# SATELLITE TRIANGULATION

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## A New Geodetic Tool

The orbiting satellite has presented the geodesist with a tool which makes possible the establishment of a triangulation scheme connecting all continents and their geodetic datums and encircles the globe with a relatively small number of earth-based points. Before the advent of artificial earth satellites, intercontinental connections and ties to islands and points remote from geodetic datums, depended upon such approaches as flare triangulation and airborne electronic distance measurements, as Hiran, which are quite limited in range. The spanning of great ocean areas was out of the question and recourse had to be made to island-hopping where this was possible, such as the North Atlantic tie between the European and North American Continents by Hiran in the 1950's.

All has changed now and there are no practical limitations on the lengths of lines which may be observed since satellites may be put in orbit at any convenient height above the earth's surface.

## The Concept

The basic concept of satellite triangulation or an explanation as to how it works is relatively simple, but the actual establishment of a triangulation scheme is another matter. The equipment, methods employed in data acquisition, and the processing and adjustment of the observations are all highly sophisticated. An explanation of all the details involved would lead us far afield and beyond the scope of this paper.

Basically the idea may be stated simply in this manner. The observations consist of photographing the satellite simultaneously from at least two ground stations against a star background. Now the stars may be considered for all practical purposes at an infinite distance. The geocentric parallax of even the nearest star is completely insensible — utterly beyond all present means of measurement. In short the lines of sight from a group of observers scattered over the earth's surface toward any given star are parallel.

But an orbiting satellite is at a finite distance and its directions as seen from widely separated points on the earth are not at all parallel and are determined by its apparent position among the stars as seen from each point (fig. 1). The trick is to determine very accurately and simultaneously the direction of the satellite at some instant by a convenient method of
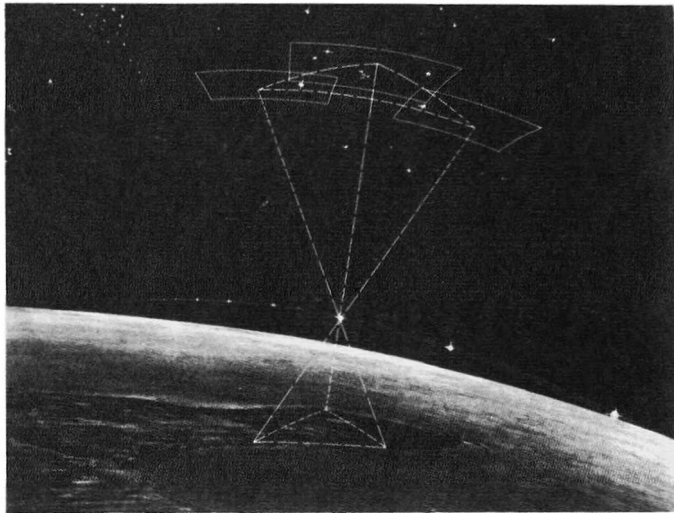


Fig. 1. — Geometry of simultaneous observations against
star background.

observation. This method turns out to be the use of a precision camera (fig. 2) which exposes on its plate a large number of star trails as well as the satellite trail. In the simple case the directions of two intersecting lines to the satellite from points A and B are determined simultaneously and form a plane in space, the orientation of which is known relative to the spin axis of the earth. If the satellite is photographed again simultaneously, in another position, from A and B, another such plane is formed and the intersection of these planes is a straight line between A and B, the direction of which is also determined, based on the star reference system. By the introduction of a third point C and with additional satellite observations similarly taken, the directions of the triangle sides of ABC are determined and we have a spatially oriented triangle. This procedure can be extended across continents and oceans until a scheme is developed which encircles the globe and closes upon itself, thus minimizing the effect of error propagation.
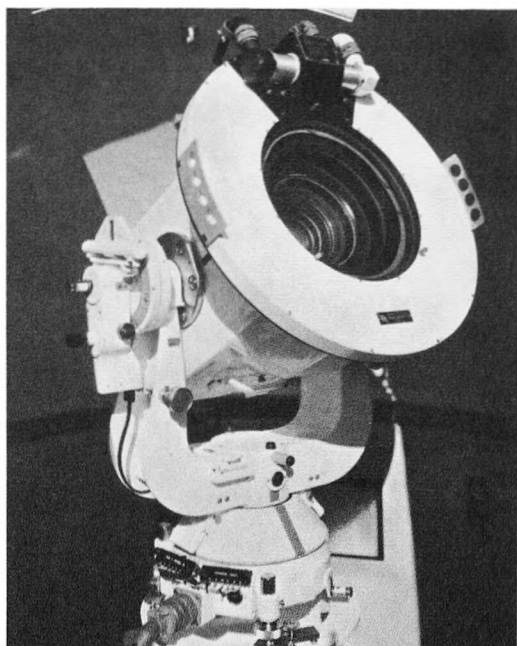
Fig. 2. — BC-4 Satellite triangulation camera.

The orbital behavior of the satellite in no way affects the accuracy of the work; the satellite is used merely as a survey beacon. However, the orbit must be known well enough to provide the means of predicting for, say, a week in advance the azimuth and elevation angles in order to preset the camera for the observations.

## Equipment

The fundamental method in obtaining data in satellite triangulation is the photographing of satellite and star images which are very accurately time correlated. The images on the photographic plates are then very accurately scaled to determine the apparent right ascension and declination of the satellite at a precisely defined epoch.

The basic instrument is a precision camera with a 450 mm focal length lens cone. The exposures are taken on glass plates with an effective size of 18 × 18 cm which corresponds to a field view of roughly 22° square.

The camera is provided with a capping shutter for chopping star trails during a pre-orientation exposure to determine the camera orientation. The azimuth and elevation of the camera are thus precisely determined since the chopped images are time correlated with WWV. The satellite trail is then photographed and its trail is chopped by a high precision internal chopping mechanism (fig. 3). After the satellite trail has been photographed, a postorientation exposure is made by again photographing the star trails in order to determine the stability of the camera in azimuth and elevation.
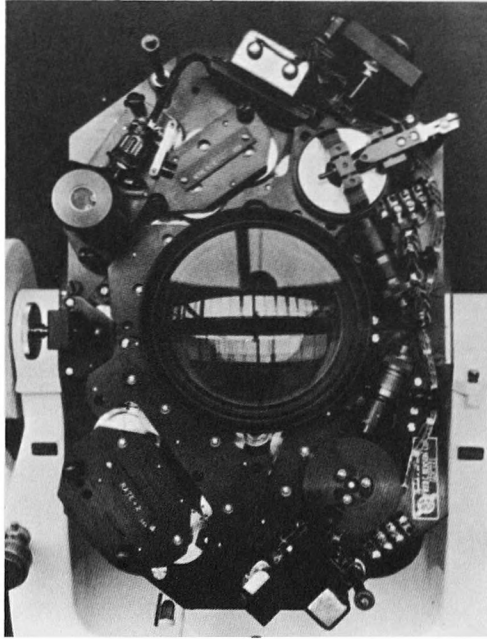
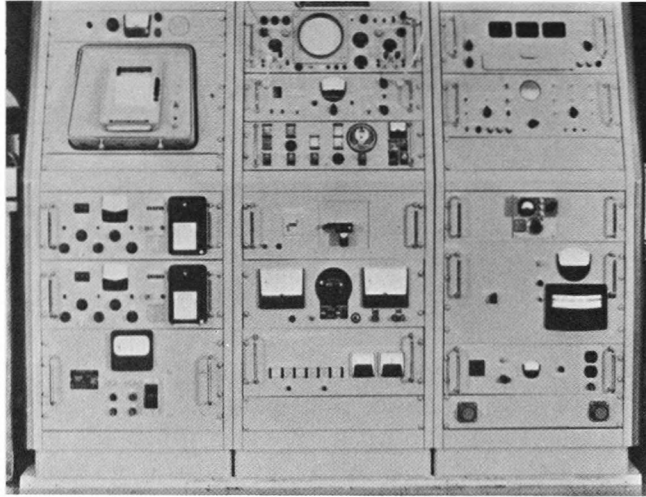Fig. 3. — Internal high precision disk shutter drive.

Fig. 4. — Electronic synchronization and timing system.

An important part of the equipment is the electronic time synchronization system (fig. 4) which provides the means of operating the camera shutter and timing the image exposures to better than 150 microseconds. Time control is maintained by a high precision portable clock (fig. 5) which is carried from station to station and back to a master clock at the National Bureau of Standards about once a month.
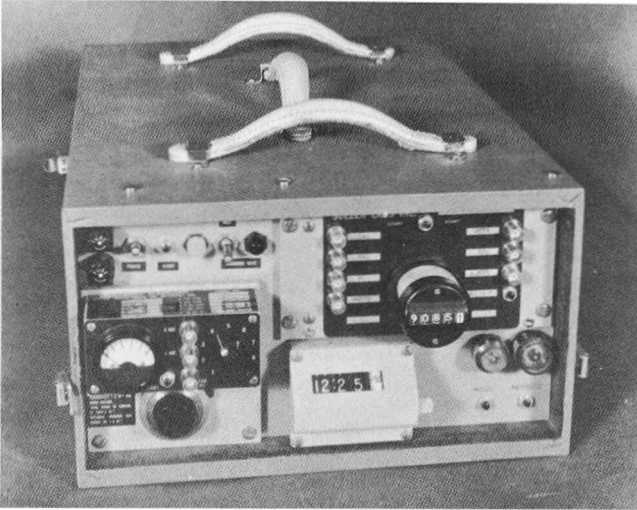
Fig. 5. — Portable quartz crystal clock.

## Plate Reduction

One of the most important steps in the reduction of data for satellite triangulation, and the most tedious and time consuming, is the scaling of the star and satellite images with a precision comparator. The plate coordinates are determined for about 1 000 star images and 600 satellite images, employing every possible method to eliminate bias errors which include those that might result from plate and lens distortion. The data are finally compressed into one fictitious image with corresponding time resulting in a space direction, the mean error of which is of the order of 0.3 second of arc. The mean error of the determination of the coordinates of a single image on the plate is about 3 microns. This is reduced statistically because of the fictitious image to something like 0.5 micron.

To give an indication of the sophisticated approach in the data processing, there are taken into consideration in correlating time with satellite images such things as difference in light travel time due to different slant ranges of the satellite, lens distortion, diurnal aberration, second-order refraction effect, and the displacement of the satellite image due to the phase angle of the sun.

## Work Accomplished

The first camera systems which include the associated timing equipment were available in 1962. During that year and the first half of 1963 many experimental observations were taken at Aberdeen Proving Ground, Maryland. These experiments consisted of simultaneous observations with three cameras placed to form small triangles of 5 and 25 metres on the side.

Knowing the geodetic relationship of the cameras by direct measurement, it was possible by determining the parallactic displacements from observations on satellite Echo I to get a feel for the expected precision. The results were quite promising.

In August 1963 the satellite triangulation program commenced on an operational basis with camera stations in Maryland, Mississippi, and Minnesota. These are points indicated in figure 6 as 002, 103 and 102. After the completion of these observations the work was extended to Florida, Bermuda and Antigua (points 104, 105 and 106).
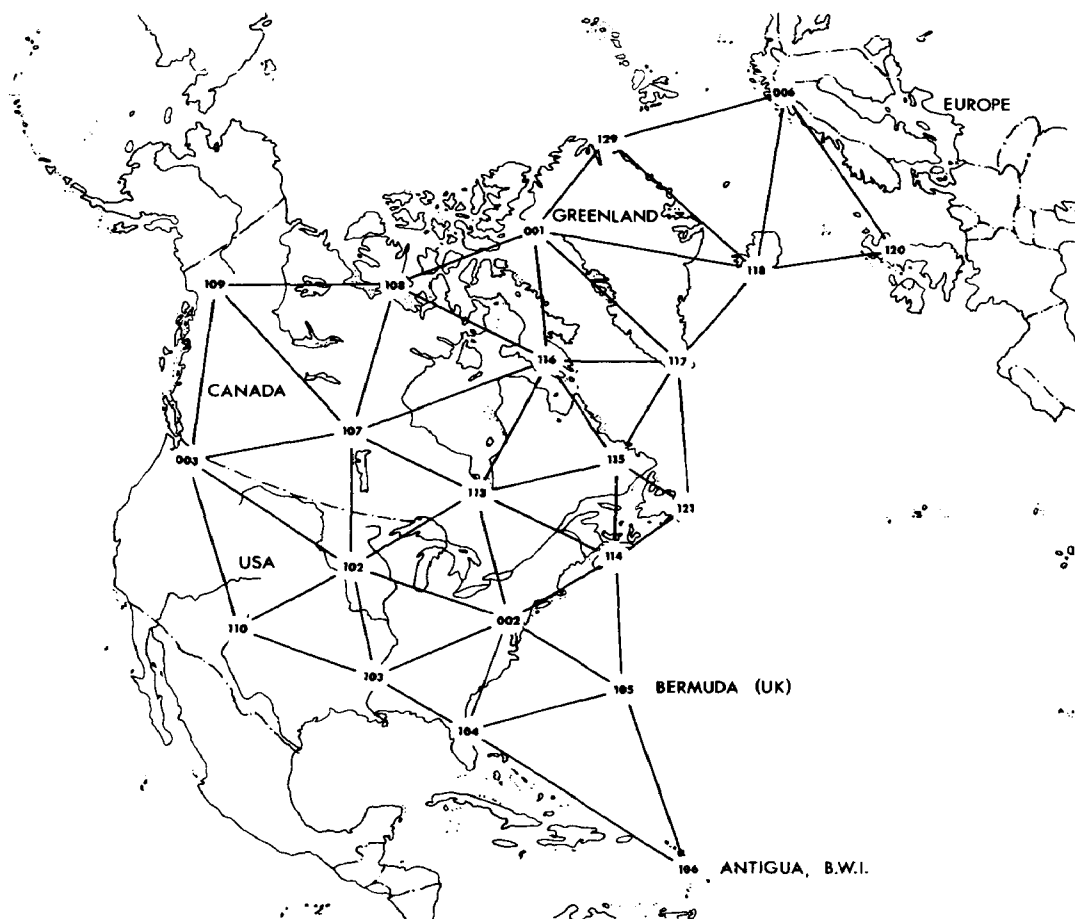


FIG. 6. — Work accomplished

Arrangements were then made with the Canadian Government for the establishment and occupation of stations in that country, and during the winter of 1964-65 points 107, 108 and 109 were observed. Later the work was extended eastward through Canada, thence to Greenland and finally to Iceland, Norway and Scotland, thus effecting a direct connection between the North American and European Continents. From figure 6 it can be seen

that the following numbers of stations have been established: United States (6); Bermuda (1); Antigua (1); Canada (7); Greenland (3), Iceland (1); Norway (1); Scotland (1); making a total of 21 stations completed.

Echo I was used solely until January 25, 1964 when Echo II was launched at about the same altitude, but in an orbit of a much higher inclination necessary for the more northerly stations. The average length of the lines of this satellite triangulation scheme is of the order of 1 500 kilometres which is about the height of the Echo I and Echo II.

Much experimentation in the computation has been done including the important phase of the assignment of relative weights between the classical geodetic work and photogrammetric observations. Simultaneous adjustments have been made of single triangles up to a network of 9 triangles. Computations have not yet been completed to the European datum; these are awaiting the scaling of the photographic plates. Lens cones of 305 mm focal length were used. These are being replaced with 450 mm cones for future work.

It is a feeling of those in the work that the mean square error of a direction in space, resulting from the observations in this network, will be of the order of 0.4 to 0.5 second of arc.

### The World Net

Planning for the world net has been underway for about three years. Several factors must be taken into consideration in planning such a net. First, geometric strength is of great importance. The earth should be covered more or less uniformly by a series of camera stations forming, ideally, equilateral triangles. Clearly such an ideal net cannot practically be achieved, but it has been approximated. Second, the decision must be made as to the average length of line desired. Involved in this decision is the total number of camera stations that are required to properly connect the continents and the existing geodetic datums, and involved also is the length required to span the vast ocean areas, particularly in the South Pacific, using available islands as stepping stones between the continental areas. Third, there are political considerations involving permission for entry into areas or onto islands that may fit the scheme. Lastly, there is the problem of accessibility by ship, rail, or aircraft. It need not be pointed out that a selection of worldwide stations, which require all these considerations, is quite a task and the process of selection might be compared in some ways to a game of checkers. Once a preliminary scheme has been decided upon and then it is found necessary for one of various reasons to move one or more points, such moves involve changes of other stations in order to conform to the general plan.

The plan now approved will hopefully not have to be changed. It includes a total of 40 stations. The average length of the line is approximately 4 000 kilometres and for the best geometry the altitude of the satellite should approximate the average length of the triangle side. Accordingly, the National Aeronautics and Space Administration is planning to launch

Pageos (*) at an altitude of about 4 250 kilometres in a near circular polar orbit. It is necessary, of course, to approximate a polar orbit in order to make acquisition of the satellite possible at all latitudes. Reference is made to figure 7 which depicts the world net and indicates what might be termed five operational phases. It can be seen that the first phase will cover the northern portion of the scheme from Japan across North America to Europe and the second phase will include an area immediately to the south of this and so forth. The cameras, of course, must be moved to different latitudes during differents parts of the year because of the change in the sun's declination. Actually all the cameras in phase 1 will not be moved at the same time to stations in phase 2, but as certain cameras with a minimum number of lines to observe will be released before the others, there will be a gradual moving of cameras between the different phases. Twelve camera systems are planned for this operation.



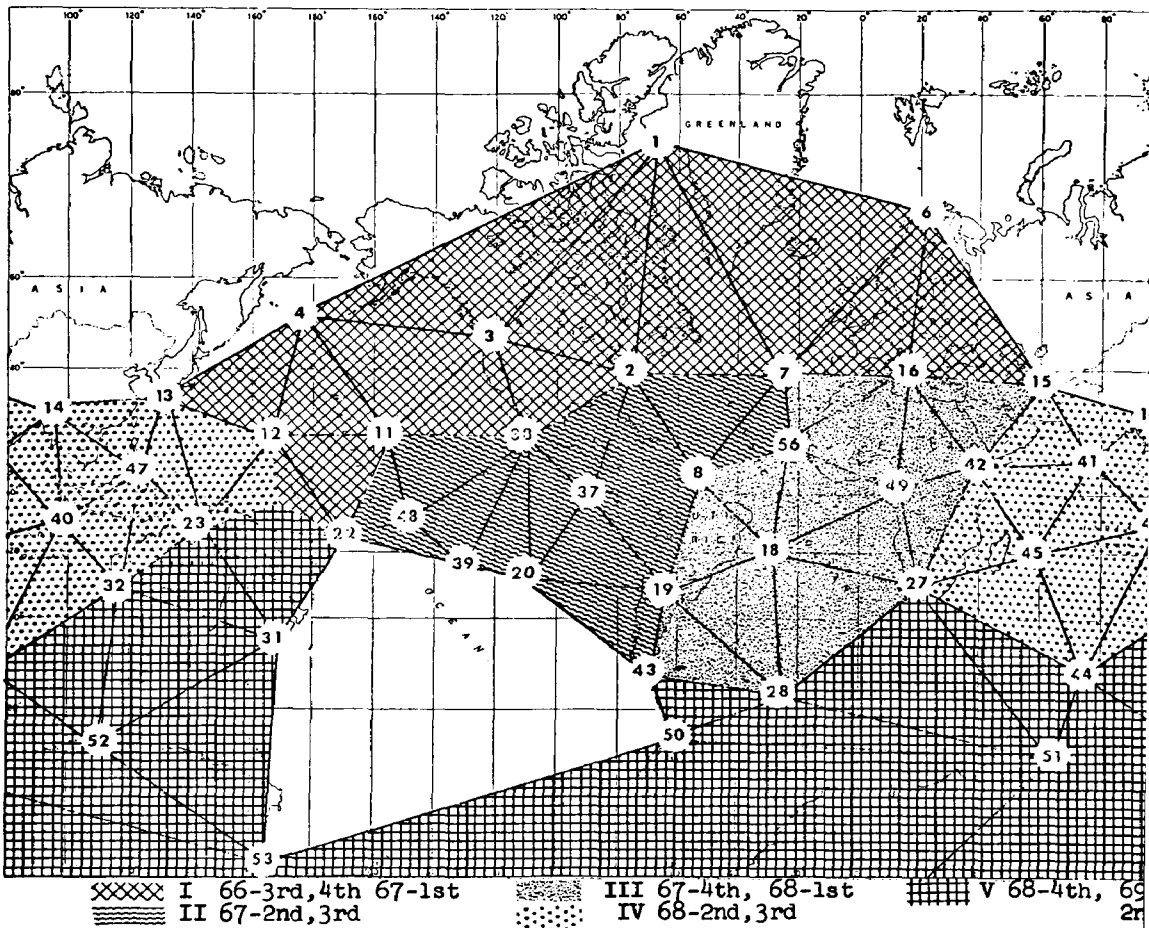| XXXXX I 66-3rd,4th 67-1st | III 67-4th, 68-1st | V 68-4th, 69 |
| ≡≡≡ II 67-2nd,3rd | IV 68-2nd,3rd | 2r |

FIG. 7. — World geometric satellite network operational phases.

(*) The launching took place at 0 h 12 m 02 s 24 June 1966 U.T.

Inasmuch as satellite triangulation in itself provides no scale what-soever, the question immediately arises as to how scalars should be introduced into the work. Some of the major geodetic datums could provide a scale between satellite camera stations, but the accuracy of the classical triangulation over these long distances is not considered sufficient. Since it has been demonstrated that the directional accuracy obtainable from the cameras to the satellite will approach, if not exceed, 0.3 second of arc then the scalars for this network should definitely be better than 1/500 000. The accuracy figures quoted herein are based on mean square errors.

To accomplish this a Geodimeter traverse is being observed between two points in the world net in the United States; one at Beltsville, Maryland and the other at Moses Lake, Washington. It is expected that the accuracy (m.s.e.) of this base line will approach, if not exceed, 1/1 000 000. Plans are underway by European countries to strengthen a chain of existing triangulation between Tromsø, Norway and Catania, Sicily to scale a line of the world net in Europe. A decision has been made that the cost of establishment of a Geodimeter traverse between these points would be prohibitive and that the introduction of a number of Geodimeter base lines and Laplace azimuths into the existing triangulation should satisfy the requirements. Another base is planned in southern Australia using the very high-grade Tellurometer traverse already established in that area. Some lengths of this traverse may be reobserved by the Australians in order to assure adequate accuracy. A selection of the fourth base line is planned for Africa. From theoretical considerations it is found that not much in the way of accuracy is gained by introducing more than four scalars and that the determination of the height is more critical in this respect than the determination of latitudes and longitudes. The coordinate system on which computations will be based is an $x$, $y$, $z$ Cartesian system in which the $z$ axis is the axis of rotation of the earth and $xy$ plane is coincident with the equator.

## Densification of Satellite Triangulation

The overall plan includes the establishment, within the large triangles of the world satellite triangulation program, of a subsidiary scheme fitted of course to the points in the world network. The first estimate is that the spacing of these points will be of the order of 1 000 kilometres employing satellites of roughly that altitude. Ideally this should be done by substan-tially the same method employed for the world net; that is, with the use of precision cameras and the photogrammetric technique. However, because of the large number of stations that will be involved and since optical observations are weather dependent, there is considerable merit in thinking in terms of accomplishing the densification networks by means of electronic ranging techniques such as DOPPLER or SECOR.

Densification of a continent can break down the world net by more closely spaced control points. The purpose of this densification is perhaps twofold. First it will provide connections to the independent triangulation datums within a continent and, second, it will provide a good overall
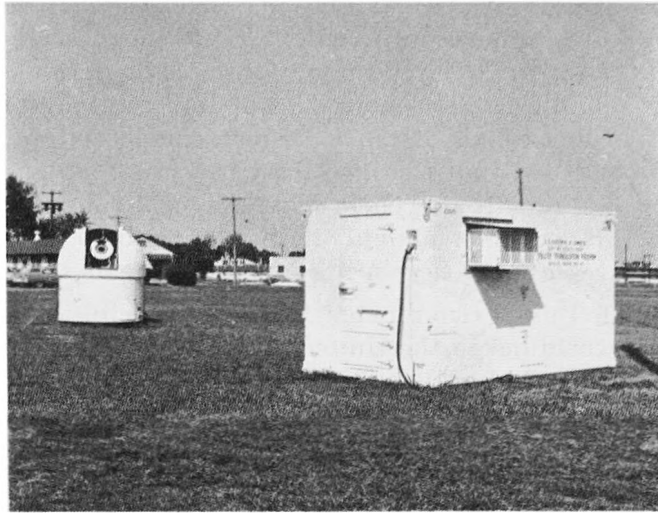
FIG. 8. — Typical station site — electronic equipment
shelter and astrodome.

control for the establishment of future classical triangulation which can be
initiated at the various points with the assurance that there will be a
reasonable fit among the various triangulation schemes when these are
joined. The same concept would, of course, apply to any continental
area.

There is a lower limit in altitude in which an orbiting satellite can be
expected to stay in orbit for any useful length of time owing to atmospheric
drag. This limit is probably something of the order of four or five hundred
kilometres. Thus consideration is being given to the use of some sort of
flare triangulation in which camera or theodolite observations can be made
simultaneously from separated points on a flare or flashing type of beacon
carried by high-altitude aircraft. This would permit the determination of
the lengths and directions of relatively short lines which may be required
to connect certain parts of existing control work, particularly in areas of
high inaccessibility.


### The Consequences


Out of all this will come a precise spatial network of triangles, the
corners of which are points on the earth's surface whose positions are
defined relative to a Cartesian coordinate system in inertial space. The
origin of this system will approximate the centre of mass of the earth.
Since there will be no input of a geophysical nature to the development of
this scheme, the product will be of no direct geophysical significance, nor
is it intended to be.

But the ultimate benefits to physical geodesy, as a result of this work,
should be of inestimable value. For the first time a geometrically consistent
system of observation stations will be placed on a common world datum.
Observations from these stations on high-density relatively low-altitude

satellites will provide data of great geophysical significance. Inasmuch as the earth's centre of mass is related to the mass distribution within it, and therefore to the gravitational field, it is logical to expect that observations affected by that gravitational field are required to refer the positions to the earth's centre of mass and therefore to a true world geodetic system.

Moreover for the first time direct measurements will have been made completely encircling the earth which will permit, in effect, a direct determination of the size and shape of the best fitting earth ellipsoid consistent with the world network. Heretofore the parameters of such an ellipsoid had to be inferred from many isolated pieces of classical triangulation networks, relatively sparse surface gravity measurements, and astro-geodetic levelling uncorrelated on a worldwide basis, although determinations thus made will likely prove to be quite accurate.

At any rate it seems reasonable to expect that, in the relatively near future, the geodesist will have approximated, to a degree heretofore unattainable, the goal for which he has striven over the centuries — a truly valid world geodetic system and a precisely defined gravitational field of the earth.