots are factors which vary slightly from one moment to another, and quite significantly for the variation in the zenithal distance of the observed star.

Let us now see what method we may use for determining the value of (8).

In order to make it clearer that the demonstration up to the present point is entirely general, let us this time consider the case where the eastern extremity of the instrument's axis of support is higher in relation to the horizontal plane, and the zero mark on the level's phial is towards the south.

Following the same reasoning as above, we shall obtain (figure 5a) :

$$x = y + \frac{t - s}{L} + \frac{2}{L \sin \alpha} (r_2 - r_1)$$

and the readings that will be taken at the ends of the bubble on the phial scale will be :

$$\left.\begin{aligned} A &= \lambda - l + \frac{y}{\sigma} + \frac{t - s}{L\sigma} + \frac{2}{L\sigma \sin \alpha} (r_2 - r_1) \\ B &= \lambda + l + \frac{y}{\sigma} + \frac{t - s}{L\sigma} + \frac{2}{L\sigma \sin \alpha} (r_2 - r_1) \end{aligned}\right\} \qquad (9)$$

Then, keeping the position of the instrument stable (figure 5b), if we rotate solely the level by 180°, so that the V, $T_1$, rests on the pivot $O_1$ and that of $S_1$ on pivot $O_2$ for the new inclination $x'$ of the level's axis in relation to the horizontal plane we shall obtain :

$$x' = y - \frac{t - s}{L} + \frac{2}{L \sin \alpha} (r_2 - r_1)$$

and the readings of the extremities of the bubble on the level's scale for this new position of ST will be :

$$\left.\begin{aligned} A' &= \lambda - l - \frac{y}{\sigma} + \frac{t - s}{L\sigma} - \frac{2}{L\sigma \sin \alpha} (r_2 - r_1) \\ B' &= \lambda + l - \frac{y}{\sigma} + \frac{t - s}{L\sigma} - \frac{2}{L\sigma \sin \alpha} (r_2 - r_1) \end{aligned}\right\} \qquad (10)$$

From (9) and (10) we obtain :

$$M = \frac{1}{4}(A+B-A'-B') = \frac{y}{\sigma} + \frac{2}{L\sigma \sin \alpha}(r_2-r_1) \tag{11}$$

Let us now turn the instrument so that without touching the level pivot $O_2$ which was to the East is now to the West, and that pivot $O_1$ which was to the West is now to the East (figure 5c), then we obtain :

$$y - x_1 = \frac{t-s}{L} + \frac{2}{L \sin \alpha}(r_2-r_1)$$

and consequently for the new readings $A_1$ $B_1$ of the ends of the bubble on the phial scale :

$$\left.\begin{aligned} A_1 &= \lambda - l + \frac{y}{\sigma} - \frac{y-x_1}{\sigma} = \\ &= \lambda - l + \frac{y}{\sigma} + \frac{t-s}{L\sigma} - \frac{2}{L\sigma \sin \alpha}(r_2-r_1) \\ B_1 &= \lambda + l + \frac{y}{\sigma} - \frac{y-x_1}{\sigma} = \\ &= \lambda + l + \frac{y}{\sigma} + \frac{t-s}{L\sigma} - \frac{2}{L\sigma \sin \alpha}(r_2-r_1) \end{aligned}\right\} \tag{12}$$

Without touching the instrument (i.e. without re-turning the pivots) we rotate solely the level on its supports by 180° (figure 5d). In this new position of the entire instrument we shall have :

$$x'_1 - y = \frac{t-s}{L} + \frac{2}{L \sin \alpha}(r_2-r_1)$$

$$\left.\begin{aligned} A'_1 &= \lambda - l - \frac{y}{\sigma} + \frac{x'_1-y}{\sigma} = \\ &= \lambda - l - \frac{y}{\sigma} + \frac{t-s}{L\sigma} + 2\frac{r_2-r_1}{L\sigma \sin \alpha} \\ B'_1 &= \lambda + l - \frac{y}{\sigma} + \frac{x'_1-y}{\sigma} = \\ &= \lambda + l - \frac{y}{\sigma} + \frac{t-s}{L\sigma} + 2\frac{r_2-r_1}{L\sigma \sin \alpha} \end{aligned}\right\} \tag{13}$$

whence :

$$M_1 = \frac{1}{4}(A_1 + B_1 - A'_1 - B'_1) = \frac{y}{\sigma} - \frac{2}{L\sigma \sin \alpha}(r_2-r_1) \tag{14}$$

Subtracting relations (11) and (14) member by member we obtain :

$$\frac{1}{4}(M - M_1) = \frac{r_2-r_1}{L\sigma \sin \alpha} \tag{15}$$

Taking relations (11), (14) and (15) into account, the relations (7) can be written :

$$\left.\begin{aligned} \frac{i}{\sigma} &= \mathrm{M} - \frac{1}{4}(\mathrm{M} - \mathrm{M}_1) \\ \frac{i'}{\sigma} &= \mathrm{M}_1 + \frac{1}{4}(\mathrm{M} - \mathrm{M}_1) \end{aligned}\right\} \qquad (16)$$

which supplies us with the values of the inclination of the axis of rotation in the two positions of the instrument in terms of the readings taken at the ends of the bubble.

To recapitulate : for determining longitude with high accuracy the inclination of the axis of rotation for the instrument's two positions must be determined by the following method, and *the operation must be carried out for each star observed* :

*a*) The telescope is set at the desired zenithal distance, the instrument is blocked, and in order to give the level time to stabilize itself the opposite sight is prepared for the second part of the observation when the axis of rotation will be reversed;

*b*) Shortly before starting the ocular micrometer observations, readings A and B at the ends of the bubble are taken and as soon as these are finished the level is reversed on its supports;

*c*) We proceed to make observations with the impersonal micrometer for the first part of the field;

*d*) Once the first part of the observation by impersonal micrometer is finished, readings A'B' of the bubble ends are made without unblocking the instrument;

*e*) The instrument's position is then reversed, and we unblock the instrument and proceed to the adjustment of the opposite sight;

*f*) The instrument being blocked at the desired position, we check that the star is in the instrument's ocular field;

*g*) We proceed to the readings $A_1$ and $B_1$ of the ends of the bubble, and immediately afterwards the level on its supports is reversed;

*h*) The impersonal micrometer observation ends with observations in the second part of the field;

*i*) Without unblocking the instrument, the ends $A'_1$ $B'_1$ of the striding level bubble are read.

Obviously if we see after operation *f*) that the star is very close to the movable thread of the impersonal micrometer, we may proceed immediately to operation *h*), carrying out operations *g*) and *i*) after the observation, *without however unblocking the instrument or changing the zenithal distance at which the telescope is set.*

In general, we should give careful attention so as :

*a*) To avoid reading the ends of the bubble immediately after any movement of the instrument. When turning the instrument the recommendations of ALBRECHT [5] should be followed. Always wait for the spirit in the level to settle, and to shorten this time the instrument must be swung very delicately, and the phial must only be lighted during the time required for the readings;

*b*) To read the ends of the striding level bubble when the small level's bubbles on the Vs of the support are centered, and this is done for the reasons mentioned.

Finally, it should be pointed out that :

$$\frac{y}{\sigma} = \frac{1}{2}(M + M_1) \tag{17}$$

that is the value of the inclination $y$ of the instrument's axis of support will be determined (if desired, although this is not required for our observations) by means of eight readings of the positions of the bubble ends on the striding level scale. Thus, in the light of the theory of probability, the value of $y$ given by the foregoing relation is far more accurate than the value provided by relation (5) by means of four readings.

Now that we have seen, in the light of the Chauvenet Problem, the appropriate way of determining the values of $i$ and $i'$ in practice, let us see in what way these values are used in the reduction computations.

## 4. — CORRECTION OF THE ERROR IN THE INCLINATION OF THE INSTRUMENT'S AXIS OF ROTATION

When determining $TU_0$, i.e. the universal time deduced from the meridian observation using a transit instrument, the inclination $i$ of the axis of rotation in relation to the horizontal plane is considered as positive if the western end is the higher.

This convention has been established in order to take into account the correction to be made to the observed time of the star's transit — a correction for the effect of inclination $i$ of the instrument's axis of support in relation to the horizontal. In fact (figure 6c), if $O_1O_2$ is inclined by an angle $i$ in relation to the horizontal plane, with the extremity $O_1$ higher and to the West, star St will be observed before its transit at the meridian, OA representing the optical axis of the telescope, i.e. at the observed instant $T_k$ ($k = 1, 2, \ldots n$) there will be an error $\varepsilon_i$ due to the inclination $i$, with a sign negative in relation to the instant $T_m$ of the actual transit of the star St at the meridian. It must be remembered that the correction always has the opposite sign to the error made. Thus to correct the observed instant $T_k$ to the actual instant of the transit of St at the meridian we must add the correcting term $C_i$ (expressed in seconds of time) whose absolute value is equal to $\varepsilon_i$ but with the opposite sign.

Using the same reasoning for the case where $O_2$ is higher than $O_1$ in relation to the horizontal plane, with $O_2$ situated to the East (figure 6) we see that the correction $C_i$ to be made has this time a negative sign since the observed transit $T_k$ of St follows the instant of the transit $T_m$ at the meridian. $T_m$ must then be affected by an error $\varepsilon_i$ of positive sign.

We may then conclude that the inclination $i$ takes the sign

*a*) positive when the western extremity of the axis of rotation is

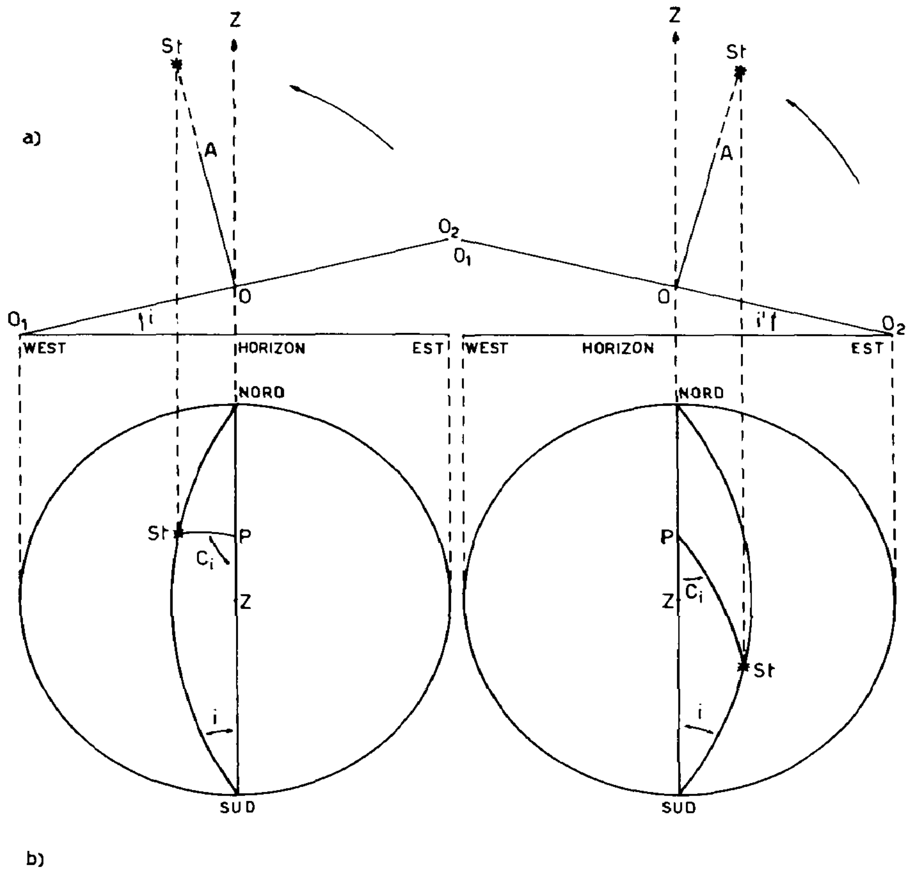


FIG. 6

higher than the eastern one in relation to the horizontal plane;

*b*) negative when the eastern extremity is higher than the western extremity.

For the rapid determination of the sign of inclination *i* during the observation, without being obliged to make a reasoning, the zero of the striding level scale in transit instruments is placed at one end of the scale. In this way when taking readings at the ends of the bubble it is only necessary to take with the positive sign those made when the zero of the scale is to the East, and with the negative sign those when the zero is to the West, to obtain by means of the general formula :

$$M = \frac{\sigma}{4}(A + B + A' + B')$$

the value of inclination *i* in seconds of time, with the sign to be given to the correction $C_i$.

In the instruments manufactured by Askania in order to make it easier to determine the sign the following indications are printed on the stand :

| + EAST | — WEST |
|---|---|

In this way, once the stand has been oriented, the observer, already sufficiently occupied, may immediately deduce the sign to be given to the various readings

$$(A, B); \quad (A', B'); \quad (A_1, B_1); \quad (A'_1, B'_1)$$

of the ends of the striding level bubble.

This being granted, we shall now determine the explicit form of the function

$$C_i = f(i)$$

which will allow us to obtain the numerical value with its sign for the correction to be made to the observed times $T_k$ in order to reduce them to times $T_m$ of the actual meridian transit of the various stars with declinations $\delta_k$ $(k = 1, 2, ..., n)$.

Let us consider figures 6b and d which represent the projected celestial sphere from the observer's zenith Z. P will be the North Pole position, St the position of the observed star, and the trace of the meridian of the place of observation will be N-S on the figure plane.

In order to make the figure clearer we have exaggerated the value of inclination $i$ in the design and we must therefore remember that the quantities $i$ and $C_i$ are so small that we may quite well take the value of the sine for the arc.

Applying the sine theorem to the spherical triangle P-St-South we have :

$$C_i = i \frac{\sin \widehat{\text{St-South}}}{\cos \delta}$$

where $\delta$ is the declination of the observed star.

But given the small value for $i$ (and thus for $C_i$) we may in any case consider the arc $\widehat{\text{St-South}}$ equal to the altitude $h$ of the star above the horizon of the place of observation, i.e.

$$\widehat{\text{St-South}} = h = 90° - z = 90° - (\varphi - \delta)$$

where the zenithal distance of the observed star is indicated by $z$ and the latitude of the place of observation by $\varphi$.

However, the preceding formula may be written :

$$C_i = i \frac{\cos (\varphi - \delta)}{\cos \delta} = i\,I \tag{18}$$

$i$ being expressed in seconds of time.

The quantity :

$$I = f(\varphi, \delta)$$

can therefore be easily tabulated.

Let us now see how in practice we may proceed in the reduction computation in order to make the $C_i$ and $C_i'$ corrections.

The observation is made with the help of an impersonal micrometer which follows the star with the movable thread FF, for example from position $F_1F_1$ (figure 7) to the position $F_2F_2$. Then the axis of rotation is reversed on its Vs (and consequently the movement of the star in the ocular field will be in the opposite sense). We wait until the star arrives at the point where the movable thread was left, i.e. at $F_2F_2$, and we continue with the thread moving up from the lower dotted line position $F_2F_2$ to the higher position $F_1F_1$. Thus in the two positions of the instrument the star's transit is observed at identical threads (in the impersonal micrometer these are replaced by electric pulses) once *before* the transit at the thread *cc* without collimation and a second time *after* the transit at this thread. Let us therefore assume that $n$ pulses were given before the star's transit at the thread *cc* and $n$ pulses afterwards. Calling the observed instants of transit in the first series $T_k$ ($k = 1, 2, \ldots, n$), and $T_k'$ ($k = 1, 2, \ldots, n$) the observed instants in the second series at the same threads, after reversing the axis of rotation on the Vs, for the instant of meridian transit we shall obtain :

$$\left.\begin{array}{l} T_k + j \sec \delta + C_i \\ T_k' - j \sec \delta + C_i' \end{array}\right\} \quad (k = 1, 2, 3, \ldots, n) \tag{19}$$

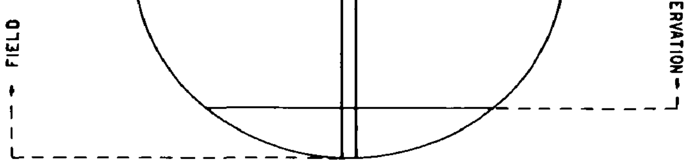


Fig. 7

where $j \sec \delta$ is the correction to be brought to instant $T_k$ observed at thread $k$ at a distance $j$ from the thread without collimation ($j$ is expressed in seconds of time) in order to reduce it to the instant of the star's transit at thread *cc*.

Adding together the $2n$ values given by the relations (19) for the instant $T_m$ of the meridian transit, we have :

$$T_m = \frac{\sum_{k=1}^{n} (T_k + T_k')}{2n} + \frac{C_i + C_i'}{2} \tag{20}$$

Thus in order to obtain the instant of the star's transit at the instrument's meridian it is necessary to average the observations, and in order to correct this mean for the errors $\varepsilon_i$ and $\varepsilon_i'$ due to the $i$ and $i'$ inclinations of the axis of rotation in the instrument's two positions it will only be necessary to sum the term :

$$C_m = \frac{C_i + C_i'}{2} \tag{21}$$

In this way the deduced instant $T_m$ is corrected either for the collimation error (eliminated by the rotation of the instrument during the observation) or the error due to the inclination of the axis of rotation and consequently for the error arising from the inequality and the irregularity of the pivots.

We should warn the reader not to make the mistake of substituting in (21) the $i$ and $i'$ inclination values deduced from (7) and then adding them algebraically. It must be considered that the signs for $C_i$ and $C_i'$ are conventional — in other words in a well-levelled instrument such combinations as

$$\frac{1}{2}(C_i - C_i'),\ \frac{1}{2}(-C_i + C_i'),\ \text{etc.}$$

may be obtained.

As far as the other corrections given by (20) to be made to $T_m$ are concerned, these will be considered in the following section which deals with the mechanical and electronic constants for an impersonal micrometer.

Before starting the observation the transit instrument must be set up in the best way possible in the meridian. Regarding the inclination $y$ of the instrument's axis of support which may easily be corrected, this inclination must be very near zero. This is why each evening before starting the observation the cupola in which the instrument is placed is opened (so that the temperature of the observation cabin may have the time to equal the outdoor temperature) and the inclination $y$ is measured by means of (17) and is corrected with the setting screws.

In short, observations of longitudes must be carried out quite as carefully as the latitude observations in the international stations, and all the same precautions as for these observations [6] must be adopted.

We have said all along that $C_i$ and $C_i'$ must be expressed in seconds of time, and this can be done if the value $\sigma$ of the level's sensitivity is known. Therefore it only remains to see how the function

$$\sigma = f(t)$$

is determined, $t$ being the ambient temperature expressed in degrees centigrade.

## 5. — DETERMINATION OF THE VALUE OF A LEVEL'S SENSITIVITY

We shall obviously not give in this section the general principles employed in order to determine the value of a level's sensitivity $\sigma$ by means of a check level, a question that is discussed in all training manuals.

We shall only speak of the particular procedures to be used to determine the sensitivity $\sigma$ of the kind of level that must be used in a transit instrument for determining longitude.

We should first of all note that it is an error to assume that this quantity $\sigma$ is a constant factor in such highly accurate levels as those used in transit instruments. The expression giving the sensitivity of a level of our type is :

$$\sigma = \sigma_0 + \alpha t + \beta\,(E_i - E_0) + \gamma\,(E_i - E_0)^2 + \ldots \qquad (22)$$

where $\sigma_0$ is the sensitivity at a temperature of 0° Centigrade, $\alpha$ a coefficient of thermic variation, $t$ the temperature expressed in centigrade degrees, and $\beta$ and $\gamma$ the so-called aging coefficients (due to encrustations on the lining caused by the liquid in the level), $E_0$ the time taken as origin of the first sensitivity measurement, $E_i$ any later time.

Since the observations for determining longitude will last for 18 months at the most it is useless to consider for the sensitivity of the level the terms

$$\beta\,(E_i - E_0) + \gamma\,(E_i - E_0)^2 + \ldots$$

and equation (22) can in our case be written :

$$\sigma = f(t) = \sigma_0 + \alpha t \qquad (23)$$

The procedure to be used for determining $\sigma_0$ and $\alpha$ is therefore as follows. At the beginning of the campaign, i.e. before starting the series of observations, a $\sigma_t$ value for a given ambient temperature $t$ is determined with a very rigorous accuracy.

For the first reduction of the observations we may quite well adopt this value found for $\sigma_t$. During the whole observational period another series of measurements is made every fortnight which thus allows determination of other sensitivity values at various temperatures $t$. At the end of this longitude campaign we shall therefore have as many equations as determinations of $\sigma_t$ carried out :

$$\begin{aligned} \sigma_1 &= \sigma_0 + \alpha t_1 \\ \sigma_2 &= \sigma_0 + \alpha t_2 \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots \\ \sigma_n &= \sigma_0 + \alpha t_n \end{aligned}$$

which, dealt with by the least squares method, will supply us with the values $\sigma_0$ and $\alpha$.

For the final reduction for obtaining the definitive value of the longitude at the observation position the following corrections are made :

$$\Delta\sigma = \sigma_A - \sigma_t$$

where $\sigma_A$ is the initial value (from the first determination) and $\sigma_t$ the value deduced from $\sigma_0$ and $\alpha$ at temperature $t$ of the observation.

This is why a precision thermometer must be placed in the observation cabin and *the temperature must be read and noted each hour during the observation.*

It must not be thought that the value of $\Delta\sigma$ is negligible, for the difference of $\sigma_t$ between summer and winter periods can reach quite considerable values, as also can the excursion of the daily temperature.

Using a Repsold-Heurtaux level-check belonging to the Capodimonte astronomic observatory [7] we carried out eighteen series of measurements on our level, each series comprising an average of 40 to 60 readings, as follows :

*a*) Bubble length $30^p$

| | |
|---|---|
| Series No. 1 | $\sigma = 1''19851$ |
| 2 | 20057 |
| 3 | 20061 |
| 4 | 20309 |
| 5 | 20283 |
| 6 | 20115 |

*b*) Bubble length $35^p$

| | |
|---|---|
| Series No. 1 | $\sigma = 1''20097$ |
| 2 | 20109 |
| 3 | 20098 |
| 4 | 20101 |
| 5 | 20124 |
| 6 | 20089 |

*c*) Bubble length $40^p$

| | |
|---|---|
| Series No. 1 | $\sigma = 1''20085$ |
| 2 | 20098 |
| 3 | 20097 |
| 4 | 20142 |
| 5 | 20081 |
| 6 | 20099 |

We may therefore deduce that the provisional values to be used for the first reductions of the observations made at Capodimonte are :

$$\sigma_A = 1''20105 \pm 0''000047$$
$$t = 25.1\ °C$$

and in seconds of time :

$$\sigma_A = 0.08007\ s$$

Great attention must be paid to ensure that the $\sigma_t$ determinations are carried out with great accuracy, and this includes the knowledge of both the periodic and the progressive errors of the check level screw. If the rate of this screw is not known it will be necessary to determine the screw's errors, and this may be done at the same time as the determination of the sensitivity $\sigma$, following the method suggested by SILVA [8], [9], [10], [11], [12].

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