ON THE OCCASIONAL FAILURE OF NEWTON ITERATION IN TIDAL PREDICTION

by A. M. SHIPLEY,
C.S.I.R. Oceanographic Research Unit,
University of Cape Town

In a previous paper (SHIPLEY 1966), the author has described the use of Newton Iteration for the solution of the equation :

$$Z'(t) = - \sum_{n=1}^{N} S_n H_n f_n \sin \{S_n t + (V_o + U_o)_n - g_n\} = 0$$

where $Z(t)$ is the instantaneous sea height and $Z'(t)$ its first time derivative. It was stated that in the case of consistently semi-diurnal tides it was sufficient when moving on to the next root to add six hours to the last iterate and then start a fresh cycle of iteration. The method worked so well for so long that it was not until recently realised that the computer programme should contain an emergency subroutine to deal with the occasional case in which convergence occurs not to next root of the equation, but back to the one which has just been computed. When this happens a conditional jump instruction causes control to step out of the main programme and into a subroutine where the root is computed by a method described below.

One could of course use the bisection algorithm, a slow but sure way of obtaining a root, but Newton iteration, being a second order process, converges very much more rapidly, and is therefore to be preferred whenever it can be used.

The first failure of the Newton Method occurred when preparing the predictions for 1968 for Port Elizabeth. A reproduction of the computer sheet for September 1968 is shown here, and it will be seen that on 16 September nearly eleven hours separates high water at 14.07 hrs from the preceding low water at 3.19 hrs. Under these conditions adding six hours to the time of low vater at 3.19 hrs merely causes convergence back to the value 3.19 hrs. The vicious circle was broken by the insertion of a conditional jump instruction which transferred control to a subroutine which uses a long-winded but sure method of calculating the next root. On entering, and again on leaving, the emergency subroutine the computer is programmed to hoot so as to alert the operator, and an asterisk is printed on the relative line of the computer sheet. In Manchester Auto Code (the language in which the programme is written), the programme is divided into chapters, on chapter at a time being held in the Immediate Access

TIDAL PREDICTION PORT ELIZABETH 1968 12 CONSTITUENTS. SEPTEMBER

245	1	4.02 L 2.3	10.42 H 3.9	16.55 L 2.7	23.26 H 4.1
246	2	6.17 L 2.2	13.00 H 4.2	19.06 L 2.4	
247	3	1.17 H 4.4	7.36 L 1.8	14.05 H 4.8	20.10 L 1.9
248	4	2.15 H 4.8	8.24 L 1.4	14.46 H 5.3	20.50 L 1.5
249	5	2.55 H 5.2	8.59 L 1.0	15.18 H 5.7	21.22 L 1.1
250	6	3.28 H 5.5	9.29 L 0.8	15.47 H 6.0	21.50 L 0.8
251	7	3.58 H 5.7	9.55 L 0.6	16.15 H 6.1	22.17 L 0.7
252	8	4.26 H 5.8	10.22 L 0.5	16.41 H 6.2	22.42 L 0.6
253	9	4.53 H 5.8	10.47 L 0.6	17.08 H 6.1	23.09 L 0.7
254	10	5.20 H 5.7	11.14 L 0.7	17.34 H 5.9	23.35 L 0.8
255	11	5.48 H 5.4	11.40 L 1.0	18.01 H 5.6	
256	12	0.02 L 1.2	6.15 H 5.1	12.06 L 1.4	18.27 H 5.2
257	13	0.29 L 1.6	6.42 H 4.6	12.31 L 1.9	18.52 H 4.7
258	14	0.57 L 2.1	7.09 H 4.1	12.53 L 2.4	19.16 H 4.1
259	15	1.28 L 2.6	7.41 H 3.6	13.10 L 2.9	19.37 H 3.6
260	16	3.19 L 3.0	14.07 H 3.5	19.50 L 3.2	
261	17	1.19 H 3.6	7.39 L 2.7	14.05 H 3.9	20.05 L 2.7
262	18	1.56 H 4.0	8.00 L 2.2	14.21 H 4.4	20.20 L 2.3
263	19	2.19 H 4.4	8.18 L 1.8	14.39 H 4.9	20.36 L 1.8
264	20	2.41 H 4.9	8.37 L 1.3	14.59 H 5.4	20.57 L 1.3
265	21	3.05 H 5.3	9.00 L 0.9	15.22 H 5.9	21.21 L 0.8
266	22	3.31 H 5.7	9.27 L 0.5	15.49 H 6.3	21.49 L 0.4
267	23	4.01 H 6.0	9.57 L 0.2	16.18 H 6.5	22.20 L 0.2
268	24	4.33 H 6.2	10.29 L 0.1	16.50 H 6.6	22.54 L 0.1
269	25	5.07 H 6.1	11.04 L 0.2	17.24 H 6.5	23.30 L 0.5
270	26	5.45 H 5.9	11.41 L 0.6	18.01 H 6.1	
271	27	0.09 L 0.6	6.26 H 5.5	12.21 L 1.1	18.42 H 5.5
272	28	0.54 L 1.2	7.15 H 4.9	13.09 L 1.8	19.32 H 4.8
273	29	1.51 L 1.8	8.24 H 4.3	14.25 L 2.5	20.51 H 4.2
274	30	3.38 L 2.3	10.36 H 4.0	17.14 L 2.7	23.26 H 4.0

Store, and the remainder in Drum Store. It was thus only necessary to add an extra chapter to the programme (Chapter 9), the subroutine being entered by means of an instruction :

ACROSS 1/9

On entering chapter 9, the programme adds two hours to the previously obtained root, the assumption being that the interval between two successive roots of $Z'(t) = 0$ will not be less than two hours. It then proceeds to calculate the sign of $Z'(t)$ at intervals of one tenth of an hour. As soon as the sign changes, the value of t is stepped down by 1/10 hour and the whole process is repeated with a step length of 1/100 hour. Control is then returned to the main programme. 16 September was the only day for the 1968 predictions for which the emergency subroutine was called in.

Chapter 9 is given in the appendix for the interest of those people familiar with Manchester Auto Code.

REFERENCE

SHIPLEY, A. M. : Tidal Predictions on a medium sized Digital Computer. *Int. Hyd. Review,* No. XLIII No. 1, Jan. 1966.

APPENDIX

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CHAPTER 9             EMERGENCY SUB-ROUTINE
VARIABLES I
1) PRESERVE
P = 1(1)10
HOOT                  ALERT OPERATOR
REPEAT
X = X' + 2            PREVIOUS ITERATE STORED IN X'
S = 0                 SET COUNTER
H' = 0.1              STEP LENGTH
K = 0                 SET COUNTER
2) A = 0
B = 0
C = 0
T = 0(1)N
B' = BT
C' = CT
E' = XB' + C'
E' = GE'              CONVERT DEGREES TO RADIANS (G = π/180)
D' = AT
W = φCOS(E')
W = WD'
A = A + W             Z(t) STORED IN A
U = φSIN(E')
U = - UGD'B'
B = B + U             Z'(t) STORED IN B
V = - WGGB'B'
C = C + V             Z''(t) STORED IN C
REPEAT
JUMP 22, B = 0
K = K + 1             STEP UP COUNTER
JUMP 3, K > 1
E = φSIGN(B)          STORE INITIAL SIGN OF Z'(t)
3) F = φSIGN(B)       COMPUTE CURRENT SIGN OF Z'(t)
H = E + F
JUMP 21, H = 0        JUMP IF Z'(t) CHANGES SIGN
X = X + H'
JUMP 2
21) S = S + 1         STEP UP COUNTER
JUMP 22, S = 2        JUMP IF COUNT EXHAUSTED
X = X - H'            STEP BACK X
H' = 0.01             DECREASE STEP LENGTH
JUMP 2
22) φ7(10000)X, 1     STORE X ON DRUM
φ7(10001)A, 1         STORE A ON DRUM
φ7(10002)C, 1         STORE C ON DRUM
P = 1(1)10
HOOT                  ALERT OPERATOR
REPEAT
RESTORE
ACROSS 5/3            RETURN TO MAIN PROGRAMME
CLOSE
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