SPECTRAL ANALYSIS OF THE PERIODIC WATER LEVEL CHANGES IN THE BALTIC

(PRACTICAL APPLICATION)

by Mieczysław LASKA
Marine Station of the Polish Academy of Sciences, Sopot

ABSTRACT

This paper deals with the practical application of the autocorrelation and power spectrum methods to the analysis of periodic water level oscillations.

A review of the theory and practical calculations of the periodic water level changes in the Baltic by means of autocorrelation and power spectrum methods is given. Both methods give the possibility to evaluate, from a superimposed series of data of different periods, that periodic component which is of greatest interest. This can be done by a series of smoothing operations, namely by a filtration which eliminates from the input data (e.g. mareogram) all unimportant additional periods.

The general discussion of the theoretical basis of the above-mentioned methods, and especially, the analysis of the water level oscillations at Sopot and Kołobrzeg (Polish coast - Southern Baltic) may provide a good tool for practical calculations to determine the periodic water level variations in various reservoirs.

INTRODUCTION

Water level changes in the Baltic are characterised by both periodic and non-periodic oscillations. The most common water level changes in this basin are the non-periodic variations, caused by anemobaric effects — mainly wind.

For periodic variations, tides and free oscillations of the water masses are of primary interest.

In paper [8] a brief description of the Baltic non-periodic oscillations was given. These phenomena, especially in their extreme form, were thoroughly analysed and the computations were presented by the author in his doctorate thesis [9]. These numerical computations have shown that it is worthwhile to determine the periods and amplitudes of the periodic variations even in such a "tideless" sea as the Baltic; they provide useful information regarding periodic phenomena which influence the range of water level during a surge.

The disrupting action of the simple periodic oscillations, in view of their "known" magnitudes and times of occurrence, might not be great. Nevertheless, the simultaneous occurrence of maximum tidal amplitude and maximum amplitude of a seiche with the dynamic set-up of a non-periodic phenomenon may yield a resultant amplitude which can cause catastrophic results.

The Baltic tide in this case plays a comparatively secondary role. The seiches in this basin, however, have on many occasions revealed negative and dangerous effects — especially in the eastern part of the Gulf of Finland.

The calculation of the periodic water level changes described will be carried out by means of the autocorrelation and power spectrum methods.

AUTOCORRELATION METHOD

The introductory analysis of the periodic water level oscillations is based on the well-known Furich autocorrelation method [1], which allows one to determine the periods by means of correlation coefficients, i.e. to analyze the dependance of a function in respect to different time intervals t and $t + \tau$.

If a series of water level values is available, and all the data plotted with respect to time, giving the function $h(t)$, then one is able to calculate the function of autocorrelation $K(\tau)$ from it as :

$$K_h(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} h(t)\, h(t + \tau)\, dt \qquad (1)$$

This function characterizes the relation between the terms of the series with a spacing between terms expressed by the value τ. It means that if there exists a periodic component with period T_0 in the series, the autocorrelation function will have a maximum value for values of $\tau = T_0$, $2T_0$ ... nT_0.

Now if there exists in the series a group of different periodic constituents, then by repeated calculation of the correlation function one will get periodic changes of the correlation function with a period corresponding to the component with the highest amplitude in our mareogram.

As an example, let us consider a simple series — representing in our case water level variations taken by a tide gauge recorder.

This series is expressed by :

$$h = \sum_{i=1}^{N} a_i \cos(\omega_i t + \varphi_i) \qquad (2)$$

where :

a_i : denotes the amplitude
ω_i : is the angular frequency
φ_i : characterizes the phase angle.

Using (2) in the equation (1) to calculate the values of $K(\tau)$ and introducing some transformations one obtains :

$$K_h = \sum a_i^2 \cos \omega_i \tau \qquad (3)$$

From the above it can be seen that if a certain component of the series (2) has a slightly greater amplitude a_i, then the corresponding component in expression (3) has a much greater amplitude in comparison to the other components because each component increases to the square value, e.g.

$$a_i = 1 , 4 , 6 \ldots \qquad a_i^2 = 1 , 16 , 36 \ldots$$

Evaluating the autocorrelation function from expression (3), by continuous repeating a periodic and fully smoothed curve of the autocorrelation coefficient is obtained.

A simple example of such calculations is shown in fig. 1 where the water level curve for the one-day period of 12 March 1963 is drawn. The lower part of the figure shows the results of the successive calculated curves of the autocorrelation function (1, 3, 4 correl.). These curves indicate a slow but distinct convergence to establish the period T of the recorded curve.

The full application of the above mentioned calculations yields fig. 2, which shows the calculated autocorrelation coefficient obtained from the mareograms of the water level recorders at Sopot and Kołobrzeg. The figure shows that successive calculations of the autocorrelation coefficients lead to the smooth curve of this coefficient exhibiting a distinct period. As may be seen, the third evaluation of the autocorrelation coefficient for the station at Sopot, already provides the possibility of a rough estimation of the periodic oscillations of the water level changes in this region. The oscillation period ranges between 12 and 13 hours. The next, or fourth, computation of the autocorrelation coefficient narrows this range to 12.3 h which corresponds in reality to the period of the semi-diurnal Baltic tide.

Similar calculations of the autocorrelation coefficient for the Kołobrzeg region did not give such satisfactory results. The fifth computation (5 correl.) gives a mean value of the period between 23.5 and 29.0 hours. One can assume that the next evaluation of the coefficient under study would give the expected period of 27 hours, which again corresponds to the Baltic seiches [7, 10, 11, 13].

It can be easily seen from the above that the calculation of periodic water level oscillations by means of the simple and normalized autocorrelation function alone whilst having some advantage has also a number of distinct disadvantages. Firstly, the calculations of the successive autocorrelation coefficients are tedious and lengthy, and it is not easy to distinguish and define the periods being sought. A more serious dis-

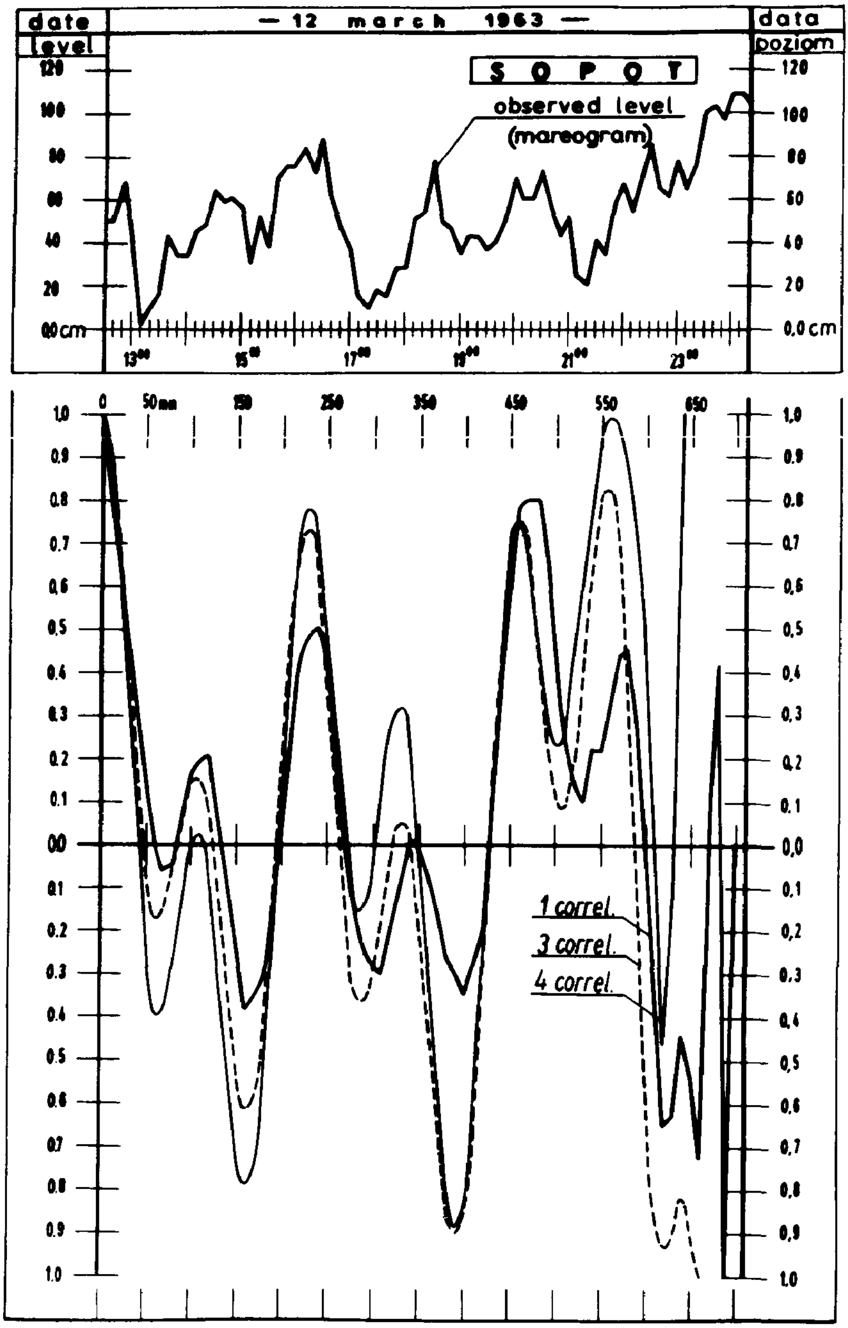

FIG. 1. — Autocorrelation coefficients of the $h(t)$ function for Sopot (12 March 1963).

advantage, however, is due to the fact that it is impossible, using this method, to find the amplitudes of the periodic oscillations. Therefore for a full and accurate estimation of periodic oscillation the method of power spectrum analysis has to be used.

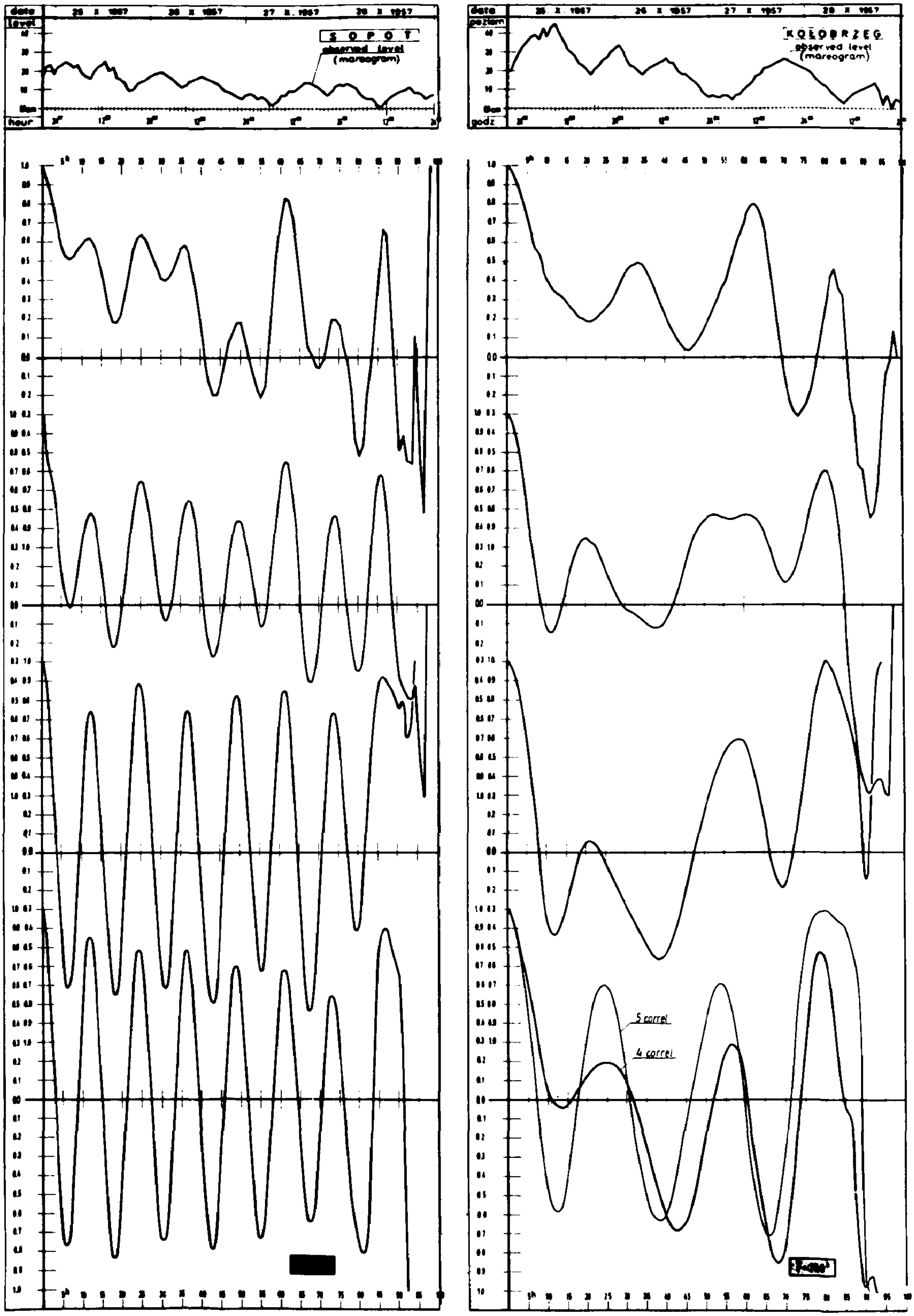

FIG. 2. — Autocorrelation coefficients of the $h(t)$ function for Sopot and Kołobrzeg (25-28 October 1957).

EXAMINATION OF THE PERIODIC OSCILLATIONS BY MEANS OF THE POWER SPECTRUM METHOD

The given series of water level variation (mareograms) against time $h(t)$ will be expressed in the form of the Fourier-Stieltjes integral [1] :

$$h(t) = \int_{-\infty}^{+\infty} dh(\omega)\, e^{i\omega t} \qquad (4)$$

where :

ω = frequency of the oscillations
t = time
$dh(\omega)$ = increments of the random function, corresponding to the spacing of the frequency $d\omega$.

Hence the autocorrelation function will be :

$$K(t_1, t_2) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} h(t_1)\, h(t_2)\, dt =$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} \left[\iint_{-\infty}^{+\infty} dh(\omega_1)\, e^{i\omega_1 t_1}\, dh(\omega_2)\, e^{i\omega_2 t_2} \right] dt \qquad (1a)$$

Dealing with the stationary random processes — e.g. stationary sea water level changes — and assuming the hypothesis of ergodicity, one can change the mean time value over the complex of realization function $h(t)$. The new mean value will be denoted by $< >$ signs.

In the expression (1a) the quantities $e^{i\omega_1 t_1}$ and $e^{i\omega_2 t_2}$ are not random. The sign of the mean value therefore refers to the random increments $dh(\omega_1)$ and $dh(\omega_2)$, namely to :

$$< dh(\omega_1)\, dh(\omega_2) > \qquad (5)$$

The autocorrelation function can express the stationarity in the case when it does not depend on the time momentum t but only on the distance between the tested points on the time axis $t_1 - t_2 = \tau$. Then expression (5) has to satisfy the following relationship :

$$< dh(\omega_1)\, dh(\omega_2) > = S(\omega_1 \omega_2)\, d\omega_1 d\omega_2\, \delta(\omega_1 - \omega_2)$$

Substituting this relationship into equation (1a) gives :

$$\iint_{-\infty}^{+\infty} S(\omega_1, \omega_2)\, d\omega_1 d\omega_2\; \delta(\omega_1 - \omega_2)\, e^{i(\omega_1 t_1 - \omega_2 t_2)}$$

$$= \int_{-\infty}^{+\infty} S(\omega_1)\, e^{i\omega_1 (t_1 - t_2)}\, d\omega_1 = \int S(\omega)\, e^{i\omega\tau}\, d\omega = K(\tau) \qquad (1b)$$

where : $t_1 - t_2 = \tau$ and $\omega_1 = \omega$.

The Fourier transform of the expression (1b) will be as follows :

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} K(\tau)\, e^{-i\omega\tau}\, d\tau \qquad (6)$$

where $S(\omega)$ is the power spectrum.

In our case $S(\omega) \cong h^2$, and represents the potential energy of the water level changes.

Plotting $S(\omega)$ as a function of the oscillation frequency, a spectrum diagram is obtained, which of course shows clear and definite peaks only when there exist periodic oscillations with frequencies ω_0, ω_1, ω_2 ..., etc.

The practical calculations are executed mostly through cosine or sine of the Fourier transform and then [1] :

$$S(\omega) = 2 \int_{-\infty}^{+\infty} K(\tau) \cos \omega\tau \, d\tau \tag{6a}$$

However the estimation of the above integral can be made by the following series

$$S_K = K_0 + 2 \sum_{\nu=1}^{m-1} K_\nu(\tau) \cos \frac{p\nu\pi}{m} + \omega_m \cos \tau_p \tag{6b}$$

where the K value is based upon :

$$K(\nu) = \frac{1}{N-p} \sum_{i=1}^{N-p} X_i X_{i+p} \tag{6c}$$

The values obtained from the expression (6b) are then smoothed through the filter proposed by TUKEY [1] :

$$U_p = 0.23\, S_{k-1} + 0.54\, S_k + 0.23\, S_{k+1}$$

$$\text{where} \quad S_{-1} = S_1 \quad \text{and} \quad S_{m+1} = S_{m-1}$$

The magnitude of the power spectrum given by the expression (6b) is calculated from the data being taken from the mareograms with a time interval Δt. It is essential to understand which frequency interval corresponds to the time interval Δt, because two periods lying close to each other may not be distinguished during the elimination of the specific power spectrum.

Before entering a more detailed discussion, let us consider two frequencies ω_1 and ω_2, and the possibility of their separation.

Let the recorded water level be :

$$h(t) = h_1 e^{i\omega_1 t} + h_2 e^{i\omega_2 t} + h'(t) \tag{7}$$

where $h'(t)$ denotes the random water level changes, and let us assume that the whole expression (7) is given in a spectral form :

$$h(t) = \int_{-\infty}^{+\infty} dh(\omega)\, e^{i\omega t}$$

where

$$dh(\omega) = d\omega\, [A_1\, \delta(\omega - \omega_1) + A_2\, \delta(\omega - \omega_2)] + dh'(\omega)$$

Considering now that the series of data from expression (7) is calculated in a finite time interval from

$$t = -\frac{1}{2} T \quad \text{to} \quad t = \frac{1}{2} T$$

and defining a function $X(t)$ as :

$$X(t) = 1 \quad \text{pour} \quad |t| < \frac{1}{2} T$$

$$0 \quad \text{pour} \quad |t| \geqslant \frac{1}{2} T$$

one obtains the following function for the Fourier transform of $X(t)$:

$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(t)\, e^{-i\omega t}\, dt = \frac{\sin \frac{1}{2}\, \omega T}{\pi \omega}$$

The Fourier transform of $h(t)$ as defined by equation (7) is evaluated and can then be expressed as :

$$h(\omega_1) = A_1\, X(0) + A_2\, X(\omega_1 - \omega_2) + h'(\omega_1)$$

$$h(\omega_2) = A_1\, X(\omega_2 - \omega_1) + A_2\, X(0) + h'(\omega_2) \tag{8}$$

These expressions confirm that the spectrum of frequency ω_1 is due to the three following factors : firstly, the spectral band which comes from the oscillation with frequency ω_1 ; secondly, the side effect caused by the presence of ω_2 with the degree of interaction depending upon the length of the observation time interval and expressed through $X(\omega_1 - \omega_2)$; and thirdly the influence of the random oscillations at frequencies close to ω_1 expressed by $h'(\omega_1)$. The same discussion can be applied to $h(\omega_2)$.

Let us return to the possibility of the practical separation of the frequencies which occur in the expression of the density spectrum, and which can be calculated by use of the series (6b).

Here the values obtained refer to the frequency distance for which the mean value is expressed according to [1] by :

$$\omega_K = \frac{2\pi p}{2m\Delta t} \tag{9}$$

or by the mean period :

$$\frac{1}{T} = \frac{p}{2m\Delta t} \tag{9a}$$

Hence it appears that the width of the frequency distance, or time frequency, being in an elementary interval, can be expressed as :

$$\Delta\omega = \frac{\pi(p+1)}{m\Delta t} - \frac{\pi p}{m\Delta t} = \frac{\pi}{2m\Delta t} \tag{10}$$

It can be easily seen that with the increase of m (which is the successive number of the correlation ordinate) the frequency distance can be a small one, and that the close frequencies can be distinctly separated. But an optional increase in the m value will probably cause an increase in the random errors to occur during the calculations of the power spectrum $S(\omega)$.

A qualitative analysis of this phenomenon can be made by assuming that the series of water levels has a "normal", or Gaussian, distribution.

In this case, the accuracy of the calculated power spectrum depends mainly upon one parameter ν, known as the number of degrees of freedom, describing the known χ^2 distribution :

$$\nu \cong \frac{2N}{m}$$

where :

N = number of observations

m = successive number of the correlation ordinates.

The above distribution allows us to calculate the confidence limits of the power spectrum, with a defined probability for the respective values of ν.

As an example, the 95 % probability confidence limits for different degrees of freedom ν are given in table 1.

The table shows, for example, that with $\nu = 10$ degrees of freedom and for the evaluated value of the power spectrum $S(\omega) = 1.0$ the "real" value of $S(\omega)$ of this density (with the probability of 95 %) lies between 0.49 and 3.10. This confirms that the value m has to be chosen by making a compromise between the great distribution of neighbouring frequencies and the accuracy of the $S(\omega)$ estimation.

TABLE 1

ν	Confidence limits	ν	Confidence limits
2	0.21 – 40.00	15	0.55 – 2.40
3	0.32 – 14.00	20	0.59 – 2.10
4	0.36 – 8.30	50	0.69 – 1.55
5	0.39 – 6.00	100	0.78 – 1.35
6	0.42 – 4.80	150	0.81 – 1.27
8	0.46 – 3.80	200	0.83 – 1.23
10	0.49 – 3.10	300	0.86 – 1.18

The error in the estimation of the $S_h(\omega)$ value can be expressed with the help of the following expression [5] :

$$\left(\frac{3/4\, m\, h^2}{N}\right)^{1/2}$$

which characterises the variance of the estimator $S_h(\omega)$.

THE RESULTS

The calculations of the power spectrum for the sea water level changes based on expression (6b) were executed with the help of an Elliott-803 computer. Besides the conditions already considered above, confirming that a very long series of data (great values of N) gives satisfactory accuracy, here the limited capacity of the computer's memory raised some additional conditions, so that the practical calculations were restricted to

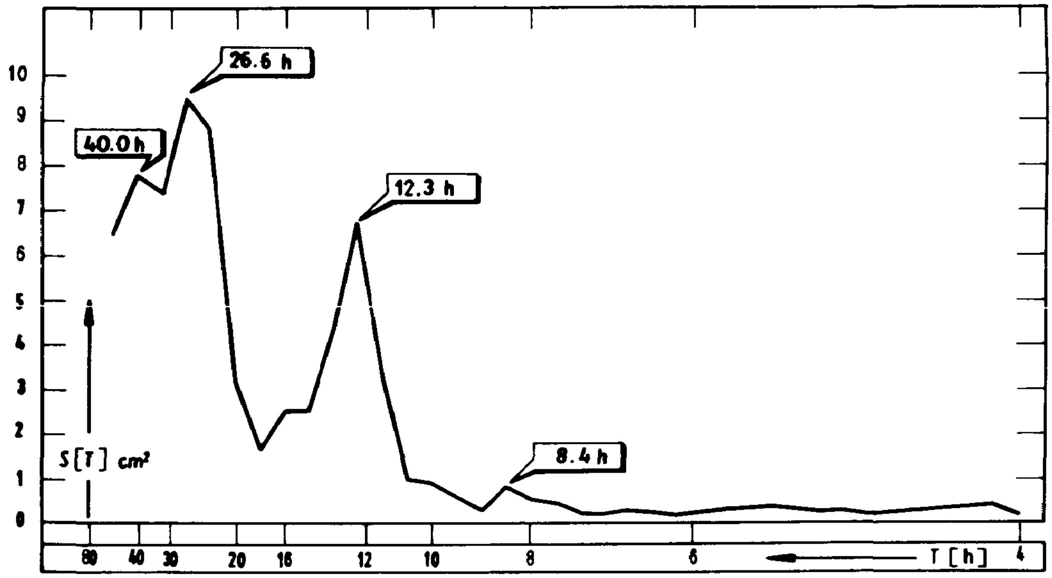

FIG. 3. — Power spectrum of the sea water level variation for Sopot (October 1957).

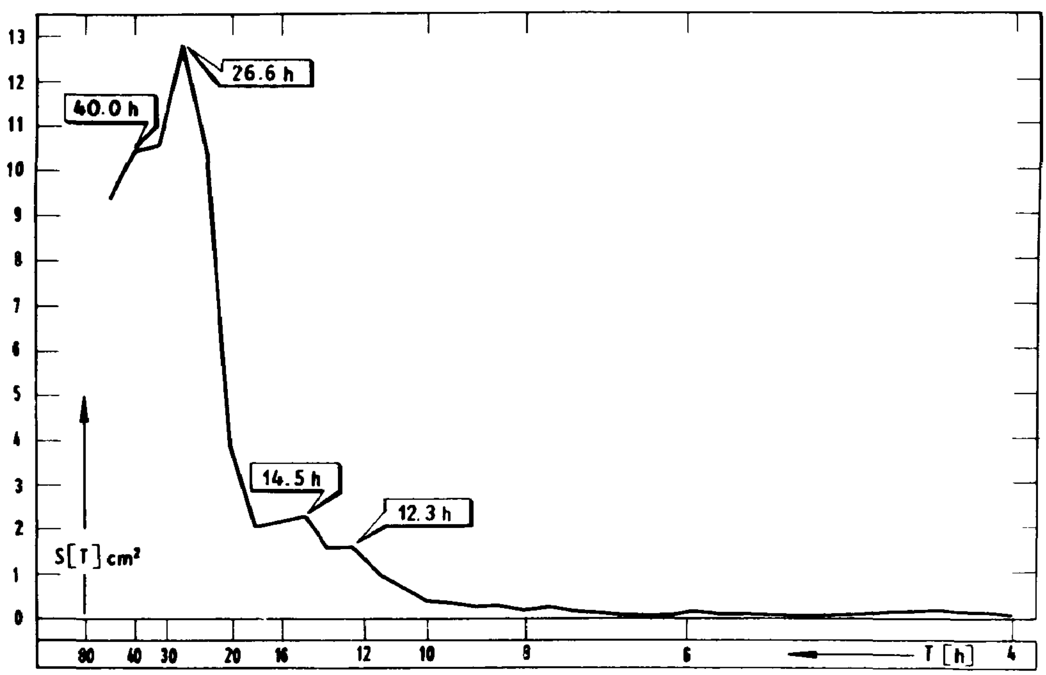

FIG. 4. — Power spectrum of the sea water level variation for Kołobrzeg (October 1957).

$\nu \approx 20$. It follows therefore that the accuracy of the calculations was

$$h\sqrt{\frac{3}{4}\cdot\frac{1}{10}} \cong 0{,}27\ S(\omega)$$

which shows that the error was 27 % of the $S(\omega)$ value.

Fig. 3 illustrates the power spectrum results of the sea water level variations in the Sopot region.

The results point out three fundamental oscillations with periods of 12.3^h, 26.6^h, and 40.0^h. Oscillations with the same period were found in the $S(\omega)$ calculations for the Kołobrzeg region (fig. 4).

Analysing both diagrams (figs. 3 and 4) one can see that besides the common oscillations of period 12.3^h, 26.6^h, and 40.0^h, two other oscillations of period 14.5^h for Kołobrzeg and of period 8.4^h for Sopot were found.

The oscillations of period 40.0^h and 26.6^h characterize the well-known [7, 10, 11, 13] uni-nodal Baltic seiches in the oscillation system for respectively the Baltic proper—Gulf of Bothnia and the Baltic proper—Gulf of Finland. The period of 12.3^h can without any doubt be referred to the semi-diurnal tide which enters the Baltic from the North Sea through the Danish Sounds. Other periods defined in these calculations require further accurate and thorough analysis.

Apart from the periods, the amplitudes of the periodic oscillations can be found from the diagrams as well, because according to the expression (6a)

$$S(\omega) \approx 2h^2 \quad \text{so} \quad h(t) = \sqrt{\frac{S(\omega)}{2}}$$

Hence the amplitude of the period 26.6^h for Sopot is

$$h(t) = \sqrt{\frac{9.5}{2}} = 2.2 \text{ cm}$$

and for Kołobrzeg it is 2.5 cm.

The amplitude of the semi-diurnal tide in the Sopot region according to the calculations is 2.0 cm.

It can be seen that the maximum amplitudes of the periodic oscillations in Polish coastal waters are very small, and their value is almost negligible in comparison with the non-periodic oscillations [9].

PREPARATION OF THE INPUT DATA AND THE OUTPUT FORM

All the data for periodic oscillations can be read straight from the water level recordings. The time interval, Δt, between the successive data is determined mainly by the type of oscillations which are concerned, and by the factors mentioned previously.

TABLE 2

Autocorrelation coefficients and power spectrum values, obtained from the mareograms (October 1957) of the water level recording station at Sopot.

p	W_p	L_p	U_p
0	1.0000	1.3700	0.1668
1	0.8425	− 1.2459	0.5670
2	0.6555	4.0203	3.2921
3	0.4634	6.1207	6.4445
4	0.2851	9.6291	7.7698
5	0.0989	5.0537	7.3531
6	− 0.0656	10.4757	9.5089
7	− 0.1504	11.6944	8.7908
8	− 0.1790	0.2887	3.2641
9	− 0.1795	1.8195	1.6641
10	− 0.1700	2.6748	2.4537
11	− 0.1440	2.5686	2.4933
12	− 0.1404	2.1352	4.3258
13	− 0.1679	11.2262	6.7044
14	− 0.2081	0.6572	3.2682
15	− 0.2529	1.4401	1.0661
16	− 0.2961	0.5971	0.8555
17	− 0.3285	0.8778	0.5618
18	− 0.3376	− 0.2152	0.3005
19	− 0.3100	0.9338	0.5248
20	− 0.2314	0.3046	0.5004
21	− 0.1316	0.5268	0.3568
22	− 0.0230	0.0101	0.1887
23	0.0696	0.2702	0.1876
24	0.1297	0.1710	0.2210
25	0.1704	0.2891	0.2103
26	0.1548	0.0645	0.1650
27	0.1052	0.2769	0.2147
28	0.0346	0.2188	0.2592
29	− 0.0310	0.3363	0.2975
30	− 0.0892	0.2851	0.3035
31	− 0.1277	0.3142	0.2714
32	− 0.1422	0.1575	0.2233
33	− 0.1288	0.2869	0.2187
34	− 0.1013	0.1200	0.1821
35	− 0.0866	0.2233	0.2028
36	− 0.0556	0.2375	0.2453
37	− 0.0255	0.2958	0.2737
38	− 0.0241	0.2814	0.3199
39	− 0.0326	0.4441	0.3255
40	− 0.0477	0.0910	0.1931
.	.	.	.
.	.	.	.
79	− 0.0955	0,0685	0.0564
80	− 0.1449	0,0484	0.0576

TABLE 3

Autocorrelation coefficients and power spectrum values, obtained from the mareograms (October 1957) of the water level recording station at Kołobrzeg.

p	W_p	L_p	U_p
0	1.0000	1.9637	0.1561
1	0.9070	− 1.9660	0.4921
2	0.7938	4.7915	4.5839
3	0.6522	10.6462	9.4025
4	0.4915	11.0934	10.4502
5	0.3209	8.7442	10.5926
6	0.1518	14.4317	12.8174
7	0.0093	13.1004	10.4538
8	− 0.1009	0.2622	3.9049
9	− 0.1970	3.2618	2.0680
10	− 0.2734	1.0710	2.2019
11	− 0.3330	3.7974	2.3403
12	− 0.3645	0.1887	1.6472
13	− 0.3850	2.9211	1.6649
14	− 0.3984	0.1917	1.0459
15	− 0.3981	1.1763	0.6499
16	− 0.3878	− 0.1277	0.3504
17	− 0.3563	0.6470	0.3361
18	− 0.3225	0.0699	0.2893
19	− 0.2800	0.4468	0.2472
20	− 0.2233	− 0.0442	0.1695
21	− 0.1542	0.3939	0.2163
22	− 0.0899	0.0597	0.1790
23	− 0.0332	0.2440	0.1381
24	0.0083	− 0.0320	0.0752
25	0.0463	0.1582	0.0711
26	0.0675	− 0.0302	0.0722
27	0.0649	0.2267	0.1164
28	0.0592	0.0042	0.0918
29	0.0382	0.1626	0.0871
30	0.0092	− 0.0072	0.0672
31	− 0.0251	0.1465	0.0871
32	− 0.0638	0.0418	0.0824
33	− 0.0938	0.1136	0.0791
34	− 0.1163	0.0353	0.0850
35	− 0.1366	0.1737	0.1324
36	− 0.1453	0.1339	0.1652
37	− 0.1528	0.2307	0.1747
38	− 0.1483	0.0841	0.1427
39	− 0.1300	0.1924	0.1174
40	− 0.1040	− 0.0253	0.0568
.	.	.	.
.	.	.	.
79	− 0.1059	0.0468	0.0435
80	− 0.1628	0.0349	0.0404

In the example described above the time step Δt between the successive values of the water level changes was chosen as 1 hour. The data for the computations on the digital computer have to be prepared in the following sequence : N, M, x_1, x_2 .... x_n; where N and M are integers and x_n are rational numbers. At the same time, to obtain a sense of those calculations the relation of $M \leqslant N\text{-}2$ has to be fulfilled. The results from the program (Elliott library program for autocorrelation and spectral analysis) are printed in a form of a table. The annexed tables (tables 2 and 3) give the main part of the results, from which the diagrams (figs. 3 and 4) were elaborated.

The different values in the tables denote :

p : 0, 1, 2, 3, 4, 5, ... M
W_p : autocorrelation
L_p : the preliminary power spectrum
U_p : the smoothed power spectrum.

The computer time needed for the calculations of the power spectrum of the sea water level changes depends mainly upon the N and M parameters. For example, the mentioned calculations with N = 72 and M = 70 occupied the computer only for 5 minutes; this, in comparison with the calculations done by the method of autocorrelation coefficient alone, is clearly advantageous.

ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to Dr Z. Kowalik for his help and criticism during this work.

Thanks are also due to Prof. S. Szymborski, Director of the Marine Station of the Polish Academy of Sciences at Sopot, for help in financing the computer runs and for permission to publish this paper.

REFERENCES

[1] Blackman, R.B. and Tukey, J.W. (1958) : The Measurement of Power Spectra. Dover Publications, New York.

[2] Conrad, V. and Pollak, L.W. (1962) : Methods in Climatology. Determination of Periods with the Aid of Autocorrelation. Chapter IV. Harvard Univ. Press, Cambridge, Mass.

[3] Defant, A. (1961): Physical Oceanography. Pergamon Press, London-New York.

[4] Hamon, B.V. (1962) : The Spectrums of Mean Sea Level at Sydney Coff's Harbour and Lord Howe Island. *Journ. of Geoph. Research*, Vol. **67**, 13.

[5] HAMON, B.V. and HANNAN, E.J. (1963) : Estimating Relation between Time Series. *Commonwealth Scient. and Industrial Research Org.*, Vol. **68**, 21.

[6] KOWALIK, Z. (1968) : Zastosowanie method widmowych do badania zmiennosci parametrow dynamicznych w morzu. *Rozpr. Hydrot.*, **22.**

[7] KRAUSS, W. and MAGAARD, L. (1962) : Zum System der Eigenschwingungen der Ostsee. *Kieler Meeresforschungen.* Inst. für Meereskunde der Univ. Kiel, **XXII**, 2.

[8] LASKA, M. (1966) : General Characteristics of the Southern Baltic Water Level Changes, ... *Intern. Hydrograph. Rev.*, Vol. **XLIII**, 2.

[9] LASKA, M. (1967) : Wahania poziomu wod Morza Bałtyckiego oraz metody ich wyznaczania. (Doctoral thesis, unpublished).

[10] LISITZIN, E. (1959): Uni-nodal Seiches in the Oscillation System Baltic proper — Gulf of Finland. *Tellus*, **II**, 4.

[11] MAGAARD, L. and KRAUSS, W. (1966) : Spektren der Wasserstandsschwankungen der Ostsee im Jahre 1958. *Kieler Meeresforsch.*, Inst. für Meereskunde der Univ. Kiel, **XXII**, 2.

[12] MUNK, W.H., SNODGRASS, F.E. and TUCKER, M.J. (1959) : Spectra of Low-Frequency Ocean Waves. *Bull. of Scripps Inst.*, La Jolla, Vol. **7**, 4.

[13] NEUMANN, G. (1941) : Eigenschwingungen der Ostsee. Aus dem Archiv der Deutschen Seewarte und des Marineobservatoriums, **61**, 4.

[14] SOUTHWORTH, R.W. (1962) : Autocorrelation and Spectral Analysis. Library of Programs, Vol. **3**, Comp. Div. Elliott Brs, London.

[15] SWIESZNIKOW, A.A. (1961) : Podstawowe metody funkcji losowych. PWN - Warszawa, 1965.

[16] SZYMBORSKI, S. (1956) : The Phenomenon of Seiche in the Gulf of Gdansk on the 17th January 1955. *Acta Geoph. Pol.*, Vol. **IV**, **I.**