

A CRITERION FOR OPTIMUM STOCKS OF NAUTICAL CHARTS

by Rear Admiral W. LANGERAAR
Hydrographer, Royal Netherlands Navy

Comments received on my previous article "Optimum stocks for nautical charts", which appeared in Volume XLIV, No. 1, January 1967 of this Review, have made it necessary to reconsider the general principle on which that article was based.

At the bottom of page 25 of the above-mentioned issue of the Review, I wrote that: "It seems a quite straightforward principle to stockpile in such a way that the gain established in equation (8) is maximized".

This, in essence, is the criterion according to which optimization was accomplished. It remains to be seen, however, whether this principle is as straightforward as it appears to be.

It is a more or less foregone conclusion that in private enterprise or for any profit-making organization the aim should be to stockpile in such a way that the ultimate gain is maximized. Whether hydrographic offices belong to this category of establishment is however a moot point.

We must take account of the fact that all hydrographic offices are incorporated in the government structure, either military or civilian, of their own nation. One of the main reasons why this has been so, and why — so far — there is no tendency to swing towards the private enterprise sector, lies in the fact that nautical chart production, with all that is entailed, simply is not — and never will be — a paying proposition.

The price of purchasing a nautical chart seems to bear no relation to the cost of surveys, and does not at all reflect or cover costs of compilation, preparation, paper, printing, storage and keeping up-to-date. This price can be taken as a token payment. Some hydrographic offices have established their prices at such a level that it becomes apparent that the price serves almost no other purpose than to avoid giving the chart away for nothing. In other offices chart prices are based on a small portion of the cost involved, for example on paper and printing costs only. Generally, however, chart prices reflect an underlying principle laid down by the Government concerned, and it is a safe conclusion to state that these prices have nothing to do with profit-making, and at the most serve only to reduce the deficit.

It does not, therefore, seem any longer straightforward to take maxim-

izing the gain as the criterion for optimum stocks of nautical charts, but rather the minimizing of all labour costs entailed after the survey at sea has been completed.

My previous article also showed another shortcoming. As soon as appreciable changes of stocks are contemplated, not only will the printing cost and the workload of keeping the stocks up-to-date be influenced, but also the number of manhours expended by cartographers and cartographic draughtsmen in the compilation and preparation of the plastic material for printing will accordingly change. This aspect was not covered in my article of January 1967.

The object of the present paper is to remedy this flaw and to take into account the costs of compilation, preparation, chart paper, printing, storage of charts and keeping the stocks up-to-date. This will be done by dividing the total cost of a stock by the number of charts ready for sale, to give an average cost per chart. The optimum stock for any particular chart will then be found by minimizing this average cost. This means that the optimum stock will be invariant in relation to the retail price of the chart.

Setting aside the question of preparing and publishing new charts, and restricting ourselves to the preparation and publication of "small corrections", "large corrections" or "new editions" of existing charts, we know from experience that for every chart there exists a statistical relation between the numbers of times the chart has to be re-issued, either as a small correction, a large correction, or a new edition.

COST OF COMPILATION AND PREPARATION

If S small corrections of a particular chart are published as against L large corrections and E new editions, and if the number of manhours involved for cartographers is respectively M_s , M_L and M_E , and for cartographic draughtsmen respectively m_s , m_L and m_E , then simple arithmetic shows that the average cost V in manhours for any issue of that chart, allowing for compilation and preparation in the hydrographic office, can be expressed by :

$$V = \frac{SM_s + LM_L + EM_E + Sm_s + Lm_L + Em_E}{S + L + E}$$

If the cartographer is paid W_c guilders per manhour and the draughtsman W_d guilders, then the actual cost for compilation and preparation, V_w , per average restocking is expressed by :

$$V_w = \frac{(SM_s + LM_L + EM_E) W_c + (Sm_s + Lm_L + Em_E) W_d}{S + L + E} \quad (1)$$

If, for the case of a fictitious chart, and ghost cartographers and draughtsmen, we assume the following values :

$S = 6$, $M_S = 1$, $m_S = 36$; $L = 3$, $M_L = 9$, $m_L = 36$; $E = 1$, $M_E = 90$, $m_E = 185$ and finally, $W_c =$ florins 13 and $W_d =$ florins 8; then equation (1) gives :

$$V_w = \frac{(6 + 27 + 90) \times 13 + (216 + 108 + 185) \times 8}{10}$$

$$= \text{florins } 567.10$$

From equation (1) it would appear that the cost for compilation and preparation of an issue is invariant in relation to the number of copies printed. This is, of course, true for the total cost but not for the average cost per chart sold. The more copies we print the lower the cost per chart will be, and the less often a re-issue will be necessary. It is necessary, therefore, to find out how many charts of an issue can actually be sold.

Out of every issue of a chart a certain percentage, F , of the stock cannot be sold because of faulty printing or cancellation of the chart before the total stock is exhausted. Furthermore, X copies are exchanged under the IHB exchange agreement, free of charge, and are thus withdrawn from sale. If the stock consists of N copies, therefore, not more than

$$N_s = (1 - F)N - X \quad (2)$$

charts of that stock can be sold. From (2) it follows that :

$$N = (N_s + X)/(1 - F) \quad (3)$$

From (1) and (2) we find that the average compilation and preparation cost per chart sold amounts to :

$$V_{av} = V_w/N_s \quad (4)$$

COST OF PRINTING AND STORAGE

The printing costs, D , for a single issue of N copies, can be represented by the equation :

$$D = O + Np$$

in which

$O =$ overhead costs unrelated to N , and

$p =$ total cost per chart for paper, actual machine time etc.

We are interested in the printing cost per chart sold, and find the average cost, D_{av} , is expressed by the following :

$$D_{av} = \frac{O + Np}{N_s} \quad (5)$$

Recent information obtained from printing firms has established that for the Netherlands the overhead cost, O , is of the order of florins 700 (US\$ 195) and that p is about florins 0.80 (US\$ 0.22). These values are of course average values since O is dependent on the number of colour separations and because p varies slightly with the format of the chart.

Storage costs will not be considered as it can be assumed that storage space will always be available and, consequently, storage costs will be constant whether or not charts are stored. This cost can therefore be considered to be invariant in relation to N .

CORRECTION COSTS

Keeping stocks of charts up-to-date is a commitment hydrographic offices have found to be costly, as correction costs will rise approximately in proportion to the square of N . If on the average C correction units per week appear on a chart of which A copies are sold weekly, during the first week AC correction units will have to be applied before selling the charts. The second week this number will have augmented to $2 AC$ units, and so forth.

From equation (2) we conclude that the total stock of this particular chart will last N_s/A weeks, so that for this last week the number of correction units to be inserted will be $\frac{N_s}{A} AC$. Consequently, the total number of correction units, T , to be made on the copies of this issue, is obtained from :

$$T = AC \left(1 + 2 + 3 + \dots + \frac{N_s}{A} \right)$$

which can be written as

$$T = \frac{1}{2} AC \frac{N_s}{A} \left(\frac{N_s}{A} + 1 \right) \quad (6)$$

If the manhours necessary to apply one correction unit are denoted by U and the hourly wages paid to the chart correctors are represented by W_U , the actual amount of money, T_w , involved in keeping the stock of one chart up-to-date amounts to

$$T_w = \frac{CUW_U}{2A} N_s^2 + \frac{CUW_U}{2} N_s$$

whereas the average cost per chart, T_{av} , for correction can be found from :

$$T_{av} = T_w/N_s = \frac{CUW_U}{2A} N_s + \frac{1}{2} CUW_U \quad (7)$$

THE OPTIMUM STOCK

From the above it has become clear that the total cost, K , for compiling, preparing, printing, storing and correcting an issue of a particular chart is found from

$$K = V_w + D + T_w$$

and that the average total cost, K_{av} , per chart sold is obtained from :

$$K_{av} = V_{av} + D_{av} + T_{av}$$

which, according to (4), (5) and (7) may be written :

$$K_{av} = \frac{V_w}{N_s} + \frac{O + Np}{N_s} + \frac{CUW_u}{2A} N_s + \frac{1}{2} CUW_u \quad (8)$$

The average cost, K_{av} , will reach a minimum with regard to the variable N , when

$$\frac{dK_{av}}{dN_s} = 0 \quad \text{and} \quad \frac{d^2K_{av}}{dN_s^2} \text{ is positive.}$$

Bearing equation (3) in mind, from equation (8) we find

$$\frac{dK_{av}}{dN_s} = -\frac{V_w}{N_s^2} + \frac{N_s p \frac{1}{1-F} - (O + Np)}{N_s^2} + \frac{1}{2} \frac{C}{A} UW_u$$

which can be simplified to read :

$$\frac{dK_{av}}{dN_s} = -\frac{1}{N_s^2} \left(V_w + O + \frac{pX}{1-F} \right) + \frac{1}{2} \frac{C}{A} UW_u \quad (9)$$

As $\frac{d^2K_{av}}{dN_s^2}$ appears to be positive, K_{av} will reach a minimum when

$\frac{dK_{av}}{dN_s}$ equals zero. In that case (9) can be written

$$\frac{1}{2} \frac{C}{A} UW_u N_s^2 = V_w + O + \frac{pX}{1-F}$$

or

$$N_s^2 = \frac{2A}{CUW_u} \left(V_w + O + \frac{pX}{1-F} \right) \quad (10)$$

Equation (10) gives the solution of the problem of minimizing the average cost per chart by expressing the optimum number to be printed in terms that are dependent solely on the following : labour costs in the hydrographic office, printing costs (overhead as well as per copy), the exchange commitments of the office, and the two parameters of the chart concerned — its weekly sales and its weekly corrections.

The hydrographic office labour costs have a particular influence on the optimum stock. If the cost of correcting the stock goes up by a factor of four (because either the number of corrections, C , goes up or there is a rise in the price of labour in the term UW_u) the optimum stock has to go down by a factor of two. If, however, the wages of cartographers and cartographic draughtsmen go up, influencing the term V_w in the right-hand coefficient, then the optimum stock has to go up also.

After having found N_s from (10), the number of copies actually to be printed, N , is found from (3). By substituting into equation (10) the values that are valid at present in the Netherlands, from equation (3) we obtain :

$$N = 55 \sqrt{\frac{A}{CUW_u}} + 32$$

However, different values will have to be assigned to the parameters given in equation (10) for other countries and other hydrographic offices. In graph No. 1 a number of curves are shown for different values ranging from 0.04 to 0.25 guilders, dollars or other currency, for the actual cost of applying one correction unit, UW_v . On the horizontal axis the fraction A/C is given, and this parameter can be considered as a constant for a particular chart but to have different values for different charts. Values of N are given on the vertical axis. The right-hand graph in this figure shows on its vertical axis these values of N for a mean value of 55 for the coefficient

$$\frac{1}{1-F} \sqrt{2 \left(W_v + O + \frac{pX}{1-F} \right)}$$

that takes into account costs of compilation, preparation, printing, exchange, cancellation, etc., all expressed in the same currency as UW_v . The left-hand graph makes it possible to relate these N values to the values of the above mentioned coefficient.

The optimum stock, N , found in the way already described minimizes the total cost. If a stock of less than N were printed, the pressure on cartographers and cartographic draughtsmen, as well as on the printing budget, would increase in such a way as to outweigh the relief it would afford the chart correctors. If a stock larger than N is printed, the cost for corrections will rise more steeply than costs of compilation, preparation and printing will decrease.

MANPOWER REQUIRED

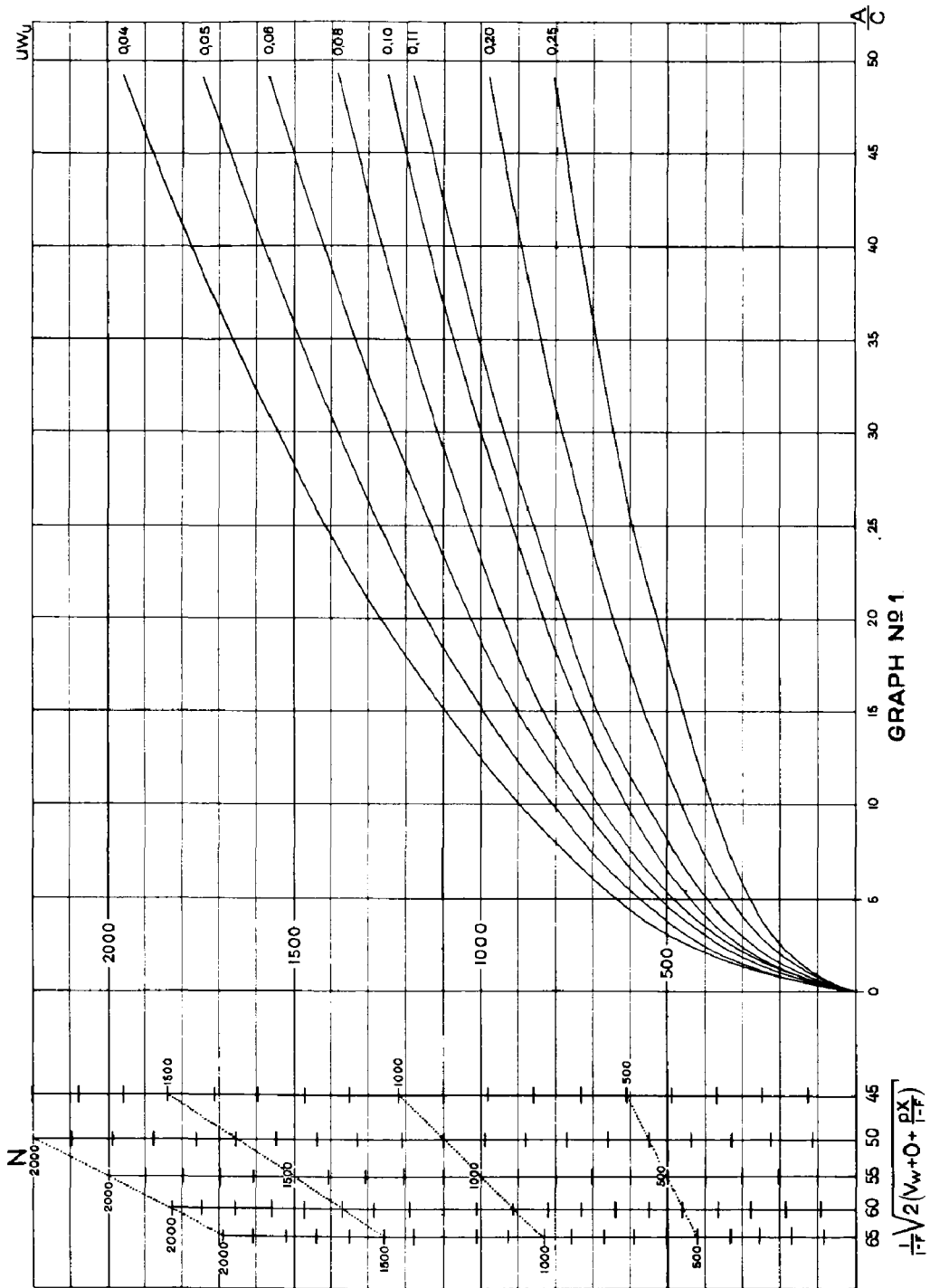
Having found equation (10) which gives a none too elegant solution to the problem of optimum stockpiling, the optimum stocks for the different charts can be found relatively quickly by means of the nomogram in graph No. 1.

When these stocks have been determined it may also be worthwhile to substitute the various values found for N_s into some of the aforementioned equations in order to find out what would be the optimum number of personnel in the different categories — cartographers, draughtsmen and chart correctors — necessary to cope with the amount of work involved in optimum stockpiling.

The optimum stock, N , of a certain chart being exhausted in N_s/A weeks, the number of reprints required annually, for a year of 50 weeks, equals $50 A/N_s$. From this, and from the introduction to equation (1), it is found that the number of manhours, V_{ca} , spent by cartographers on these reprints is obtained from :

$$V_{ca} = \frac{50A}{N_s} \frac{SM_S + LM_L + EM_E}{S + L + E} \quad (11)$$

If we consider the second coefficient of the right-hand side of the equation to be a constant for the *average* issue of every chart, then the



optimum stockpiling of all the charts published by a certain hydrographic office requires from its cartographers a number of manhours, V_{ca}^T , which can be represented by the following equation

$$V_{ca}^T = \frac{50(SM_S + LM_L + EM_E)}{S + L + E} \sum_{i=1}^q \frac{A_i}{N_{Si}} \quad (12)$$

assuming that the hydrographic office in question publishes a total of q nautical charts requiring regular revision and correction.

In a similar way we may arrive at the total number of manhours, V_{dr}^T , required by the draughtsmen to provide the hydrographic office with the optimum stocks continuously. We find :

$$V_{dr}^T = \frac{50(Sm_S + Lm_L + Em_E)}{S + L + E} \sum_{i=1}^q \frac{A_i}{N_{Si}} \quad (13)$$

The total number of manhours, V_{cc}^T , required to keep the optimum stocks of charts corrected by chart correctors can be found from equation (6), bearing in mind that U denotes the manhours required to apply one correction unit. From equation (6) we find that the total number of corrections, T_y , on a certain chart over one year is :

$$T_y = 50 \frac{A}{N_S} T = 25C(N_S + A) \quad (14)$$

From equation (14) the manhours, V_{cc} , required to apply these corrections, is found to be :

$$V_{cc} = 25CU(N_S + A) \quad (15)$$

Finally, the total number of manhours, V_{cc}^T , required to keep optimum stocks of all the nautical charts of a hydrographic office corrected during a whole year will be :

$$V_{cc}^T = 25U \sum_{i=1}^q C_i(N_S + A)_i \quad (16)$$

If we divide the manhours required from the different categories of personnel, and obtained from equations (12), (13) and (16), by the number of net manhours for a whole year, we shall find how many men of different specializations are required to keep a hydrographic office supplied with charts that are compiled, drawn, printed and corrected in an optimum way, because the overall cost involved in these different activities has been minimized.

The number of net manhours in a year will differ from country to country and even differs slightly within a country, depending on the type of labour employed, for it is well known that women require more time for personal care than men. The percentage of illness is also slightly larger in the case of women. But, generally speaking, the number of net manhours per year will be between 1 600 and 1 750. This figure in the Netherlands amounts to 1 670 (men and women averaged), an average of three weeks' vacation being also taken into account.