A NEW TECHNIQUE
FOR ECHO SOUNDING CORRECTIONS

by T.V. Ryan and P.J. Grim
Environmental Science Services Administration,
Pacific Oceanographic Research Laboratory,
Seattle, Washington.

The argument that reasonably complete maps of the oceans are the first requisite for studies leading to an understanding of oceanic structures and processes was advanced by the U. S. National Academy of Sciences Committee on Oceanography in 1959 (NASCO 1959). The U. S. Coast and Geodetic Survey (USC&GS) accepted this thesis and as a pilot project in 1961 launched a survey of unparalleled scope in the North Central Pacific. As of January 1967, 33 ship months had produced the areal coverage illustrated in figure (1).

The basic plan, now known as SEAMAP (*S*cientific *E*xploration *A*nd *MA*pping *P*rogram) provides for continuous echo sounding and magnetic and gravity observations on a line spacing of 10 nautical miles (18.52 km) under Loran-C control. Oceanographic station observations (temperature, salinity, oxygen) and geological samples (dredge hauls and cores) are also being obtained. In addition to the basic plan, many additional observations and special investigations have been carried out concurrently by Environmental Science Service Administration (ESSA) scientists and by other research laboratories. These supplementary programs include studies of deep currents, light transmissivity, biological populations and primary productivity, time changes in properties, bottom photography, natural and man-induced radio-activity, etc.

In view of the immense quantity of echo soundings which were to be obtained from the survey, it was apparent that a computerized technique was essential for processing position data and correcting the soundings for sound velocity. With a computer (IBM Model 1620) committed to the task, and the relatively large volume of oceanographic station data obtained during the survey, it was felt that an improved method for correcting echo soundings could be divised.

Three prominent factors enter into the problem : (1) The basic relationship among the independent variables, i.e., temperature, salinity and pressure, and the dependent speed of sound; (2) The approximations used to describe the speed of sound in the specific water volume in which the echo soundings are made (it should be noted that the speed of sound usually

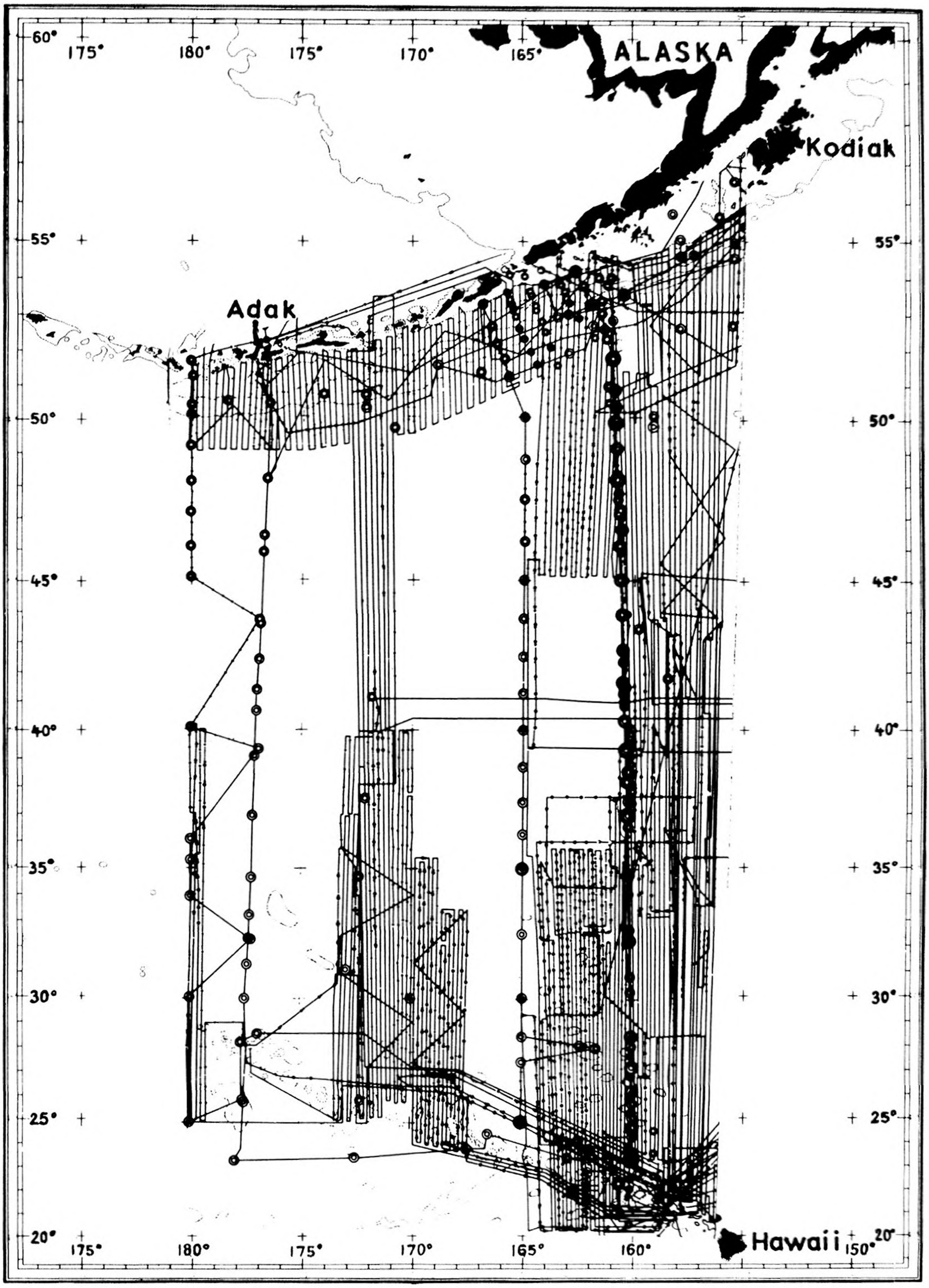

FIG. 1. — North Pacific SEAMAP Surveys.

varies in a non-linear fashion with latitude, longitude, depth and time); and (3) The computational technique one uses to compute the correction and apply it.

1. — THE BASIC RELATIONSHIP

Modern studies and field tests (DEL GROSSO, 1952; BYERS, 1954; MCKENZIE, 1960; WILSON, 1960) argue that the well known tables and equations which were developed by MATTHEWS (1939) and KUWAHARA (1939) from theoretical considerations relating temperature, salinity, and pressure to sound velocity (*), although remarkably accurate, are systematically different from the results of empirical studies. BYERS (1954) attributes the discrepancy to an error in the published values for the isothermal compressibility of water, a factor which both MATTHEWS and KUWAHARA recognized as a weak point in their calculations. The values used by MATTHEWS and KUWAHARA for compressibility are based on measurements made in 1893

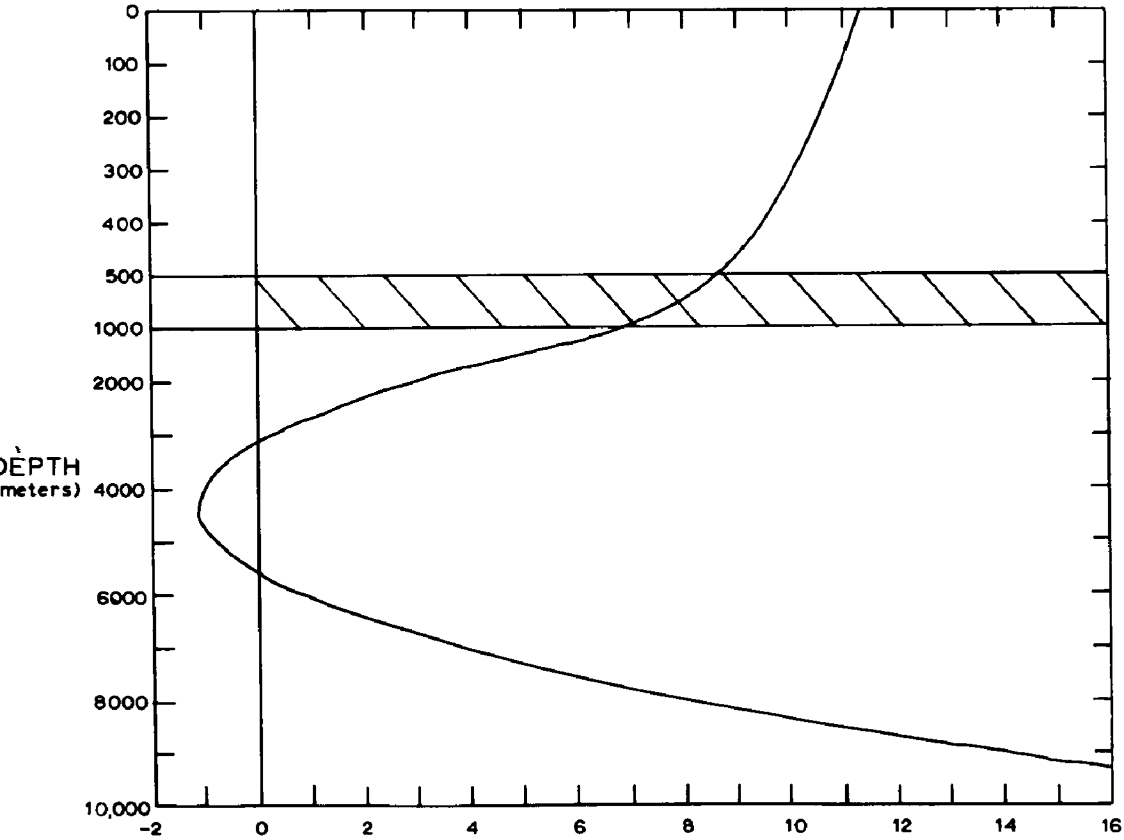

FIG. 2. — Velocity difference (ft/sec), Wilson minus Kuwahara. Comparison of sound velocity values, Surface to 9 000 metres, given by Wilson's equation and Kuwahara's tables (from SOWER, 1961).

(*) Popular usage favors the term 'velocity' for 'speed' of sound although the value is a scalor not vector quantity. In accordance with the custom the term 'velocity' will be used hereafter instead of 'speed'.

and 1908. Figure 2 from SOWER (1961), presents a comparison of KUWAHARA's velocities (which are essentially identical with those of Matthews) with values computed from the WILSON (1960) equation for a typical oceanographic condition. Acceptance of Wilson's equation by marine scientists was confirmed when in 1963 the U.S. National Oceanographic Data Center replaced Kuwahara's equation with Wilson's equation for computing the velocity of sound from oceanographic station data. Thus with regard to the basic relationship among the independent variables, we decided that the system must based on Wilson's equation (1960).

2. — CLASSIFICATION OF THE ENVIRONMENT

MATTHEWS' tables (1939) divide the world's oceans into 52 areas on the basis of echo sounding conditions. It is not evident what criteria MATTHEWS used for drawing the area boundaries, but is certain that in 1939 he had only a small fraction of the oceanographic station data which are available today for evaluating the acoustic characteristics of the ocean. In the SEAMAP area the C&GS has occupied 151 stations well distributed with regard to space and the period of the survey. With this relatively dense and appropriate body of data available, we chose to use it exclusively to study the sound velocity structure of the region. The spacial and temporal variability in Mean Vertical Sound Velocity (*) (MVSV), (computed by a method to be described later) throughout the SEAMAP region was examined by plotting geographically the Mean Vertical Sound Velocity, at each of the 151 oceanographic stations, to 200, 1 000 and 5 000 metres. These depths were chosen to provide a basis for (a) evaluating the maximum effect of seasonal variability (200 metre level), (b) representing a depth of a large portion of the soundings (5 000 metres) and (c) providing a look at an intermediate depth (1 000 metres). The plotted values were then contoured. As expected from the known physical characteristics of the North Pacific, the contours were found to have a decided east-west trend. The region was then divided into a sufficient number of latitudinal zones with boundaries such that the variability in each zone, resulting from geophysical and seasonal changes, did not exceed an acceptable tolerance. The tolerance (due to both areal and seasonal effects) deemed acceptable was 4 metres at 200 metres (fathometer resolution is about 3.6 m), and 0.5 % of depth at depths greater than 1 000 metres. The zone boundaries and the Mean Vertical Sound Velocity, in terms of "R", which is the Mean Vertical Sound Velocity divided by the fathometer velocity (**) at 200, 1 000 and 5 000 metres are illustrated in figures 3, 4 and 5. To reduce clutter, 1.0000 has been subtracted from "R" and the decimal point omitted. The "R"

(*) Mean Vertical Sound Velocity (MVSV) is the mean velocity of the sound wave from the ocean surface to the depth of interest, assuming the wave travels vertically downward, and the velocity changes in a linear fashion within each layer.

(**) Fathometer Velocity is the assumed velocity of sound in sea water used by the instrument manufacturer when building the readout device. The USC&GS fathometers use 1 463.43 m/sec (800 fms/sec) as a fathometer velocity. See 'Fathometer Depth' below.

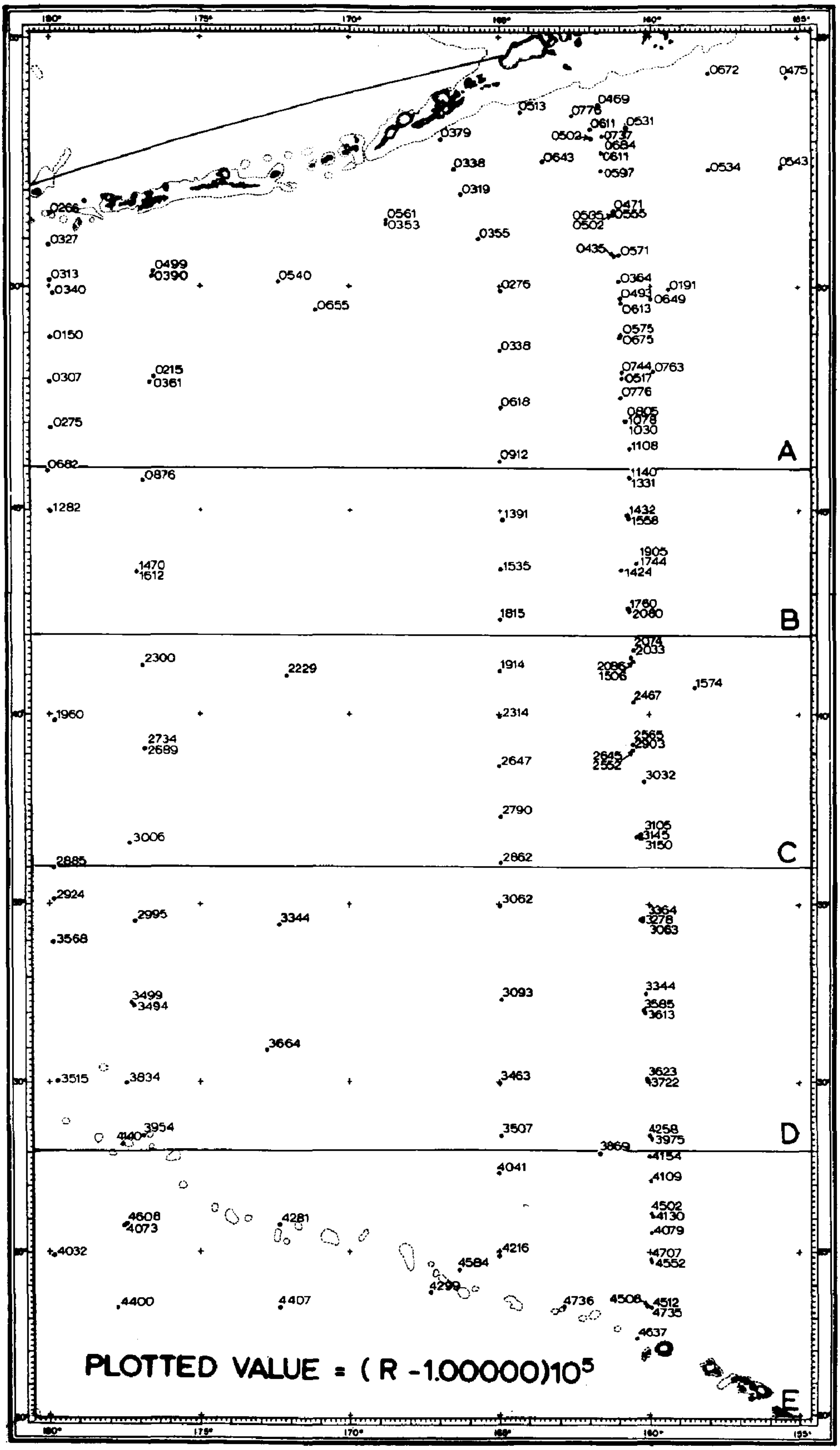

Fig. 3. — Ratio R for 0 to 200 metres.

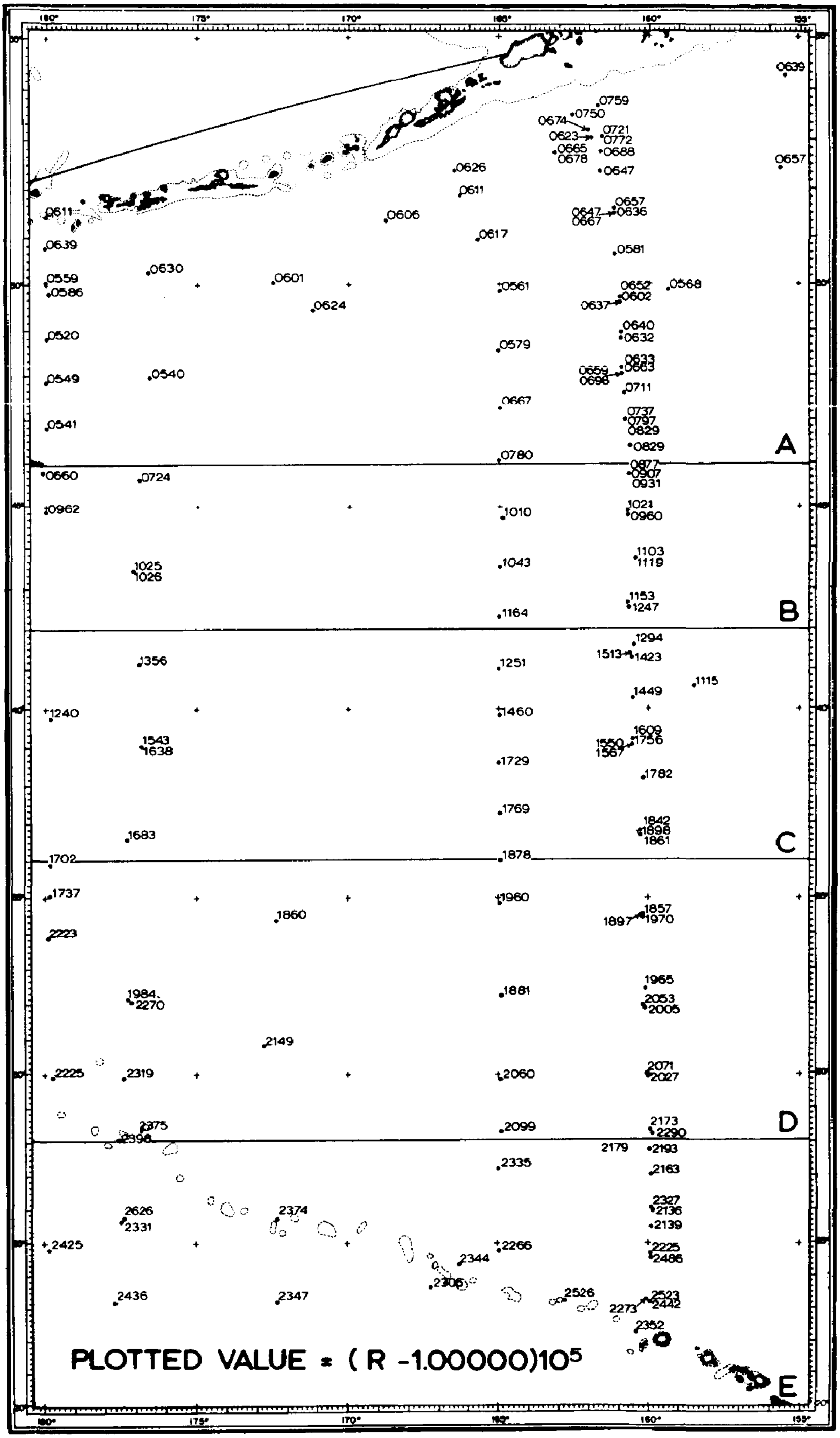

FIG. 4. — Ratio R for 0 to 1 000 metres.

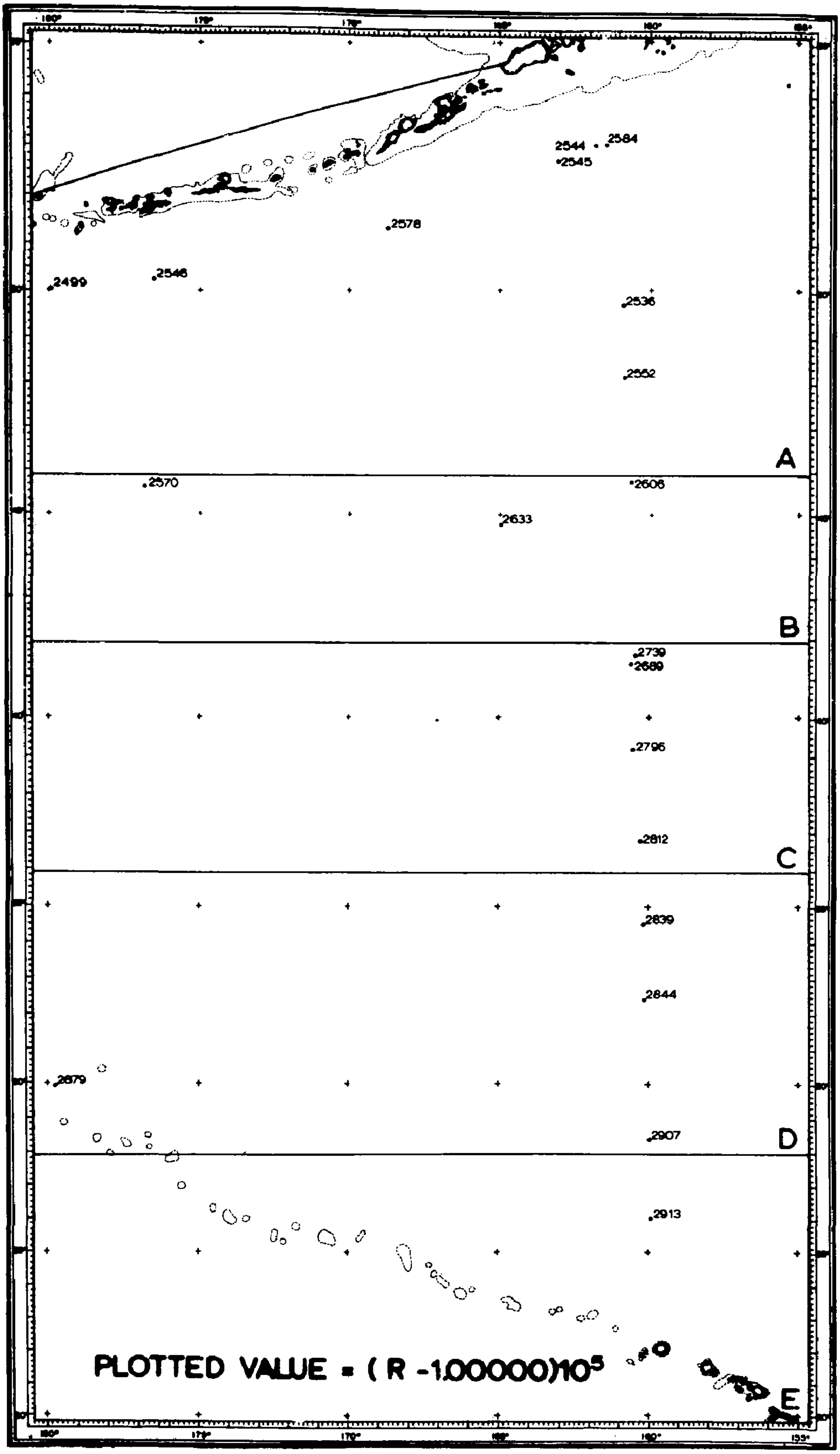

FIG. 5. — Ratio R for 0 to 5 000 metres.

value, therefore, shows the proportional error in the "raw" fathometer depth (*). For example, in the upper right corner of figure 3, the value 0475 means that a fathometer reading of 200 m would be 0.475 % too shallow; at the same location at 1 000 m we learn from figure 4 that the reading is 0.639 % too shallow; and from figure 5 we see it would be 2.534 % too shallow at 5 000 m.

Our original intent was to multiply the raw soundings by the appropriate "R" values, to yield the corrected sounding. Difficulties in fitting an equation to the plot of "R" versus depth, forced us to develop an alternate approach which will be discussed later. Having found that the variability was such that the SEAMAP region could be divided into 5 zones, each with an acceptably low variability in "R", the problem remained to characterize each zone by a single Mean Vertical Sound Velocity curve.

3. — THE COMPUTATION OF MEAN VERTICAL SOUND VELOCITY AND APPLICATION OF THE SOUND VELOCITY CORRECTION

The oceanographic station data "detail" punch cards, prepared routinely by the U. S. National Oceanographic Data Center (NODC), list sound velocity computed from Wilson's equation for each oceanographic standard depth (**). A program (identified as VEL I) was written to compute certain statistical properties of the sound velocity for each oceanographic standard depth based on all stations lying within each zone : The statistical values computed are the lowest velocity, the average velocity, the highest velocity, the standard deviation of the velocities from the average value and the number of observations used in the calculations. The number of stations available in each zone is fixed; however, as the stations sample to various depths, the number of available observations decreases with increasing depth.

A reproduction of the output of this program is shown in the appendix. The average velocities computed by this program for the oceanographic standard depths comprise the data which are used to compute the Mean Vertical Sound Velocity for the zone in which the echo soundings are made. The maximum and minimum values found within each zone at each depth are used to compute the maximum error in the sounding correction which would result from natural variability in the zone. To provide sound velocity data for depths which exceed the depths of the oceanographic stations (in all zones stations to at least 5 000 metres were available) values of

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(*) Fathometer Depth is the depth scaled from the fathometer. It is a product of 1/2 the travel time for the sound wave from the surface to bottom to surface, times the velocity assumed by the instrument manufacturer.

(**) Oceanographic Standard Depth. These are depths at which the oceanographic parametres, measured and derived, are routinely listed by most oceanographic laboratories When necessary, the independent variable is determined at the standard depth by an interpolation formula. The oceanographic standard depths used by the National Oceanographic Data Center are 0, 10, 20, 30, 50, 75, 100, 125, 150, 200, 250, 300, 400, 500, 600, 700, 800, 900, 1 000, 1 100, 1 200, 1 300, 1 400, 1 500, 1 750, 2 000, 2 500, 3 000, and at 1 000 m intervals to bottom.

RESULTS (PARTIAL) FROM VEL I PROGRAM FOR ZONE B

DEPTH IN M.	LOW VELOCITY	AVERAGE VELOCITY	HIGH VELOCITY	STANDARD DEVIATION	NUMBER OF OBS.
10.	1486.2	1502.08	1523.2	14.074	10
20.	1486.3	1501.28	1521.5	13.159	10
30.	1486.3	1497.60	1513.9	10.115	10
50.	1484.1	1488.05	1495.2	3.530	10
75.	1477.2	1483.77	1489.5	3.713	10
100.	1476.5	1484.02	1489.5	4.171	10
125.	1476.4	1484.07	1489.1	3.495	10
150.	1477.7	1483.75	1488.6	3.125	10
200.	1480.0	1482.90	1486.6	2.116	10
250.	1477.5	1480.21	1483.1	1.980	10
300.	1475.6	1478.14	1481.5	1.908	10
400.	1473.9	1475.29	1477.6	1.219	10
500.	1473.9	1474.64	1475.8	.619	10
600.	1474.9	1475.23	1475.9	.337	10
700.	1475.5	1476.05	1476.7	.398	10
800.	1476.4	1476.88	1477.3	.286	10

temperature, salinity, and pressure were extrapolated to 9 000 metres. We now have a means of characterizing the environment by a "mean station" and can express quantitatively the maximum error that our approximation can cause.

Given the data from the "mean station", which we assume expresses within allowable tolerances for areal and temporal variability the velocity of sound for each oceanographic standard depth, it is still necessary to compute the effective velocity for the sound wave travelling vertically from the surface to the reflecting bottom. We have called this the Mean Vertical Sound Velocity. Various techniques (*) have been used; however, in essence all divide the water column into a number of layers in each of which a constant, average velocity is assumed to apply. In the real ocean the speed changes continously with depth and thus with a finite number of layers sound is travelling through layers in which the speed is continously changing. A simple calculation will show that the true Mean Vertical Sound Velocity through a layer in which the velocity changes is something less than the arithmetic average of the velocity in the bounding surfaces of the layer. Use of the simple averaging technique thus yields a velocity which is too fast.

With a digital electronic computer it is practical to compute an average speed utilizing the equation $dt = \frac{dz}{v}$ which recognizes the fact that the speed changes in each layer. Integration of the equation over each layer yields the time required to transit the layer. By summing the times and dividing by the total depth, a more accurate Mean Vertical Sound Velocity is obtained. The mathematics of this procedure are given in the appendix.

Given an acceptable method for computing the Mean Vertical Sound Velocity to each oceanographic standard depth, one can compute the fathometer depth which when corrected equals the oceanographic standard depth, by the equation :

$$\text{fathometer depth} = \frac{\text{(fathometer velocity)}}{\text{MVSV}} \text{(oceanographic standard depth)} \qquad (1)$$

The correction to be applied to the fathometer depth is :

$$\text{correction} = \text{oceanographic standard depth} - \text{fathometer depth} \qquad (2)$$

This important point is illustrated as follows :

The fathometer in reality simply records a figure based on one half of the time for the sound wave to reach the sea bottom and return. It expresses this time as its product with the constant fathometer velocity and reads out in fathometer depth. Therefore, the time, t_z, required for the

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(*) MATTHEWS averaged the temperature and salinities from the upper and lower surface of each layer and computed a speed from the 'average' temperatures, salinities, and pressures. Since the speed of sound is not a linear function of temperature and salinity, strictly speaking the simple average of the bounding temperatures and salinities does not yield precise values for determining the average velocity within the layer. If the range in temperatures and salinities across the layer is small, the error due to this simplification is trivial.

sound wave to reach the bottom at depth z, can be computed from the equation :

$$t_z = \frac{\text{fathometer depth}}{\text{fathometer velocity}} \qquad (3)$$

and since

$$\text{true depth} = t_z \, (\text{MVSV}) \qquad (4)$$

by combining equations (3) and (4), we derive

$$\text{true depth} = \frac{\text{fathometer depth}}{\text{fathometer velocity}} \, (\text{MVSV})$$

or rearranging terms,

$$\text{fathometer depth} = \frac{\text{fathometer velocity}}{\text{MVSV}} \, (\text{true depth}) \qquad (5)$$

As oceanographic standard depths are "true" depths, we can solve for the fathometer depth which corresponds to each oceanographic standard depth, and then, by computing the correction at each fathometer depth according to equation (2), produce a table of corrections to be applied to those fathometer depths. Note that this procedure departs from MATTHEWS (and others) who computed the correction from oceanographic data listed at "true" depths and then prepared correction tables which in effect apply the correction to "true" depths. In practice one enters a table (or other correction technique) with a fathometer depth to "look up" the correction. Since fathometer depths are usually too shallow, the correction obtained is for a column of water shallower than the real column. The refinement incorporated by the present method eliminates another very small but systematic error, which varies in different regions but generally results in too large a correction above 1 000 metres and too small a correction below. At 5 000 metres it can amount to 3 metres. A computer program (VEL II) was written to compute the corrections at each fathometer depth corresponding to an oceanographic standard depth, utilizing the accepted Mean Vertical Sound Velocity of each zone. The output from this program is illustrated in the appendix. It includes (1) the correction as computed from the *accepted* Mean Vertical Sound Velocity, and (2), the corrections computed from the maximum and (3), minimum Mean Vertical Sound Velocities found in that zone. Also computed are the ratios of the Mean Vertical Sound Velocity to the fathometer velocity for all three types of Mean Vertical Sound Velocities.

The corrections computed from the sets of maximum and minimum Mean Vertical Sound Velocity for each zone are used to establish the maximum error in the correction that could result within a zone, as a result of the variability in the environment. Figure 6a indicates the standard deviation from the mean sound velocity for all depths for all five zones, and figure 6b illustrates the maximum probable error in the echo sounding correction which one might expect as a result of the variability indicated in figure 6a.

As a result of Program VEL II we have a listing, for each zone, of the corrections to be applied to the fathometer depths which when corrected are the oceanographic standard depths from 10 metres to 9 000 metres.

RESULTS (PARTIAL) FROM VEL II PROGRAM FOR ZONE B

ACCEPTED MVSV			MINIMUM MVSV			MAXIMUM MVSV			TRUE DEPTH
1	2′	3	1	2	3	1	2	3	4
9.740	2.599	1.02668	10	0	1.01583	10	0	1.04112	10
19.481	5.187	1.02662	20	0	1.01650	19	1	1.04084	20
29.238	7.617	1.02605	30	0	1.01630	29	1	1.03968	30
48.839	11.609	1.02377	49	1	1.01586	48	2	1.03512	50
73.454	15.461	1.02105	74	1	1.01458	73	2	1.03005	75
98.089	19.105	1.01948	99	1	1.01334	97	3	1.02703	100
122.668	23.322	1.01901	123	2	1.01260	122	3	1.02522	125
147.308	26.918	1.01827	148	2	1.01210	146	4	1.02399	150
196.624	33.756	1.01717	198	2	1.01179	196	4	1.02219	200
245.999	40.007	1.01626	247	3	1.01158	245	5	1.02073	250
295.454	45.464	1.01539	297	3	1.01119	294	6	1.01947	300
394.526	54.737	1.01387	396	4	1.01039	393	7	1.01741	400
493.699	63.014	1.01276	495	5	1.00980	492	8	1.01580	500
592.868	71.321	1.01203	594	6	1.00948	591	9	1.01477	600

COLUMN IDENTIFICATION:

1. Fathometer depth
2. Correction for sound velocity to fathometer depth
3. Ratio = $\frac{\text{MVSV}}{\text{Fathometer velocity}}$
4. True depth

Columns 1, 2, 4 in metres.
Column 2′ in decimetres.

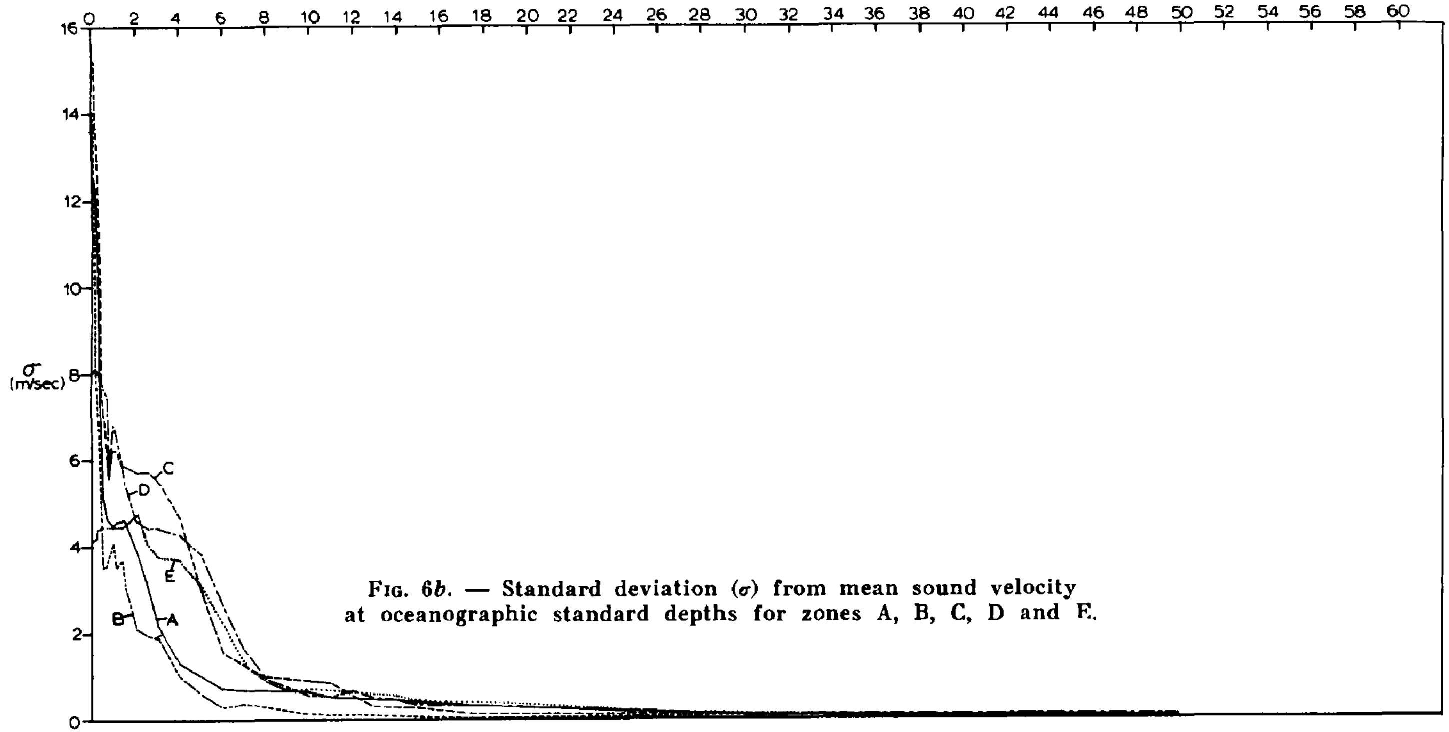


Fig. 6*b*. — Standard deviation (σ) from mean sound velocity at oceanographic standard depths for zones A, B, C, D and E.

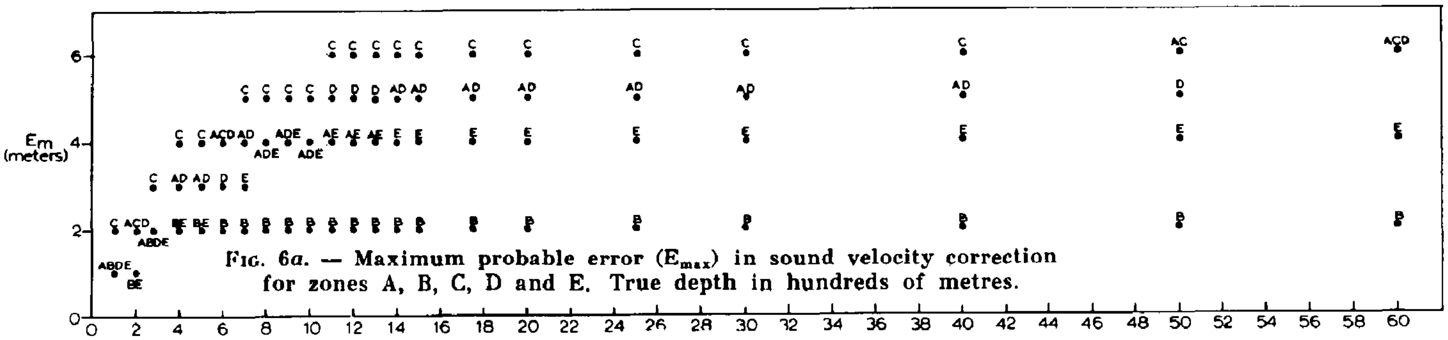


Fig. 6*a*. — Maximum probable error (E_{max}) in sound velocity correction for zones A, B, C, D and E. True depth in hundreds of metres.

TABLE I

Fathometer corrections for North Central Pacific SEAMAP Survey Surface to 1 500 metres

(fathometer velocity = 1 463 m/sec)

Equation : $\text{Corr} = bM + cM^2 + dM^3 + eM^4$

Where M = Fathometer depth in metres

Corr = Correction in metres to be added to fathometer depth

ZONES	CONSTANTS			
	b	*c*	*d*	*e*
Zone A (46° to 56° N)	$+ 6.0424 \times 10^{-3}$	$- 3.3660 \times 10^{-6}$	$+ 5.2830 \times 10^{-9}$	$- 1.2858 \times 10^{-12}$
Zone B (42° to 46° N)	$+ 2.04564 \times 10^{-2}$	$- 2.2860 \times 10^{-5}$	$+ 1.7232 \times 10^{-8}$	$- 3.9494 \times 10^{-12}$
Zone C (36° to 42° N)	$+ 2.52993 \times 10^{-2}$	$- 2.2670 \times 10^{-5}$	$+ 1.4598 \times 10^{-8}$	$- 3.0225 \times 10^{-12}$
Zone D (28° to 36° N)	$+ 4.3265 \times 10^{-2}$	$- 4.0874 \times 10^{-5}$	$+ 2.2439 \times 10^{-8}$	$- 4.1810 \times 10^{-12}$
Zone E (20° to 28° N)	$+ 5.3085 \times 10^{-2}$	$- 5.4265 \times 10^{-5}$	$+ 3.0317 \times 10^{-8}$	$- 5.7826 \times 10^{-12}$

TABLE II

Fathometer corrections for North Central Pacific SEAMAP Survey 1 500 metres to 10 000 metres

(fathometer velocity = 1 463 m/sec)

Equations : $\text{Corr} = a + bM + cM^2 + dM^3 + eM^4$

Where M = Fathometer depth in metres

Corr = Correction in metres to be added to fathometer depth

ZONES	CONSTANTS				
	a	*b*	*c*	*d*	*e*
Zone A (46° to 56° N)	− 0.36	$+ 2.800 \times 10^{-3}$	$+ 3.244 \times 10^{-6}$	$+ 3.530 \times 10^{-10}$	$- 1.203 \times 10^{-14}$
Zone B (42° to 46° N)	+ 4.64	”	”	”	”
Zone C (36° to 42° N)	+ 7.64	”	”	”	”
Zone D (28° to 36° N)	+ 14.64	”	”	”	”
Zone E (20° to 28° N)	+ 17.64	”	”	”	”

These are the data by which the echo soundings are corrected. The corrections for each zone are plotted against the fathometer depths to which they apply and smooth curves drawn through the data points. By means of a least square curve-fitting procedure, a polynomial expression is derived to match the empirical curves. This expression, therefore, yields a correction as a continuous function of depth for each zone. We have found that a fourth order equation will fit the curves with departures less than 0.3 metre above 1 500 metres. Below 1 500 metres a single polynomial expression will fit the curves for all zones merely by changing the a_0 term, with departures not exceeding 1 metre. The coefficients of the polynomials for the SEAMAP zones are listed in tables I and II. The correction versus depth curve and polynomial expressions for zone B are shown in figure 7.

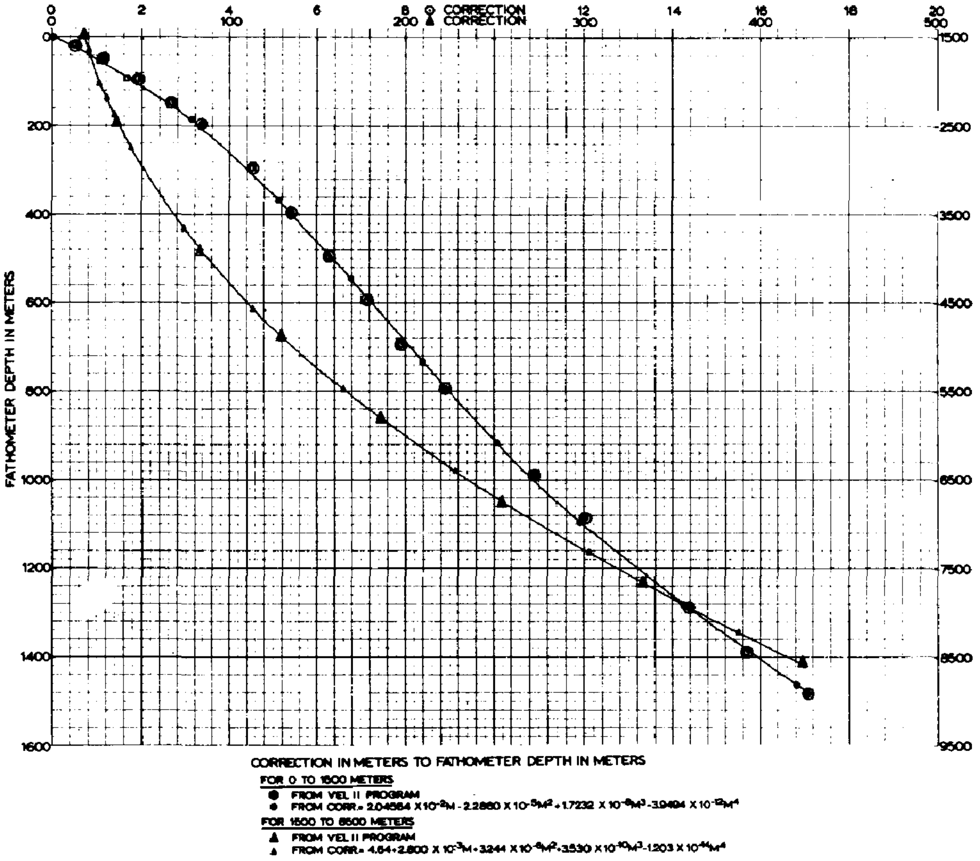


FIG. 7. — Corrections to fathometer depths for sound velocity for Zone B.

A simple computer program provides a look-up routine whereby the coefficients appropriate to the zone in which a sounding is made are used with the fathometer depth to 'solve' the polynomial for the correction. This correction is then added to the fathometer depth to yield the corrected depth.

As a supplement to the computer method, conventional tables have been prepared from the polynomial coefficients which provide for 'manual' correction of fathometer readings when the situation does not warrant computer assistance. The tables have been prepared in two parts to yield corrected depths in metres and fathoms.

4. — SUMMARY

The method described here embodies the following features.

1. Echo sounding corrections for sound velocity are available as a continuous function of depth, and are calculated and applied without manual effort.

2. The procedure is based on Wilson's formula which is generally accepted as superior to the MATTHEWS-KUWAHARA values. The differences, though small, are systematic, resulting in a slightly deeper ocean.

3. The procedure is designed to utilize all data on the environment available from the U.S. NODC, and yields quantitative information as to the probable error in the corrections.

4. A minor refinement in calculating the Mean Vertical Sound Velocity is achieved by means of an equation which recognizes that the velocity changes within each layer.

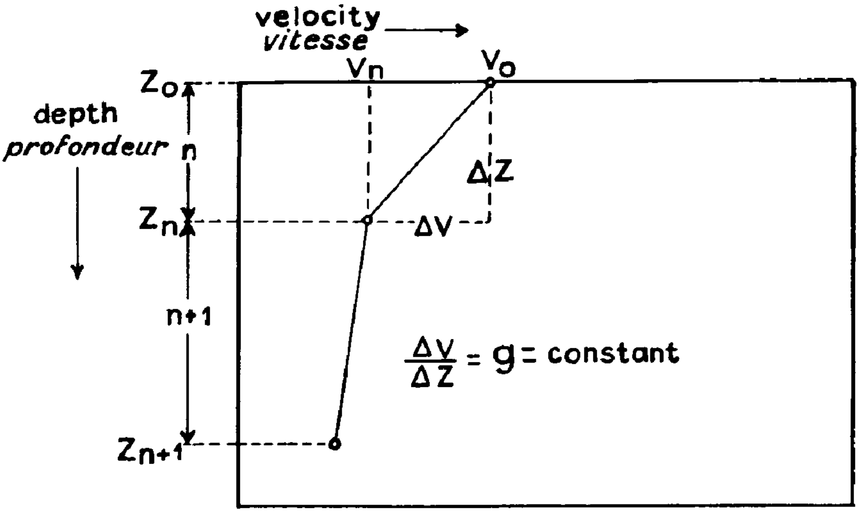
5. A minor refinement in applying the correction is achieved by developing the corrections as a function of fathometer depths rather than true depths.

6. Although corrections are made as a continuous function of depth, " scarps " as large as 7 metres in water depths of 5 660 metres can occur when track lines cross zone boundaries. These artificial scarps could be eliminated by using an interpolation technique between zones. As the boundary location is known, the cartographer can compensate for artificial scarps if they should occur.

We are indebted to Mr. R. K. REED for reviewing the manuscript and Mr. James L. STEPHENS for drafting the figures. The developmental work was accomplished on the IBM 1620 computer which was made available by Admiral H. J. SEABORG, Director, Pacific Marine Center, Seattle, Washington.

METHOD FOR COMPUTING THE MEAN VERTICAL SOUND VELOCITY (MVSV) FROM VELOCITIES GIVEN AT THE OCEANOGRAPHIC STANDARD DEPTHS

The MVSV is the mean velocity of a sound wave travelling from the ocean surface to any depth, Z_n where Z_n is one of the oceanographic standard depths. As the actual sound velocity usually changes continuously with depth, the derivation of a MVSV is complex; as a simplification the water column is considered to consist of a number of layers bounded by the oceanographic standard depths, within each of which the sound velocity gradient is constant.



In each layer, the time required for the sound wave to pass vertically through the layer is

$$t_n = \frac{z_{n+1} - z}{v} = \frac{\Delta z}{v} \tag{1}$$

where Z_n is an oceanographic standard depth, Z_{n+1} is the next deeper oceanographic standard depth, and V is the sound velocity. Within each layer the sound velocity is assumed to vary as a linear function of depth. It is clear that the velocity at any depth Z within the layer is

$$v_z = v_0 + \left(\frac{\Delta v}{\Delta z}\right) z = v_0 + gz \tag{2}$$

(note g has a negative sign)

where g is the gradient of velocity within the layer.

Substituting the expression V from equation (2), into equation (1), and taking the derivative of equation (1)

$$dt = \frac{dz}{v_0 + gz}$$

and integrating over the depth interval n from layer Z_0 to Z_n,

$$\int dt = \int_{z_0}^{z_n} \frac{dz}{v_0 + gz} = t_n = \frac{1}{g} \ln(v_0 + gz) \Big]_{z_0}^{z_n}$$

$$t_n = \frac{1}{g} \ln(v_0 + gz_n) - \frac{1}{g} \ln v_0$$

but $v_0 + gz_n = v_n$, so

$$t_n = \frac{1}{g} \ln(v_n - v_0) = \frac{1}{g} \ln \frac{v_n}{v_0}$$

which yields the time for the sound wave to pass through the first layer.

Similarly for all other layers to a depth Z_N :

$$T_N = \Sigma t_n + t_{n+1} + t_{n+2} \dots t_N$$

the mean velocity for the sound wave to get to depth Z_N is therefore

$$VMVS_N = \frac{Z_N}{T_N}$$

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