A NOTE ON THE APPLICATION OF IMAGE INTERFERENCE TO BOTTOM CONTOURING

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INTRODUCTION

Image interference in the sea is a well known phenomenon produced by phase interference between the acoustic waves transmitted to the sea bottom via a direct path and those via a path which includes one surface reflections. The effect has been extensively studied by a number of investigators [1-8] and is also discussed in several general works on underwater acoustics [9-11]. Various approximation methods have been applied to the analysis of this phenomenon. The simplest is the isovelocity approximation used by the Research Analysis Group of the U. S. National Research Council [1] and by **Chesterman** *et al.* [8]. More refined theories include those of **Young** [2], **Sanders** and **Stewart** [3], and **Pedersen** [6]. **Young** noted that it is the difference in propagation time along the two possible ray paths that determines the relative phases of the interfering waves. He thus justified the replacement of the curved rays propagating with a depth-dependent velocity by straight ones, and developed an equation for the transmission anomaly in which linear velocity gradients were taken into account. **Stewart** made calculations using the actual curved ray paths for the case of an almost isothermal sea, and in the analysis of **Pedersen** the additional effect of refraction on the pressure amplitudes of the interfering waves was also taken into consideration.

Since image interference patterns of underwater sound are very sensitive to water depth and other geometric parameters, they offer a variety of potential applications. By measuring the fringe position, **Christensen** [12] was able to deduce the physical size and location of the scatterers responsible for the interference. **Haines** [13], in a review article, first pointed out the possibility of making use of this phenomenon to derive bottom contours from the facsimile records of a side-scan sonar. However, it was **Chesterman** *et al.* [8] who made the first quantitative studies on bottom contouring by this method by plotting the reciprocal range for successive fringe maxima (or minima) against the fringe order. From the slope of the resulting graphs, information on exact bottom depths and hence on bottom contours was deduced. Where the bottom slope is constantly varying, the interference condition itself was used. The greatest advantage of this method is that by making a single traverse, continuous coverage of a sea-bottom strip a few hundred yards wide is achieved.

The purpose of this note is to demonstrate that in the application of underwater image interference to bottom contouring, the assumption of isovelocity propagation (by CHESTERMAN *et al.* [8], for example) is definitely incompatible with the accuracy desired. Temperature and pressure refraction must be taken into account, and the effect of other factors (which will be mentioned later) should also be considered.

REVIEW OF THE BASIS OF ANALYSIS

Let us now re-examine the basis of the analysis of CHESTERMAN *et al.* [8]. Using the notations of figure 1, the path difference between the interfering rays at B is

$$\Delta = (\mathrm{SC} + \mathrm{CB}) - \mathrm{SB} = \mathrm{IB} - \mathrm{SB}$$

i.e.,

$$\Delta = [(\mathrm{D} + t)^2 + x^2]^{1/2} - [(\mathrm{D} - t)^2 + x^2]^{1/2} \qquad (1)$$

If we put

$$\Delta = m\,\lambda/2$$

and recall that there is a phase reversal on reflection at the surface because of the much lower acoustic impedance of the reflecting medium [14], then we see that interference is constructive when m assumes an odd, integral value and is destructive when m is even, integral.

From equation (1), we have

$$\frac{\mathrm{D}^2}{\left(\frac{m\lambda}{4}\right)^2} - \frac{x^2}{t^2 - \left(\frac{m\lambda}{4}\right)^2} = 1 . \qquad (2)$$

Thus, points for which the ray paths differ by a constant lie on a hyperbola with the sound source at one focus. It is customary to expand equation (2) and neglect terms of order (λ/t), (λ/D) and higher. This gives the linear relation of

$$x = \frac{4t\mathrm{D}}{m\lambda}$$

which is an adequate approximation since it only introduces errors that are considerably less than 1 % for ranges greater than (say) 7 times the water depth.

A second approximation used is the identification of the horizontal and slant ranges (see [8]). Thus, with the last equation, this gives the familiar relation of

$$\mathrm{R} = \frac{4t\mathrm{D}}{m\lambda} \qquad (3)$$

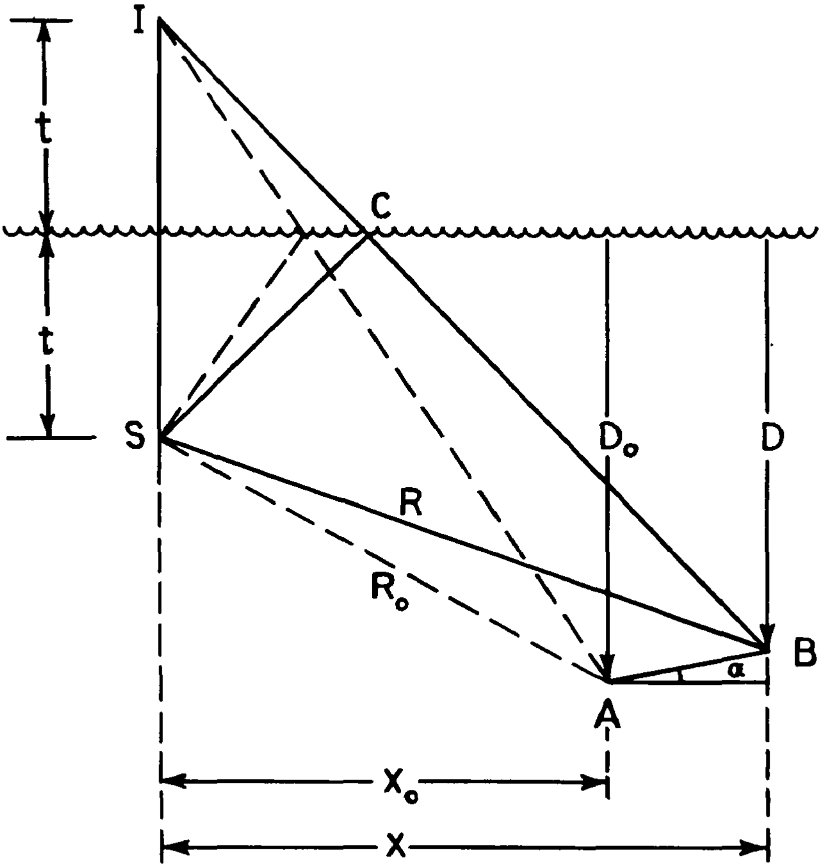

FIG. 1. — Schematic representation of the geometry for image interference

S = source of acoustic energy submerged at depth t
I = image source
R = slant range from S to point B on sea bottom
x = horizontal range from S to B
A = point on sea bottom, slope of AB is α

In figure 2, the range difference is plotted against water depth for various slant ranges. It can be seen that this approximation is quite inadequate for seabed contouring applications where measurements to the order of a yard are involved.

THE ANALYSIS

The first step in the accurate analysis of side-scan interferograms is the determination of the fringe order m. This can be done by tracing the fringe to a flat seabed or to a place where a precise depth measurement has been made, and then proceeding with equation (3). With m thus determined, the position of interference fringes can be used to deduce the

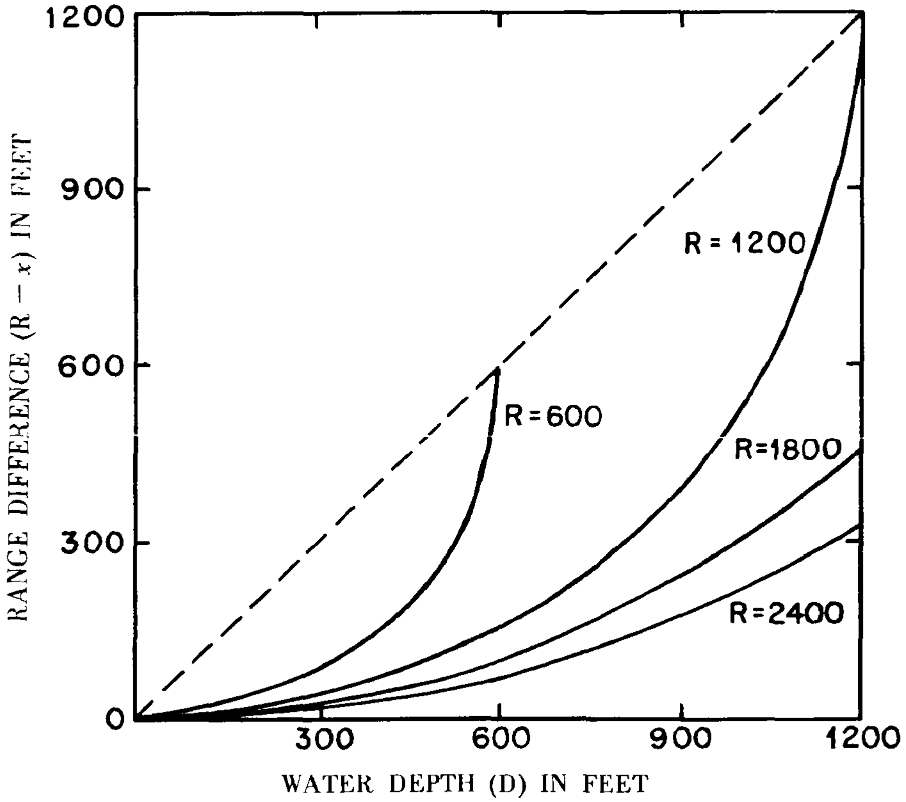

Fig. 2. — *Variation of the difference between horizontal and slant ranges with water depth.*

R = slant range
x = horizontal range

water depth at specific points. Alternately, if a seabed of constant slope α is assumed, we have (figure 1)

$$D = D_0 - \alpha(x - x_0) \tag{4}$$

Let us for the moment consider isovelocity propagation only and identify R with x.

From (3), we have

$$m = \frac{4tD}{\lambda R} - \left(\frac{4t}{\lambda}\right)\frac{D_0 - \alpha(x - x_0)}{R}$$

$$= \left(\frac{4t}{\lambda}\right)\frac{D_0 - \alpha(R - x_0)}{R} = \frac{4t}{\lambda}(D_0 + \alpha x_0)\frac{1}{R} - \frac{4t\alpha}{\lambda}$$

or

$$m = \frac{4tf}{c}(D_0 + \alpha x_0)\frac{1}{R} - \frac{4tf\alpha}{c} \tag{5}$$

where f = frequency of the projected underwater sound and c = its velocity of propagation.

Thus, a plot of m versus $1/R$ is a straight line, from the slope and intercept of which the depth at any range may be calculated.

If we now relax equating R and x, the m versus $1/R$ plots will no longer be straight lines. They will become curves with a slope given by :

$$\frac{dm}{d\,(1/R)} = \frac{4tf}{c}\left[D_0 + \alpha x_0 + \frac{\alpha^2\,(D-t)\,(t-D_0-\alpha x_0) - 2\alpha x_0\,(D-2D_0)}{\alpha^2\,(D-t) + (D-D_0) - \alpha x_0} \right] \qquad (6)$$

The first two terms of (6) are identical with the corresponding ones of (5), but the last is a correction term arising from the differences between R and x. The importance of this term depends on the values of range and bottom slope in question, but its order of magnitude may well be comparable to the term just preceding it.

Let us next relax the isovelocity propagation condition and take, as a better approximation, YOUNG's theory of acoustic refraction. According to this theory, the interference pattern is given by [3] :

$$R^2\left[\frac{\Delta c}{c\,(t+D)}\right] + R\left[\frac{cm}{4ft}\right] - D = 0 \qquad (7)$$

instead of equation (3).

Here

$$\Delta c = c_0 - c_1$$

$$\frac{1}{c_0} = \frac{1}{t}\int_0^t \frac{dy}{v(y)}$$

$$\frac{1}{c_1} = \frac{1}{D-t}\int_t^D \frac{dy}{v\,(y)}$$

where y is the depth measured from the sea surface downwards and $v(y)$ gives the vertical variation of the sound velocity function.

Hence

c_0 = harmonic mean velocity between the surface and the transducer depth

c_1 = harmonic mean velocity between the layers at the transducer depth and at the bottom

$$c \simeq c_0 \simeq c_1$$

This depth—range relationship together with the isovelocity approximation of equation (3) is plotted in figure 3 for the typical case considered by CHESTERMAN *et al.* [8]. The dotted lines correspond to isovelocity conditions, and the solid lines are for upward refraction due to pressure alone as predicted by YOUNG's theory.

It can be seen (figure 3) that at a range of 2 400 feet, the difference in deduced depth between the two sets of curves can be as large as 15 feet. Also, at a water depth of 50 feet (say), pressure refraction alone increases the range of the interference minimum of order 6 by 1 000 feet and that of fringe order 12 by some 100 feet. Thus, for the application of image interference to bottom contouring, the isovelocity theory is clearly inadequate.

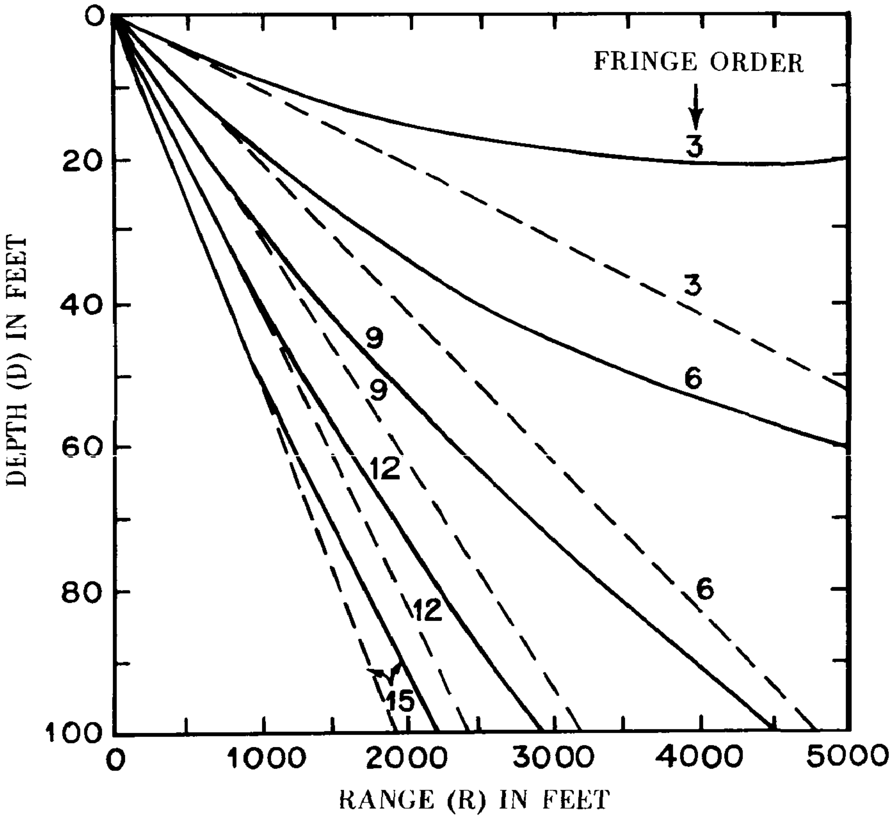

FIG. 3. — Positions of interference minima for a transducer depth of 14.5 feet. The dotted lines are for isovelocity conditions, and the solid lines for upward refraction due to pressure alone as predicted by YOUNG's theory.

Frequency $f = 48$ kcps
Average sound velocity $c = 4\,850$ feet per second
Uniform velocity gradient = 0.0182 (foot per sec) per foot.

From equations (7) and (4), if we again identify R with x, we can deduce that :

$$\frac{dm}{d(1/\mathrm{R})} = \frac{4ft}{c}\left[\mathrm{D}_0 + \alpha x_0 + \frac{x^2\,\Delta c}{c\,(t + \mathrm{D})} + \frac{\alpha x^3\,\Delta c}{c\,(t + \mathrm{D})^2}\right] \tag{8}$$

Hence refraction introduces two additional correction terms to the slope of the m versus 1/R curves. For a positive velocity gradient, these curves are bent upwards away from the isovelocity straight line, and for a negative velocity gradient, they are bent downwards.

OTHER COMPLICATING FACTORS

There are other complicating factors. Heaving, yawing and pitching of the transducer (if placed within a towed body which is not stabilized) and the action of waves would constantly alter the phase difference between

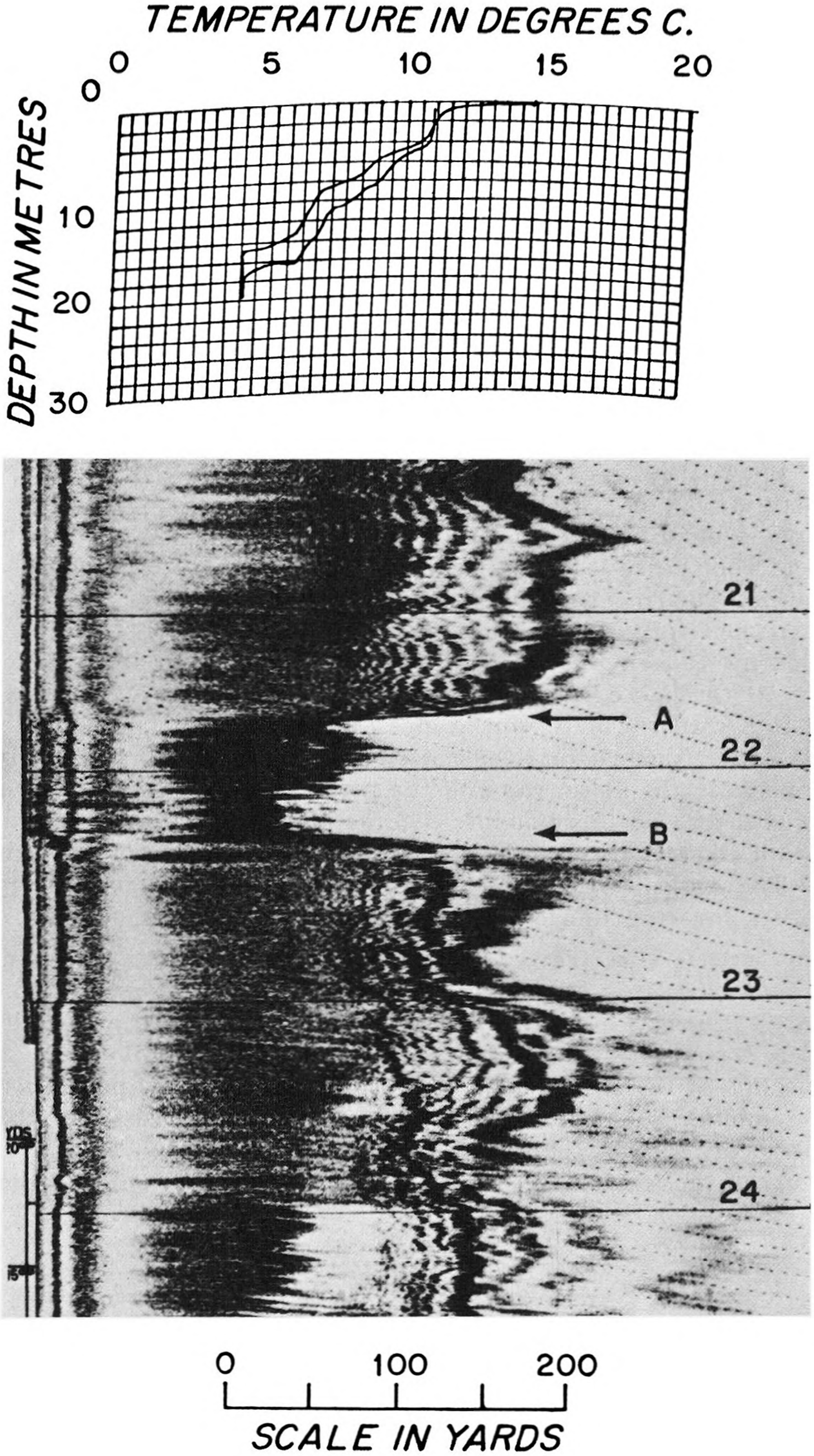

Fig. 4. — Image interference of direct ray and ray reflected at the boundary between the isothermal layer and the thermocline.

the direct and surface-reflected rays, and the depth dependence of tidal currents would set up velocity gradients. The effects of these factors must be evaluated, and taken into account if they are likely to be significant. More refined theories than that of YOUNG are probably not necessary, except where considerably great depths or long ranges or severe refraction conditions are encountered. Experimentally, a method of stabilizing the fringes would be essential. This can be done by placing an artificial reflector (of dimensions say 3 feet by 20 feet) just underneath the surface waves, so that a stable reflected beam is assured.

IMAGE INTERFERENCE OF OTHER ORIGIN

There are other physical conditions which can give rise to interference patterns and must be recognized during analysis. Figure 4 shows a fringe system (accompanied by severe downward acoustic refraction) together with the bathythermogram taken just prior to the survey. The isothermal layer extended through the first 5 metres (5.5 yards) of water, and is succeeded by a steep thermocline. The transducer was initially towed at 8 yards. At the point A, it was raised to 4 yards; at B it was again submerged to 8 yards. The disappearance of the interference fringes between points A and B showed that the pattern in this case was caused by interference between the direct ray and the ray reflected at the interface of the isothermal layer and thermocline. The persistance of the fringes indicated that abrupt changes in the thickness of the isothermal layer were probably absent. The acoustic beam was pointing one degree up for this traverse, thus rather favoring image interference.

Finer interference fringes superimposed on a pattern produced in the normal way have been reported to exist in a stratified ocean bottom [15]. These are formed as a result of phase interference between the direct ray and the ray which has penetrated the top layer of the bottom and suffered one reflection at the interface of the bottom strata. Short-period irregularities resembling interference fringes (also superimposed on the main pattern), as are seen in figures 33 and 35 of reference [8], can also be due to heaving of the transducer, which changes the path lengths of the sound rays.

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NOTE AND REFERENCES

[1] Bergmann, P.G. and Yaspan, A. (ed.) (1946) : Physics of sound in the sea, Part 1 : Transmission. NDRC Summary Technical Rept. Div. 6, Vol. 8, Washington, D.C. (Research Analysis Group), Sections 2.6.3. (p. 31 et seq.) and 5.2.1 (p. 95 et seq.).

[2] Young, R.W. (1947) : Image interference in the presence of refraction. *J. Acoust. Soc. Am.* **19**, No. 1, 1-7.

[3] Sanders, F.H. and Stewart, R.W. : Image interference in calm, near-isothermal water; with an appendix : "Lloyd mirror" interference in the presence of slight uniform refraction — an approximate theory. *Canad. J. Phys.* **32**, 599-619.

[4] Ahrens, E. (1957) : Use of horizontal sounding for wreck detection. *Int. Hydr. Rev.* **XXXIV**, 73-81 (p. 77).

[5] McKenzie, K. V. (1961) : Bottom reverberation for 530- and 1030-cps sound in deep water. *J. Acoust. Soc. Am.* **33**, 1498.

[6] Pedersen, M. A. (1962) : Comparison of experimental and theoretical image interference in deep water acoustics. *J. Acoust. Soc. Am.* **34**, No. 9, 1197-1203.

[7] Stubbs, A.R. (1963) : Identification of patterns on Asdic records. *Int. Hydr. Rev.* **XL**, No. 2, 53-68 (pp. 60-63).

[8] Chesterman, W.D., St. Quinton, J.M.P., Chan Y. and Matthews, H.R. (1967) : Acoustic surveys of the sea floor near Hong Kong. *Int. Hydr. Rev.* **XLIV**, No. 1, 35-54.

[9] Horton, J.W. (1957) : Fundamentals of Sonar. U.S. Naval Institute, Annapolis, Maryland. 387 pp. (Art. 3C-4, 113-117).

[10] Richarson, E.G. (1957) : Technical Aspects of Sound, Vol. II. Elsevier Publishing Co. (Chapter 1, section 3).

[11] Albers, V.M. (1965) : Underwater Acoustics Handbook II. Pennsylvania State University Press, 365 pp. (Section 4.3.3m, 49-52).

[12] Discussion by Christensen in Hersey, J.B. (1954) : Acoustic Instrumentation as a tool in oceanography. National Research Council Publ. No. 309, Washington, D.C.

[13] Haines, R.G. (1963) : Developments in ultrasonic instruments. *Int. Hydr. Rev.*, XL No. 1, 49-57 (p. 53-54).

[14] This phase reversal was inadvertently neglected in the analysis of Chesterman *et al.* [8].

[15] Brown, M.V. and Ricard, J.H. (1965) : Interference pattern observed in reflection from the ocean bottom. *J. Acoust. Soc. Am.* **37**, No. 6, 1033-1036.