VERTICAL DEFLECTION FROM DIP OBSERVATION

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1. — INTRODUCTION

Needless to say, knowledge about vertical deflection is indispensable to research in geodesy and geology. Around a small island, for example a volcanic islet, we may expect a certain amount of vertical deflection due to the attraction of the island and its subterranean magma mass. For this reason, particular consideration should be given to vertical deflection on the islands of and around Japan on account of their geological characteristics (fig. 1). These islands contain a number of volcanoes and they are situated at the edge of the Eurasian Continent which in the Northern Pacific Ocean is fringed by deep trenches. In order to determine the deflection in islands, GOUGENHEIM (1959) tried applying data on the dip of the horizon, and this was followed by a discussion by BHATTACHARJI (1961).

Far from an island, the general geoid can be approximated as a part of a spherical surface; it is at this surface that we aim for observations referred to the horizon. On the other hand, near the island the geoid is distorted, generally upwards, due to subterranean structure; the vertical at a station on the island is the normal to this distorted geoid. Then, the dip at the station in the line determined by the observation station and the centre of the island, and the dip in a perpendicular direction should differ by the amount of the vertical deflection at the station. It should be emphasized that the deflection so obtained is that relative to the general geoid, not to a reference ellipsoid as defined in the usual sense in geodesy.

The present paper gives the results of work carried out to investigate the distribution of vertical deflection in some of the southern islands of Japan (Izu Syoto).

2. — SELECTION OF OBSERVATION STATION

There are two requirements in selecting a suitable station for observation of the dip of the sea horizon for the present purpose.

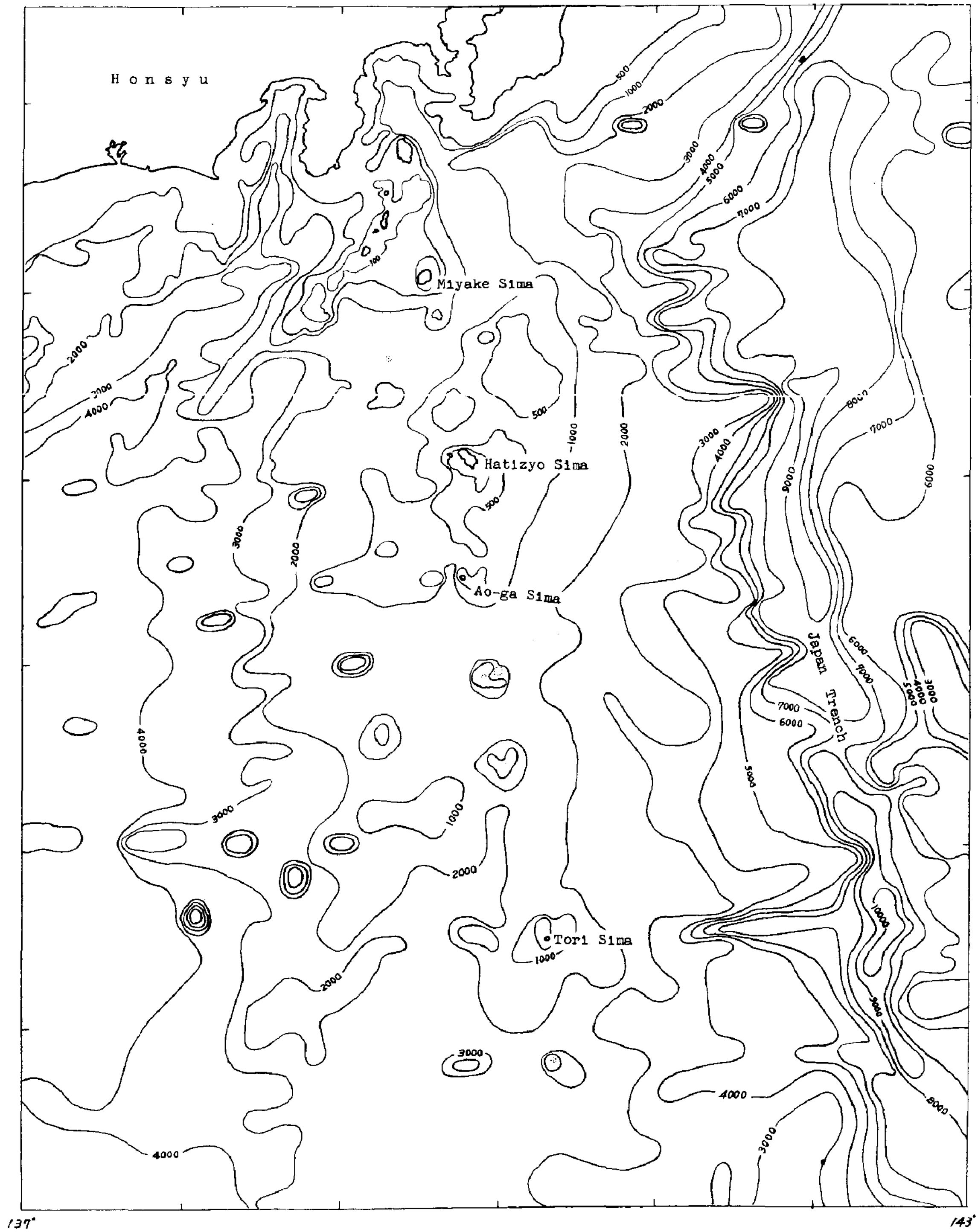

FIG. 1. — Bottom Topography around the Izu Syoto.

Firstly, the horizontal field of the visible horizon must be larger than 180°. In practice, even for a small islet of conical shape, it is fairly difficult to find a spot from which the horizon can be observed over a field greater than two right angles, owing to obstructions by bushes or adjacent cliffs.

Secondly, the height of the observation station above the sea surface must be selected so as to enable clear observation of the horizon image in the telescopic field. If the height of the observation station is too low, the undulation of the sea surface at the observed horizon disturbs measurements. If, on the contrary, the height of the observation station is too great, the distance to the (true) observed horizon may be greater than the visibility at that time. Hence, the horizon image will be too vague to measure.

TABLE 1

Visible range

h	d	ω	ω''
m	km	10^{-4}	
10	12.25	0.816	16″.8
20	17.32	0.577	11.9
30	21.21	0.472	9.7
40	24.49	0.408	8.4
50	27.39	0.365	7.5
70	32.40	0.309	6.4
100	38.73	0.258	5.3
150	47.43	0.211	4.4
200	54.77	0.182	3.8
300	67.08	0.149	3.1

Table 1 shows the relation between the height of observation station h, the distance to the horizon d, its reciprocal ω, and this value expressed in

TABLE 2

Frequency of visibility in 1963 at Hatizyo Sima lighthouse

	5	10	20	30	40	50	60	70 km
	(Days)							
January	1	3	9	16	2			
February		3	11	14				
March	4	7	14	6				
April	5	7	4	12	2			
May	9	11	6	5				
June	3	5	9	6	5	2		
July	3	8	5	10	3	1	1	
August		6	13	6	2	1	2	
September	2	10	3	12	1	1	1	
October	5	6	4	9	5		1	1
November	1	1	10	11	6	1		
December	1	2	6		11	10	1	
Sum	34	69	94	107	37	16	6	1

seconds of arc. The last quantity, ω'', denotes the angle subtended by a length of 1 metre at the horizon as viewed from a place h metres above the sea surface. Thus, a difference in height of 1 metre at the sea horizon will disturb the observation by about $10''$ when observed at a height of 30 metres. We may provisionally assume the magnitude of the vertical deflection to be of the order of $10''$, from the examples given by GOUGENHEIM (1959) and from experiences in geodetic surveys in Japan. Consequently, a height of 30 metres may be considered to be the lower limit for an observation station. Table 2 gives data on visibility observed at a lighthouse in Hatizyo Sima, where one of the present observations was made. In general, visibility ranges between 20 and 40 km and is better in winter than in summer. Visibility over 50 km is exceptional. Hence, from Table 1, we find the upper limit for an observation station to be about 100 to 150 metres.

We must also consider another restriction necessitating the choice of an observation station at a sufficiently low height. The dip of the sea horizon consists of two parts, geometrical and optical. The method by which vertical deflection is determined from dip observations can be applied on condition that the optical dip observed at the station is the same in every direction. Therefore, we must first assume the function representing the refraction to be identical in all directions. In these circumstances, the amount of refraction depends merely on the refractive indices at the observed horizon, or, more strictly speaking, immediately above sea surface at the observed horizon and at the eye position. FREIESLEBEN (1948) gives a strict formula for dip :

$$D = 5'.04\sqrt{0.1123h + T_0 - T_h} \qquad (1)$$

where D is expressed in minutes of arc, and h is the height of eye in metres, and T_0 and T_h, respectively, temperatures in centigrade immediately above sea surface and at the eye piece. By means of this formula, SINZI and OWAKI (1961) derived a correlation between $T_0 - T_h$ and $T_h - T_w$, where T_w denotes water temperature measured at the sea surface. The resulting formula is

$$D = 5'.04\sqrt{0.1123h - \alpha(T_h - T_w) + \beta} \qquad (2)$$

α and β being constants. Formula (2) can be approximated by the usual formula,

$$D = 1'.776\sqrt{h} + b(T_w - T_h) \qquad (3)$$

which has been employed classically.

The value of the constant b, which is called the coefficient of temperature difference, has been determined from the pioneer work of Koss (1899) by various authors. We will not here look into this question. Table 3 gives some reference data. In the North Pacific, the value of b seems to decrease with decreasing latitudes. The second value in Table 3, i.e. $b = 0'.20$, was derived from observations made along the southern coast of the mainland of Japan by the former Hydrographic Department of the Imperial Navy (AKIYOSHI, 1933), and the value has been adopted extensively by Japanese vessels for sight reduction.

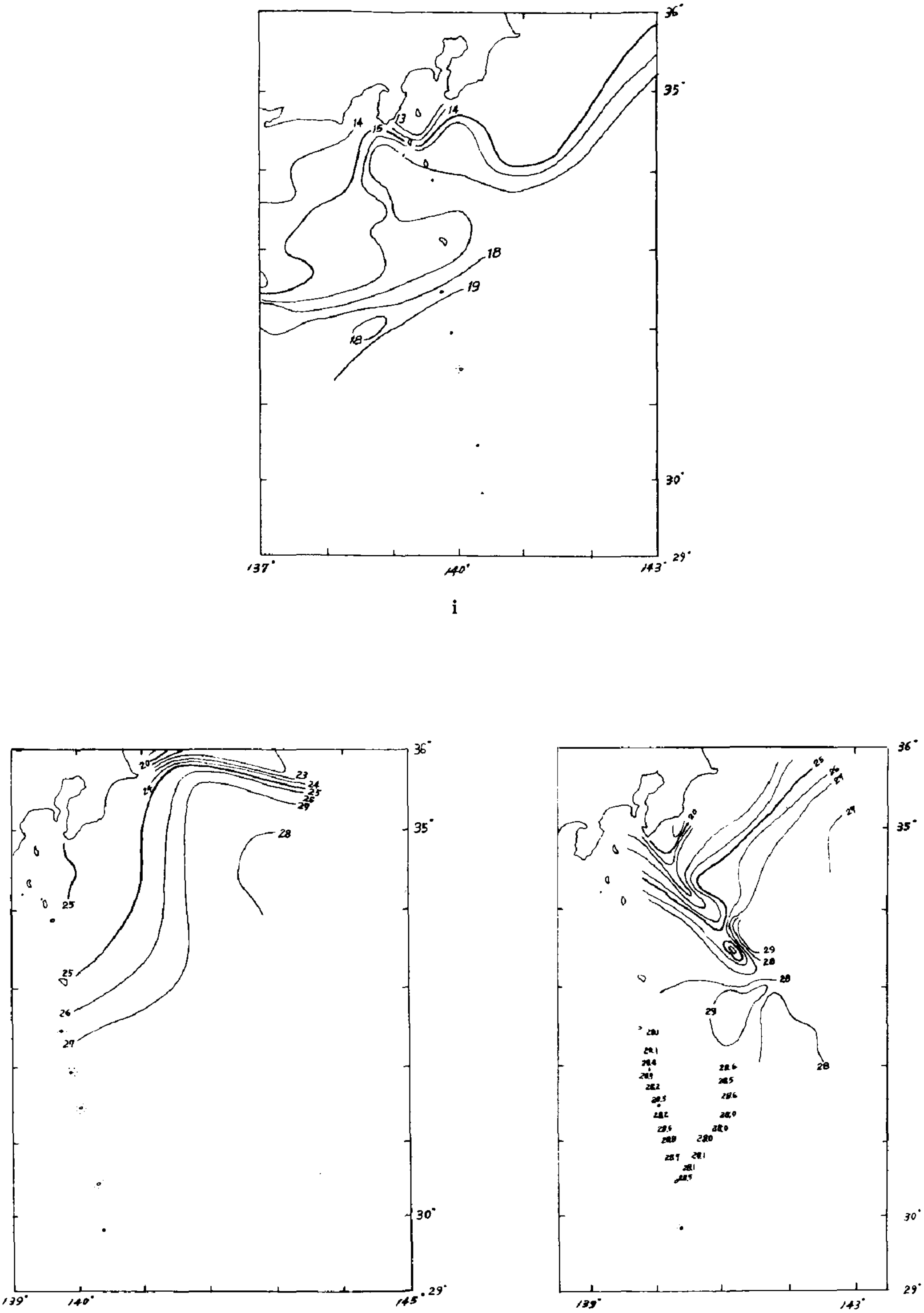

Fig. 2. — Distribution of Surface Temperature.

i. Feb.-Mar. 1963
ii. Jul. 1963
iii. Jul. 1964

TABLE 3

Coefficient of temperature difference

Author	Year	b	Sea area
Kohlschütter	(1903)	0ʹ.39	Adriatic Sea
Hydrogr. Dept. Japan	(1933)	0.20	Kuroshio region
Akiyoshi	(1936)	0.11	Yokosuka to E. Caroline Is.
Sinzi and Sugimoto	(1961)	0.01	Around Mariana Is.

Therefore, if we assume that expression (3) with $b = 0'.20$ holds good for the observation of dip for determining vertical deflection, the distribution of surface temperature at the observed horizon must also be taken into account. A difference of 1 °C in surface temperature for different points along a visible horizon causes a difference in observed dip of about 10″, which is large enough to affect the determination of vertical deflection. We can conclude that the observation station has to be chosen sufficiently low in order that all the surface temperatures in the visible area should be practically equal. Fig. 2 shows surface temperatures in the Kuroshio region where the present observations were made. In this region, as can be seen from Table 1, it is preferable to select an observation station not higher than 100 metres above sea surface. The effect of refraction will be mentioned again in a later section of this article.

3. — OBSERVATION

Up to the present, observations of dip have been carried out for vertical deflection investigations involving four islands situated in the volcanic belt of Mt. Fuji.

(1) Hatizyo Sima (33°.1 N, 139°.8 E);
11 to 18 March, 1962, 4 stations.
Observers : SINZI, OWAKI, TANO and NISHIMOTO.

(2) Miyake Sima (34°.1 N, 139°.5 E);
3 to 13 March, 1963, 4 stations.
Observers : SINZI, JOJO and FUKUDA.

(3) Ao-ga Sima 32°.4 N, 139°.5 E);
16 August to 18 September, 1963, 5 stations.
Observers : SUZUKI, UNIWA and MORI.

(4) Tori Sima (30°.5 N, 140°.3 E);
27 June to 25 July, 1964, 4 stations.
Observers : SINZI, YAMAZAKI, MORI, NISHIMURA and KAJI.

These islands, composed mainly of tholeiite, lie on a ridge between a miogeosyncline extending south of Sikoku and Kyusyu, and a parasio-geosyncline called the Japan Trench and which forms the eastern border of the transitional zone between the continent and the oceanic bed. It has been recently stated, however, that this ridge is a volcanic arc running between oceanic beds.

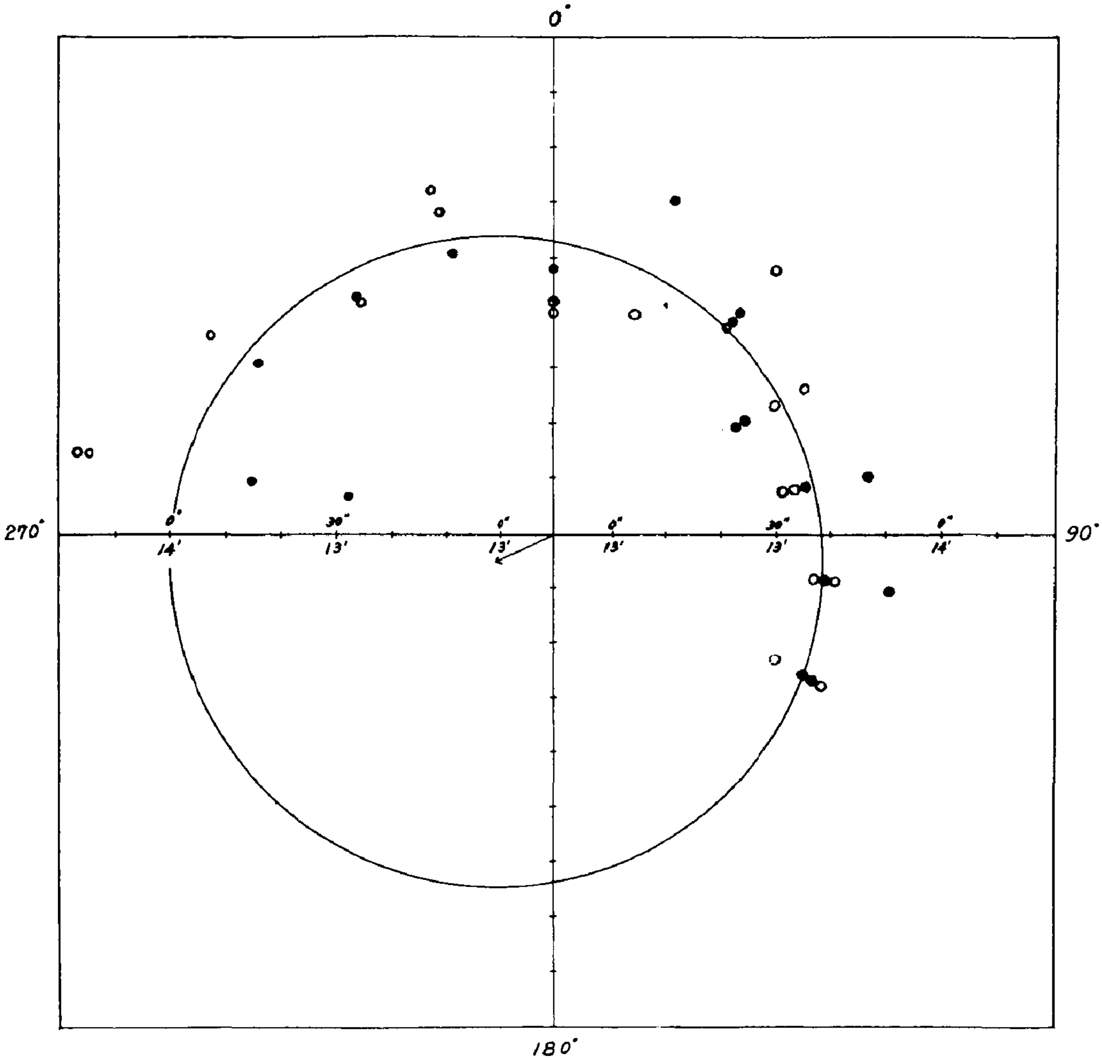

FIG. 3. — Dip Observation at Aka Saki, Hatizyo Sima.
(Obs. No. 1-8 of Table 4)

A Wild T3-theodolite was used. Each set of observations at a particular station consists of two series : one with the clump to the right and the other with the clump to the left. In each series, dip is measured for every 20° in azimuth (in a few cases for every 30°) except at the beginning and end of each set. Fig. 3 shows an example of observations, the circles and dots denoting the values of dip observed with the clump respectively to the right and to the left.

4. — REDUCTION OF DATA

In fig. 4, the set of observations is diagrammed. The plane OXY is perpendicular to the vertical at the observation station O, and each dot denotes an observed value of dip S_i for azimuth A_i. If we can draw a circle passing through all the S_i and A_i dots, the centre of this circle will represent the direction of the normal to the plane defined by the observed horizon. The distance and the direction of the centre of this circle from the origin of the coordinate represents the deviation of the vertical at that observation station referred to the normal to the locally undulated geoid around the station. Hereafter we will refer to this circle as the " dip circle ".

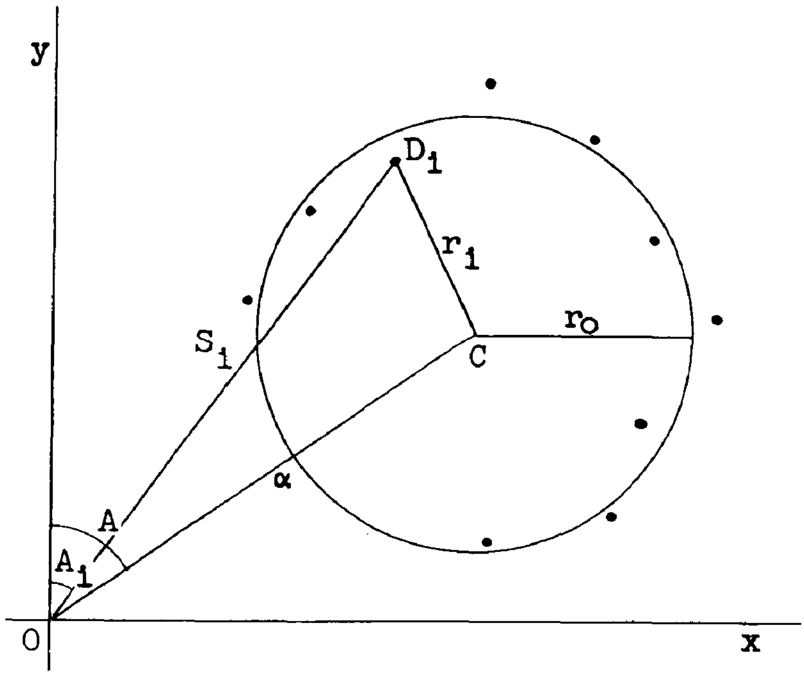

FIG. 4. — Dip Circle.

In practice, a dip circle may be drawn in such a way that it passes as close as possible to each dot. This circle can be expressed by the formula :

$$\Sigma_i \, w_i \, (r_i - r_0)^2 = \text{min.}$$

where r_0, r_i and w_i denote, respectively, the radius of the dip circle, the distance between the centre of the dip circle and the i-th dot (S_i, A_i), and the weight affecting the measurement. We now take rectangular coordinates whose Y-axis is directed northwards, and whose X-axis is perpendicular to the Y-axis and towards the east.

We have :

$$x_i = S_i \sin A_i \qquad y_i = S_i \cos A_i \tag{4}$$

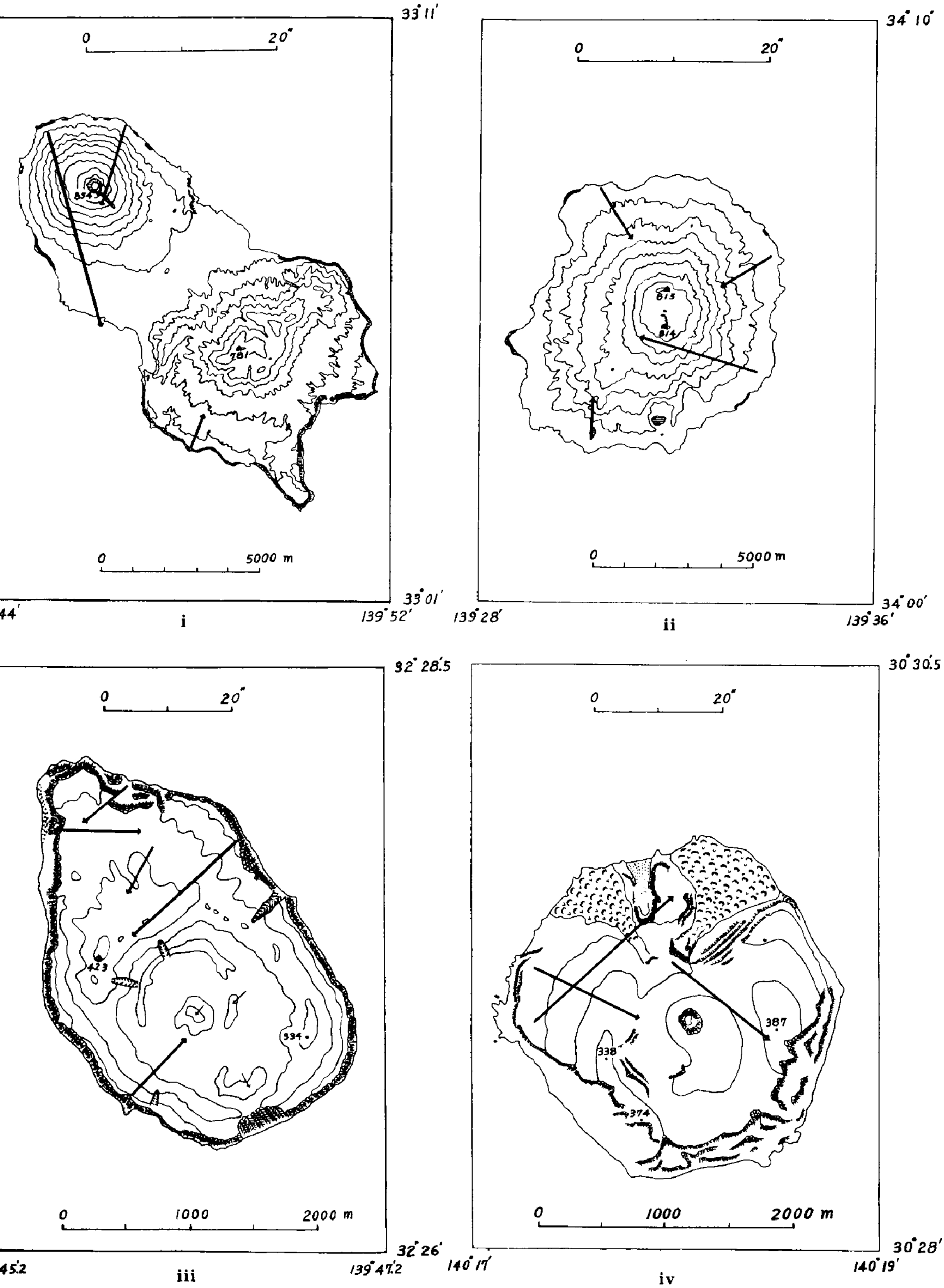

Fig. 5. — Vertical Deflection and Topography.

i. Hatizyo Sima
ii. Miyake Sima
iii. Ao-ga Sima
iv. Tori Sima

TABLE 4

Results of Observations

I. — Hatizyo Sima

No.	Obs. spot	Date J.S.T.	α	p.e.	A	p.e.	Te	Wind Sp.	Wind Dir.
	Sueyosi λ 139°51′30″E	1962 Mar.	″	″	°	°	°C	m/s	
	Light φ 33 04 32 N								
I-1		11^d $9^h.7$	19.0 ±	4.0	243 ±	7	14.9 ~ 16.3	6 ~ 7	WSW
3	h 84^m	11 13.5	18.2	3.2	276	6	15.8 ~ 16.8	6 ~ 8	SW
4		13 8.7	1.7	2.1	132	46	16.2 ~ 18.0	7 ~ 8	W
14	Z 17° ~ 207°	17 16.2	6.2	1.8	249	14	10.5 ~ 11.6	10 ~ 12	SW
15		18 9.1	6.2	2.8	286	13	10.8 ~ 11.8	3 ~ 6	WSW
16		18 10.2	7.9	2.5	318	15	11.5 ~ 12.5	3 ~ 5	WSW
17		18 11.4	18.1	4.5	335	15	12.0 ~ 13.0	4 ~ 5	WSW
18		18 13.7	4.3	1.6	101	11	12.1 ~ 12.9	5 ~ 6	WNW
19		18 16.4	4.4	1.5	37	32	11.7 ~ 12.5		
20		18 15.2	1.8	3.1	285	50	10.6 ~ 11.8		
21		18 16.0	9.3	1.2	222	11	10.2 ~ 11.1		
		Mean	4.0 ±	1.2	259 ±	15			
	Ototiyo- λ 139°47′58″E								
	Hama φ 33 03 32 N								
I-5		13^d $13^h.9$	5.5 ±	2.8	121 ±	29	17.4 ~ 19.0	5 ~ 6	W
6	h 40^m	13 15.4	16.9	1.2	112	8	16.6 ~ 17.3	4 ~ 8	WSW
12		17 10.6	7.7	1.3	246	10	11.3 ~ 12.6	3 ~ 5	W
13	Z 12° ~ 300°	17 11.6	0.4	3.9	359	7	11.5 ~ 12.0	6 ~ 9	W
		Mean	3.6 ±	0.9	20 ±	32			
	Aka- λ 139°46′38″E								
	Saki φ 33 09 12 N								
I-8		16^d $11^h.6$	11.7 ±	1.5	235 ±	8	13.0 ~ 14.8	6 ~ 8	W
9	h 61^m	16 13.2	4.9	3.8	181	27	13.4 ~ 15.4	6 ~ 7	W
10		16 14.5	11.7 ±	1.0	159	4	14.4 ~ 15.7	6	SW
	Z 279° ~ 119°								
		Mean	8.8 ±	0.9	197 ±	22			
	Ōgoe- λ 139°45′02″E								
	Hana φ 33 09 02 N								
I-11		16^d $16^h.5$	21.1 ±	2.5	164 ±	5	12.0 ~ 13.3	8 ~ 10	SW
	h 84^m								
	Z 224° ~ 62°								

and for the centre of the circle

$$x_0 = \alpha \sin A \qquad y_0 = \alpha \cos A \tag{5}$$

α and A being, respectively, the amount and the direction of vertical deflection required.

If the observation is made at a station of sufficient height, the amount of dip will be sensibly larger than that of vertical deflection, i.e. x_i, $y_i \gg x_0$, y_0. Hence,

$$r_i - r_0 = S_i - \sin A_i \, . \, x_0 - \cos A_i \, . \, y_0 - r_0 \tag{6}$$

TABLE 4
Results of Observations
II. — Miyake Sima

No.	Obs. spot		Date J.S.T.	α p.e.	A p.e.	Te	Wind Sp.	Wind Dir.
	Oki-no-Taira	λ 139°33′39″E	1963 Mar.	″ ″	° °	°C	m/s	
		φ 34 03 58 N						
II-3			15^d $9^h.8$	12.6 ± 2.2	279 ± 7	11.8 ~ 14.7	2.5	S
4		h 45^m	15 11.8	12.6 ± 2.2	294 ± 8	17.8 ~ 20.6	3.3	WSW
		Z 17° ~ 211°						
			Mean	12.6 ± 1.5	287 ± 5			
	Akaba-Gyō	λ 139°33′58″E						
		φ 34 05 58 N						
II-5			15^d $15^h.6$	6.0 ± 2.0	238 ± 16	11.8 ~ 15.9	3.8	SW
		h 40^m						
		Z 330° ~ 180°						
	Sin-mio	λ 139°30′16″E						
		φ 34 02 48 N						
II-7			17^d $10^h.3$	4.6 ± 0.8	3 ± 8	11.0 ~ 13.1	5.8	E
		h 50^m						
		Z 99° ~ 285°						
	Izu	λ 139°30′29″E						
		φ 34 07 07 N						
II-8			17^d $13^h.8$	8.8 ± 1.3	145 ± 7	12.0 ~ 13.0	6.3	ESE
9		h 40^m	17 15.1	2.4 ± 1.5	150 ± 31	11.2 ~ 12.6	4.2	E
		Z 232° ~ 75°						
			Mean	6.5 ± 1.0	145 ± 8			

For convenience, for r_0, we may take an approximate value q_0, with its correction q, namely :

$$r_0 = q_0 + q \tag{7}$$

and put

$$m_i = S_i - q_0 \tag{8}$$

which is a known quantity. We then obtain the observation equations :

$$\sin A_i . x_0 + \cos A_i . y_0 + q - m_i = 0 \tag{9}$$

By means of the method of least squares we can evaluate the most probable values for x_0, y_0 and q. Hence, employing expression (5), the amount and the direction of vertical deflection can be obtained.

Table 4 gives the results of observations; reading from left to right the columns give the name of the observation station and its geographical position and height, the observed field of the horizon, the date of the observation, the magnitude and azimuth of vertical deflection with probable errors, the air temperature, the wind speed and wind direction. Except in the case of Ao-ga Sima, the last two quantities have been adopted from the data supplied by the meteorological observatories on the respective

TABLE 4

Results of Observations

III. — Ao-ga Sima

No.	Obs. spot		Date J.S.T.	α p.e.	A p.e.	Te	Wind Sp.	Wind Dir.
	Miko-no-	λ 139°45′53″E	1963 Aug.	″ ″	° °			
	Ura	φ 32 27 59 N						
III-1			23^{d} $11^{h}.5$	9.4 ± 0.6	234 ± 4			E
2		h 58^{m}	23 16.0	10.3 2.0	209 9			E
		Z 310° ~ 98°						
			Mean	9.5 ± 0.6	231 ± 4			
	Naka-	λ 139°46′02″E						
	hara	φ 32 27 44 N						
III-3			25^{d} $11^{h}.5$	6.2 ± 2.5	208 ± 21			SE
4		h 282^{m}	26 11.5	10.2 3.0	217 15			SE
5			26 16.0	7.8 1.9	210 20			SE
		Z 312° ~ 117°						
			Mean	8.5 ± 2.0	208 ± 11			
	Sanbō	λ 139°45′55″E	1963 Sep.					
		φ 36 26 40 N						
III-6			3^{d} $14^{h}.3$	12.2 ± 1.6	43 ± 7			
		h 48^{m}						
		Z 124° ~ 238°						
	Nisi-	λ 139°45′31″E						
	Ura	φ 32 27 49 N						
III-7			4^{d} $10^{h}.5$	13.5 ± 2.6	92 ± 5			SW
		h 81^{m}						
		Z 200° ~ 350°						
	Uti-	λ 139°46′26″						
	Ura	φ 32 27 45						
III-8			6^{d} $15^{h}.5$	14.0 ± 1.1	242 ± 4			SW
9		h 48^{m}	17.0	25.4 0.5	223 2			SW
		Z 333° ~ 123°						
			Mean	22.3 ± 0.5	227 ± 2			

islands. As can be seen in fig. 3, the standard deviation of an observed value of dip for a single direction in relation to its dip circle is 3″ to 5″. Fig. 5 shows the topography of the 4 islands, vertical deflection being indicated by an arrow positive towards the nadir. The scale for deflection is given in the upper part of each figure. It is evident from these data that, in most cases, the nadir of the vertical points to the central part of the island, or, in any case, points inland.

TABLE 4

Results of Observations

IV. — Tori Sima

No.	Obs. spot	Date J.S.T.	α p.e.	A p.e.	Te	Wind Sp.	Wind Dir.
	λ 140°18′00″E	1964 Jul.	″ ″	° °	°C	m/s	
	φ 30 29 14 N						
IV-1		$1^d\ 16^h.8$	18.4 ± 1.3	131 ± 4	26.6 ~ 28.4	2.5	SSE
2	h 195^m	17.8	20.6 1.7	124 5	26.8 ~ 29.4	2.5	SSE
	Z 151° ~ 86°						
		Mean	19.3 ± 1.5	128 ± 5			
	Meteorol. Obs. λ 140°17′20″E						
	φ 30 28 58 N						
IV-3		$2^d\ 9^h.0$	29.8 ± 1.0	48 ± 2	27	2.5	SW
	h 81^m						
	Z 157° ~ 359°						
	λ 140°17′21″E						
	φ 30 29 11 N						
IV-4		$6^d\ 15^h.4$	18.3 ± 1.0	114 ± 2	28.8 ~ 29.8	2.5	S
	h 60^m						
	Z 205° ~ 45°						
	λ 140°18′26″E						
	φ 30 29 19 N						
IV-5		$22^d\ 15^h.8$	0.7 ± 0.3	129 ± 16	28.2 ~ 28.8	2.5	SSW
	h 180^m						
	Z 5° ~ 165°						

5. — DISCUSSION

Prior to discussing vertical deflection in connection with the attraction of the island mass, we must consider other possibilities which can seemingly cause an equivalent effect on the vertical deflection through the processes just described. These possibilities are abnormal refraction, and the effect of wind on the theodolite.

If the island is mantled by air layers or an air mass, where meteorological conditions are different from those above the sea surface, then abnormal refraction will occur when light rays pass through the boundary between the air mass and the general atmospheric layer. This may occasionally result in phenomena consisting of an apparent vertical deflection whose nadir points either outwards from or inwards toward the island. Due to radiation from the island's surface, temperatures in the air mass will, in most cases, be higher than in the general layers. For simplicity, we may suppose the form of island to be conical. At the boundary of the warmer air mantle, the incident angle of a light ray from the horizon to the observation station along the line between the observed horizon and the centre of the island is larger than the incident angle of another light ray in a perpendicular direction. This makes the plane defined by the dip circle tilt outwards, resulting in apparent deviation of the vertical with the nadir pointing in the opposite direction to the centre of the island. However, this abnormal refraction effect seems to be insignificant, and, in general, we may ignore the existence of such an air mantle when interpreting the observational data shown in Table 4. Furthermore, this effect can be avoided by making observations in strong winds, in which case air masses of different temperatures are blown away. Strong winds, however, disturb dip observations, as will be described later.

We now also have to examine the effect of both the irregular distribution of temperature at the sea surface and of changes of air temperature with time. From Tables 1 and 4 we see that in the present observations the distances to the horizon from the observation stations range from 25 to 65 km, and most are between 25 and 40 km. In fig. 2 we see that, for each set of dip observations, it is difficult in practice to expect a difference in surface temperature of less than 1 °C at each observed point on a horizon whose field is greater than 180° in azimuth. Moreover, the change of air temperature with time is, in most cases, more than 2 °C (see Table 4). In Section 2 we found that a difference of 1 °C in water- or air-temperature resulted in a difference of about 10″ in dip, adopting the classical formula (3), with $b = 0\overset{\prime\prime}{.}20$. Therefore, the above difference in surface temperature or changes in air temperature with time are likely to disturb the dip observation by an amount of 10″ or more. Since this value is about the same as the amount obtained for the vertical deflection itself, the dip observation data would seem to have been completely disturbed.

Nevertheless, fig. 5 clearly shows an inward tendency for the vertical deflection, and the standard deviation of dip in a single direction is only about 3″ to 5″. Neither the influence of irregular distribution of surface temperatures nor the effect of changes of air temperature with time are to be found. We might, therefore, infer that the amount of dip is practically insensitive to changes in temperature with time and space over short time intervals.

On the other hand, it is a fact that the value of dip in one direction varies in relation to changes in air temperature at the observation station, when the observation lasts 2 or 3 hours, or, in other words, that a diurnal variation of dip is clearly recognizable. One might expect this to be a natural conclusion arising from expression (3). In fact, we obtain a fairly accurate value for the coefficient of temperature difference from such an observation. However, this value is not applicable to an observation made on another day. The values of the coefficient b determined by many authors (e.g. those shown in Table 3) were derived from statistical analysis of various observational data obtained under various conditions. Further observations for investigating this effect are much needed, in order to clarify the phenomena of insensitivity of dip, mentioned above.

As can be seen in Table 4, strong westerly monsoons are predominant in Japan throughout the winter. During the observations in March it was frequently found that the bubble of the theodolite level moved in the direction from which wind was blowing. If the observation is made on the island's west coast, the matter is worse, for the monsoon blows more heavily than on the east coast. A tilt of the theodolite towards the interior of the island causes an apparent deviation of the vertical with the nadir pointing towards the centre of island. This effect amounts to about 2″ or 3″. However, this can be avoided by making observations carefully.

We can now consider the effect of the attraction of the island, i.e. the gravity anomaly, due to submarine geological features around the island. For simplicity, let us assume that the mass under the island has the same effect as if it were concentrated at a point called " point mass ". It should be remarked that the magnitude and azimuth of vertical deflection from dip observations do not supply the location and the magnitude of this mass, but merely yield a relation between these two quantities, because vertical deflection indicates the direction of the resultant vector of gravity and the attraction of the local mass. The absolute value of the resultant vector can be obtained by combining the data from the gravity observations.

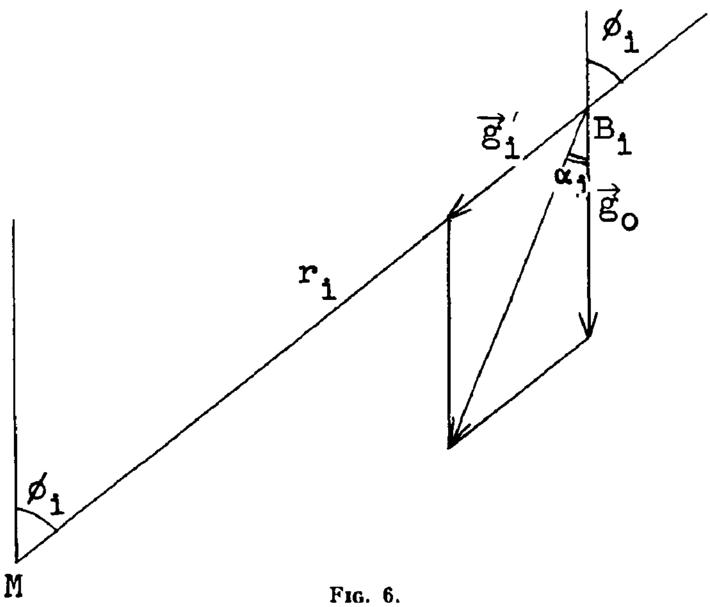

FIG. 6.

Let x_0, y_0, z_0 be the position of the point mass M, where z is measured from mean sea level downwards and y is taken towards the north. (See fig. 6). The components of the sum of the attractive force g' exerted by the point mass and the force g_0 exerted by the earth itself on a unit mass are :

$$X_i = -\frac{GM(x_i - x_0)}{r_i^3} \tag{10}$$

$$Y_i = -\frac{GM(y_i - y_0)}{r_i^3} \tag{11}$$

$$Z_i = -\frac{GM(z_i - z_0)}{r_i^3} - g_0 \tag{12}$$

where x_i, y_i, z_i are the coordinates of the i-th observation station B_i, g_0 the gravity acceleration, G the constant of gravitation, and r_i the distance between the point mass M and the station B_i.

$$r_i^2 = (x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2 \tag{13}$$

As shown in fig. 6, the deflection of the vertical α_i at the observation station B_i is the angle between the vectors $\vec{g}_0$ and $\vec{g}_0 + \vec{g}_i'$. Thus, geometrically, we have

$$\sin \alpha_i = \frac{|g_i'|}{|g_0|} \sin(\Phi_i + \alpha_i) \tag{14}$$

where Φ_i is the angle between vectors $\vec{g}_0$ and $\vec{g}_i'$ and is expressed by

$$\sin \Phi_i = \frac{\sqrt{X_i^2 + Y_i^2}}{g_i'} \tag{15}$$

Since α_i is smaller than the unit and hence sufficiently small relative to Φ_i, expression (14) reduces to

$$\alpha_i = \frac{|g_i'|}{|g_0|} \sin \Phi_i$$

which, taking expression (15) into account, again reduces to

$$\alpha_i = \frac{\sqrt{X_i^2 + Y_i^2}}{g_0}$$

and, according to (10) and (11),

$$\alpha_i = \frac{GM}{g_0} \cdot \frac{\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}}{r_i^3} \tag{16}$$

On the other hand, it is obvious that the line of attractive force $\vec{g}_i'$ projected on the x-y plane is coincident with the direction of the vertical deflection. Therefore, we have the relation :

$$X_i \cos A_i - Y_i \sin A_i = 0$$

or

$$(x_0 - x_i) \cos A_i + (y_0 - y_i) \sin A_i = 0 \tag{17}$$

where A_i is the azimuth of the vertical deflection measured eastwardly from the north.

From a set of observed A_i values, the horizontal position (x_0, y_0) of the mass can be obtained from expression (17). Then, by using equation (16) the α_i observed values give the relation between M and r.

To take an example, the following is the procedure for the data obtained at Miyake Sima and shown in Table 4-II. For horizontal coordinates we employed the UTM grid of Japan, taking as origin a grid point close to the centre of the island, and shown by a black dot in fig. 7.

Using (17) and the method of least squares, the horizontal coordinates of the point mass are determined as

$$\begin{aligned} &\qquad\text{(p.e.)} \\ x_0 &= -\ 0.8 \pm 0.3 \\ y_0 &= -\ 0.5 \pm 0.6 \end{aligned} \qquad \text{in km}$$

in km, and these are the coordinates of point C in fig. 7. We can assume each value of z_i to be zero, since all the observation stations are at a sufficiently low height. Hence from expression (16)

$$r^3 = 1.00 \times 10^{-15}\ \text{M}$$
$$\pm\ 0.14$$

or, from (13)

$$(z_0^2 + 17.6)^{3/2} = 1.00 \times 10^{-15}\ \text{M}$$
$$\pm\ 1.8 \qquad\qquad \pm\ 0.14$$

where M is in gr and r and z_0 in km. To take an example,
$\text{M} = 1.37 \times 10^{17}$gr for $z = 3$ km, and $\text{M} = 1.27 \times 10^{18}$gr for $z = 10$ km.

If we replace this point mass by a homogeneous sphere of radius R, its mass is

$$\text{M} = \frac{4}{3}\,\pi\,\text{R}^3\,\Delta\rho \tag{18}$$

where $\Delta\rho$ is the density difference between the sphere and its surrounding layer. Further, we assume the magnitudes of R and r to be about identical. Then, from expression (16), we obtain

$$\Delta\rho = \frac{3g_0}{4\pi\text{G}}\,\frac{\alpha}{\sqrt{(x - x_0)^2 + (y - y_0)^2}} \tag{19}$$

Applying the respective mean values for the α_i's and for $\sqrt{(x_i - x_0)^2 + (y - y_0)^2}$ that were adopted in the above determination of the $z_0 \sim$ M relation, we obtain

$$\Delta\rho = 0.28 \pm 0.04\ \text{gr/cm}^3$$

This is a very suitable value for andesite which is predominant in these volcanic islands.

6. — COMPARISON WITH ASTRONOMICAL OBSERVATIONS

Astronomical observations of longitude and latitude are now in progress along the Izu Syoto (Izu Islands) (Suzuki and Harada, 1966, Suzuki and Sugimoto, 1967) employing a Tsubokawa photoelectric Astrolabe (Tsubokawa, 1959 and 1963). The observational data for Miyake Sima (shown in table 5) can be compared with the data of vertical deflection

Table 5

Vertical Deflection from Astronomical Observations at Miyake Sima

ξ : Meridian, η : Prime Vertical

Station	Elev.	Epoch	Geodetic Lat. Long.	Astro-Geod. ξ η p.e.	Total
		1965			
Satado	25^m	Mar. 13	+ 34°05′23″.1 − 139 34 06.4	+ 17″.7 ± 0″.3 + 5.3 0.3	18″.5 16°.7
Ako	4	Mar. 18	+ 34 04 34.2 − 139 28 59.3	+ 13.3 ± 0.6 − 13.7 0.4	19.1 314.2
Meteor. Obs.	54	Mar. 20	+ 34 07 15.0 − 139 31 31.0	+ 21.0 ± 0.6 − 6.5 0.6	22.0 342.8

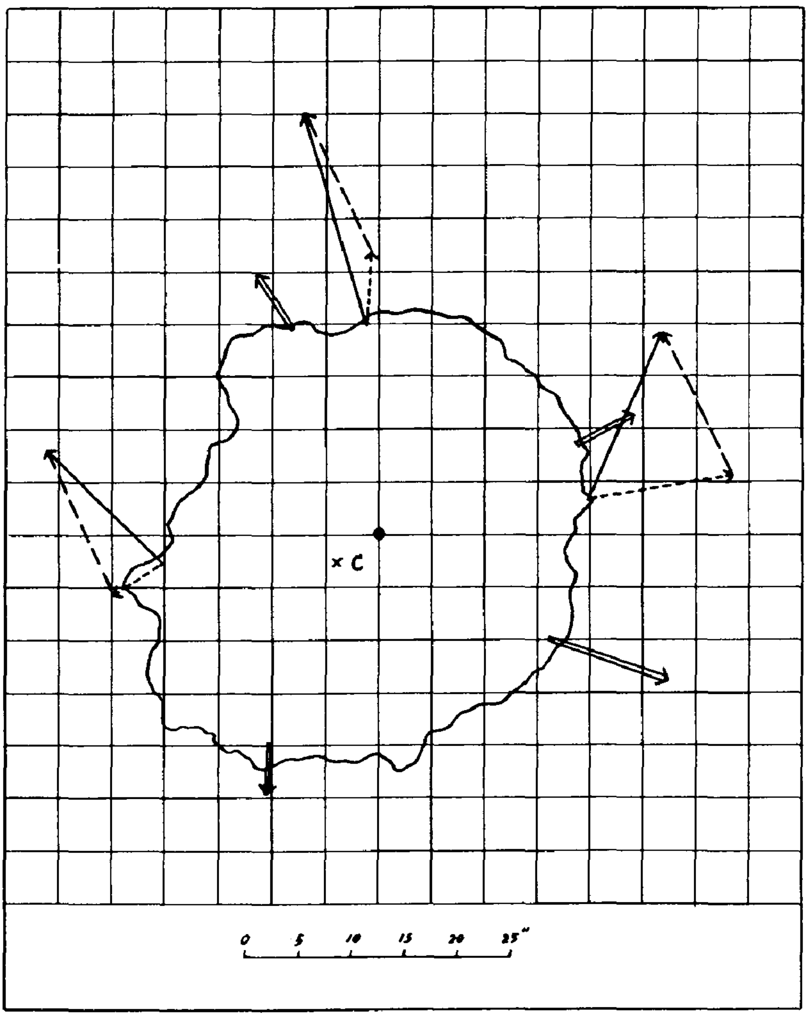

Fig. 7. — Comparison of Vertical Deflections from Dip and those from Astronomical Observations.

from dip observation, although, for most of the islands, the number of astronomical observations is insufficient for such a comparison.

In fig. 7, solid-line vectors show the observed deflection of the vertical, i.e. Astro.-Geod. It has been suggested that the geodetic datum of Japan should be amended by — 20″ in longitude and — 10″ in latitude (Hirose, 1948 and 1959; Omori, 1956; Dambara, 1963). The general northward trend of the solid-line vectors may be attributed partly to this datum error and partly to any local effects peculiar to the island.

Each of the dashed-line vectors in fig. 7 shows the rough mean of solid-line vectors. Dotted-line vectors are obtained by subtracting the above common mean vector from the respective solid-line vectors. Hence, we may assume that each of these dotted-line vectors indicates the vertical deflection proper to each observation station. For comparison, vectors from dip observations (fig. 5-ii) have been shown by a double-line, the direction to the zenith being taken as positive.

We find that the tendency of the dotted-line vectors is fairly similar to that of the double-line vectors both in magnitude and in azimuth. We may therefore assert that the vertical deflection from dip observations suffers little from the effect of abnormal refraction. It may also be possible to estimate the character of datum errors by comparing vertical deflections from dip observations with those obtained from astronomical and geodetic surveys.

7. — CONCLUSION

Vertical deflections obtained from dip observations agree well with those of the classical process (Astro-Geod.). This suggests that the effect of abnormal refraction on dip observations can be ignored, and consequently we can assert that the procedure for dip observations is relevant for the determination of local vertical deflection.

We can evaluate the horizontal coordinates of the point mass as well as the relation between the mass and its depth. By replacing this point mass by a homogeneous sphere, the density of this sphere can be obtained. The resulting value of the density of the island fits well with its geological structure.

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