INTRODUCTION

Users of radiopositioning chains with carrier waves of about 2 MHz, and first of all hydrographic surveyors are well aware by experience that land paths of electromagnetic waves have to be as short as possible. Most often this operating requirement comes up against contrary requirements governing the setting up of shore antennae — easy access and industrial power supply system near at hand for example. The choice of transmitter sites must therefore be a compromise.

Unavoidable land paths then cause anomalies in measurements; such anomalies are determined at a limited number of positions called "calibration positions". Then these anomalies are estimated for any position in the area covered by the chain — by means of sometimes generous extrapolation — and shown as corrections to be applied to the records on board.

Systematic accuracy tests of a Decca Hi-Fix radiopositioning chain were made in 1964 in the vicinity of Brest. Unavoidable land paths were the cause of serious trouble. In compensation, the solving of these difficulties, besides permitting the trials themselves to be carried out successfully, brought about the following:
— a quantitative evaluation of the velocity attenuation of electromagnetic waves over land;
— the perfecting of an easy and efficient calibration method;
— finally, an approach to a critical study of the lane identification process.

At a time when radiopositioning chains are tending to come into general use for coastal coverage, the results of general consequence obtained during the trials are an interesting contribution to the experimental knowledge of radiopositioning systems.
PRINCIPLE OF THE TRIALS

Topographical situation of the chain: The chain consisted of two patterns, the North and the South ones. In the following study, only the results relating to the North pattern will be given since these are the most significant. Figure 1 (situation of the North pattern) shows up the importance of land paths from the slave antenna.

Trial procedures: These consisted of three entirely different steps:
--- initial verifications and adjustments (three or four days);
--- calibration (about one week);
--- checking (about two weeks).

All reference positions were obtained from three simultaneous sightings by theodolite to the receiving antenna on board, the ship being stationary. Simultaneous optical sightings and observations on board were ensured by radio "pips". All the computations were made in Lambert projection and all the corrections made in connection with this projection system were taken into account.

Initial adjustment consisted only in observing and computing a few positions, with a view to adjusting roughly the phase difference between slave and master transmitters, so that the number zero limit hyperbola (baseline extension) passes through the master antenna. The purpose of this operation was to minimize merely by a manual display the error resulting from anomalies of propagation due to the limited land paths between master and slave antennae.

Calibration consisted in determining an empiric correction curve in hundredths of lane, based on observations made at about fifteen observation positions:

\[
\text{correction} = \text{function of } \Phi_{\text{obs}}
\]

\(\Phi_{\text{obs}}\) being the hyperbola number (or "phase") observed on board.

THEORETICAL MEANS OF MAKING USE OF CALIBRATION RESULTS

Symbols

\(\Phi_{\text{obs}}\) : phase observed on board (fraction of a unit and also whole unit number)
\(\Phi_{\text{th}}\) : corresponding theoretical phase (id.)
M, S, B : master antenna, slave antenna, antenna on board ship
\(f\) : basic frequency
\(V\) : propagation velocity over sea

Adopted velocity: \(V = 299650\ \text{km/s}\).
Classical means: The "calibration curve" is drawn up from calibration observations:

\[ \Phi_{th} - \Phi_{obs} \text{ plotted versus } \Phi_{obs} \text{ (or } \Phi_{th}, \text{ which is only slightly different)} \]

It is assumed that this curve yields the corrections to be brought to \( \Phi_{obs} \) to obtain \( \Phi_{th} \) during the survey. Let us recall that

\[ \Phi_{th} = \frac{f(BM - BS)}{V} + f \cdot \frac{MS}{V} = \frac{f(BM - BS)}{V} + \Phi_0 \]

If \( V \) contains an error \( \Delta V \), and if transmission from the slave antenna is not exactly correct (error concerning the constant term \( \Phi_0 \)), the error of \( \Phi_{th} \) is as follows:

\[ \Delta \psi = -\frac{\Delta V}{V} \Phi_{th} + \Delta \Phi_0 \]

The calibration curve will therefore be close to a straight line defined by:

\[ \Phi_{th} - \Phi_{obs} = \alpha \cdot (\Phi_{th} - \Delta \Phi_0) \approx \alpha \cdot \Phi_{obs} - \Delta \Phi_0 \]

Fig. 1. — Topographical situation of the North pattern and distribution of calibration positions.

Let us note that:

— \( \Delta \Phi_0 \), the \( y \) intercept, does not exceed a few hundredths in the case of a careful initial adjustment;
— $\alpha$ should be insignificant, since $V$ has been determined for years with good accuracy for the most often used geographical areas;
— the differences between the actual calibration curve and its average straight line account for “local propagation anomalies”.

**Difficulties encountered**: The observations made at the fifteen regularly spaced calibration positions located near the baseline of the North pattern (see figure 1) were used to draw up the calibration diagram shown in figure 2. The scattering of the points obtained in this manner made the plotting of a smooth curve rather risky. But, above all, the slope of the best straight line passing through these points ($\Delta V/V = 0.25 \%$ approx.) yielded an aberrant result: it implied an error in the propagation velocity of about 750 km/s.

[Diagram of North pattern calibration diagram obtained by classical method]

Looking at the formula

$$\Phi_{th} = \frac{f(BM - BS)}{V} + \Phi_0$$

one can see that an error in $V$ being *a priori* excluded, the error has to be sought in either $f$ or $(BM - BS)$. Consequently, the crystal frequency was checked and a coarse geodetic error — of about 15 metres — was sought for in the antennae coordinates by checking the optical observations. This search was not fruitful.

**Proposed means**: It then seemed necessary to examine again the possible errors in $V$, even though these were considered most improbable.

Some land paths reach 2 km. Tentatively, the corrections to be brought to $\Phi_{obs}$ were quickly computed, assuming that propagation velocity over land was 1$\%$ (or 3 000 km/s) lower than propagation velocity over sea. The great effect of the length of land paths was then plainly clear since the residual scattering of the differences

$$\Phi_{obs} + \text{correction} - \Phi_{th}$$

amounted on the average to 2 hundredths of a lane around a value ($\Delta \Phi_0$).
It was therefore admitted that the propagation velocity over land — which will be designated \( V_t \) — was considerably different from \( V \). This simple assumption led to a new method for making use of calibration observations, which excludes the above-mentioned "calibration curve" notion. This last point is noteworthy, for the assumption that the correction due to anomalies of propagation is solely dependent on the number designating the hyperbola is justified only in the case of short baselines.

Path BM is divided into a land path \((BM)_m\) and a sea path \((BM)_s\). BS is divided in a similar way, and a propagation velocity over land \( V_t \) — defined as a constant for convenience' sake (we shall come back to this point later on) — has to be determined. Assuming the existence of \( V_t \) amounts to admitting that \( \Phi \) obs has the following form:

\[
\Phi \, \text{obs} = \frac{f \left[ (BM)_m - (BS)_m \right]}{V} + \frac{f \left[ (BM)_s - (BS)_s \right]}{V_t} + \Phi_0 - \Delta \Phi_0
\]

If we put:

\[
V_t = V - \Delta V,
\]

we may write:

\[
\Phi \, \text{obs} = \frac{f \left( BM - BS \right)}{V} + \frac{f \left[ (BM)_s - (BS)_s \right]}{V} \cdot \frac{\Delta V}{V} + \Phi_0 - \Delta \Phi_0
\]

\[
= \Phi \, \text{th} + \left\{ \frac{f \left[ (BM)_s - (BS)_s \right]}{V} \cdot \frac{\Delta V}{V} - \Delta \Phi_0 \right\}
\]

In principle, two \( \Phi \) obs values relating to the North pattern are now sufficient to determine quantities \( \Delta V/V \) and \( \Delta \Phi_0 \) by comparison with \( \Phi \) th. All calibration observations concerning both patterns have of course been used.

From the excellent results obtained by making use of calibration and then of calibration verification, the above assumption and its subsequent deduction have, \textit{a posteriori}, largely proved both assumption and interpretation to be very good. Let us emphasize two facts of particular interest:

1. Chain calibration was reduced to the determination of three constants only: \( \Delta \Phi_0 \) for each one of the two patterns, and \( \Delta V/V \).
2. The value found was

\[
\Delta V/V = 0.0126
\]

\[
V - V_t \simeq 3800 \text{ km/s}
\]

This value is considerably higher than is generally admitted.

\textit{Notes:} It has been assumed that \( V_t \) was independent of the length of land paths. The validity of this assumption is not to be taken into consideration for the first 250 metres, since all antennae are set up at least 250 metres inland. The correction of the possible omnidirectional error is included in the term \( \Delta \Phi_0 \).

On the other hand, no land path exceeded 3 kilometres. It may be that \( V_t \) increases considerably with the land path length when this exceeds
3 kilometres: such a case does not occur often because of the great values of calibration corrections this would entail.

Finally, the assumption \( V_f = \text{constant} \) implied that ground conductivity is the same in the vicinity of all transmitter antennae. This condition was actually met, for in this particular case the nature of the ground was homogeneous and its moisture was governed by atmospheric conditions.

**PRACTICAL MEANS OF MAKING USE OF CALIBRATION RESULTS**

*Practical effect of \( \Delta V / V \):* A decrease of propagation velocity over land results in an increase of the transmitter-to-ship travel time, this increase being proportional to the land path, i.e. in a phase advance of the signal received on board. Two errors make the phase difference \( \Phi \text{obs} \) erroneous: the one concerning the land path of the waves generated at \( M \) has to be subtracted, the other relative to the land path of waves originating in \( S \) has to be added. It is therefore convenient to write

\[
\Phi \text{th} = \Phi \text{obs} - \text{correction}
\]

with

\[
\text{correction} = \left[ -(BM)_r + (BS)_r \right] p + \Delta \Phi_0.
\]

and

\[
p = \frac{f \cdot \Delta V}{V}
\]

Computations have been made in order to obtain directly the proportionality factor:

\( p = \text{gain or loss of lanes per land kilometre} \)

It was found that:

\( p = 0.08 \text{ lane/km} \ (\pm 0.005 \text{ lane/km}) \)

The wave length \( V/f \) being 158 metres, it was found that:

\[
\frac{\Delta V}{V} = p \times \frac{V}{f} = 0.08 \times 0.158 = 1.26 \% \ (\pm 0.1 \%)
\]

*Measurements of land paths when in small number:* The drawing of the shoreline in the vicinity of each antenna was only imperfectly given by outdated charts. This was achieved with sufficient accuracy by using the French Institut Géographique National's 1/25 000 air photographs, upon which antennae sites had been carefully pinpointed.

A 1/50 000 "calibration chart" was then drawn up in Lambert projection. This chart included:

- the Lambert rectangular grid allowing the rapid plotting of the various receiving ship positions;
- the antennae positions;
- the drawing of the shoreline in the vicinity of each antenna. For convenience' sake this was achieved by a homothetic drawing in the ratio
of 2/1, i.e. at scale of 1/25 000. Thanks to this artifice, the reduction of photographic data and the drawbacks it involves were avoided.

The calibration chart allows an easy measurement of land paths, either in kilometres or, more directly, with a suitably graduated ruler, in hundredths of lane.

**Measurement of land paths when in great number:** The above operating method has been actually used for the 70 checking positions. For the thousands of positions usually determined during a survey it would, however, entail a heavy loss of time. The method can be easily improved in the following way:

— Using the calibration chart, cut the survey area into sectors, the centre of which is master antenna M, so that the land path correction has the same value to within one half a hundredth of lane in each sector (or with a better approximation). The sector limits are determined from the distance between antenna and shoreline, so that the sector definition is based on the land path difference values equal to $1/p$ kilometre (i.e. in the present case, $1\text{ km}/8 = 125$ metres).

— Make a similar cut from slave antenna S. The survey area is then cut into quadrilaterals. In each one of these is written the whole number of hundredths obtained by subtraction. In this way the complete calibration corrections are shown for the whole pattern, to within an accuracy of one-hundredth corresponding to the accuracy of the data acquired on board.

— Apply the same method to the second pattern, using different drawing symbols or colours.

— From the figures obtained two families of equal-correction curves may be drawn up, the values of which are expressed in hundredths. It should be possible to use these results in a computer by cutting the survey area into curvilinear quadrilaterals limited for each pattern by regularly spaced hyperbolae — with the largest possible spacing.

**CHECKING — RESULTS OBTAINED**

The geographical distribution of the 70 checking positions is shown in figure 3. The above calibration correction is applied to each observed phase $\Phi_{\text{obs}}$, and checking for each position consists in comparing the corrected phase

$$\Phi_{\text{cor}} = \Phi_{\text{obs}} + \text{correction}$$

with the theoretical phase.

For each position the deviation ($\Phi_{\text{cor}} - \Phi_{\text{th}}$) represents the error in the phase measurement. By totalling the positions where the deviation ($\Phi_{\text{cor}} - \Phi_{\text{th}}$) is the same, it is found that:

- for 8 positions, $\Phi_{\text{cor}} - \Phi_{\text{th}} = 0.00$
- for 7 positions, $\Phi_{\text{cor}} - \Phi_{\text{th}} = + 0.01$
- etc.
Let \( N \) be the number of positions where the value of the deviation \( (\Phi \text{ cor} - \Phi \text{ th}) \) is the same. The experimental function of the distribution of deviations

\[
N = f (\Phi \text{ cor} - \Phi \text{ th})
\]

is determined graphically by a certain number of plotted points and is shown as the best curve passing through them. This curve is roughly similar to the typical bell-shaped curve showing the density probability of the Laplace-Gauss law, this curve being referred to an appropriate origin for the abscissae (+ 0.005, i.e. one half a hundredth) and to suitable abscissa and ordinate scales. Those curves are indicated in figure 4.

Fig. 3. — Geographical distribution of checking positions.
Fig. 4. — Smoothed curve showing the distribution of deviation $N = f(\Phi_{\text{cor}} - \Phi_{\text{th}})$. Crosses indicate raw results.

The following table shows the observed percentage of positions where the difference ($\Phi_{\text{cor}} - \Phi_{\text{th}}$) remains between certain typical limits:

<table>
<thead>
<tr>
<th>Deviation between</th>
<th>Percentage of positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.03$ and $+0.03$</td>
<td>$73%$</td>
</tr>
<tr>
<td>$-0.05$ and $+0.05$</td>
<td>$96%$</td>
</tr>
</tbody>
</table>

Let us mention that results obtained for the South pattern were fairly similar. Such results show that in spite of the bad topographic situation of the antennae, the accuracy of the hyperbolic fix after correction has remained highly satisfactory.

We have not entered into an investigation of the causes of residual deviations. Such a study should be made with great care on account of the diversity of the causes.

The causes may either be determined or merely assumed.

The deviations may be due:

- to the geodetic reference (position determined by theodolite);
- to insufficient knowledge on propagation (for example, virtual position of the receiving antenna due to interference on board the ship and generating "head effect");
- to the equipment itself (for example, resolving power of the receivers);
- etc.
A STUDY OF THE LANE IDENTIFICATION PROCESS

Lane identification processes of hyperbolic radiopositioning chains with carrier waves of about 2 MHz very often yield indications which are not easily used. These indications are reliable, but sometimes there is an error of one lane or more. The examination of calibration results of Hi-Fix trials has opened the way to a critical study of the lane identification process.

Principle of the process: Let us briefly recall this principle with an example. The chain is using two frequencies at the same time, HF and LF; the LF frequency is exactly equal to 9/10 ths of the HF frequency.

Example:

— reading of HF receiver: ............... 45
— reading of LF receiver: ............... 72

The lane identification box subtracts

0.45 — 0.72 = 0.73 \text{(mod. 1)} (*)

and multiplies the result by 10, i.e. 7.3. This figure is the phase of a hypothetical chain using a frequency equal to 1 tenth of the HF frequency, thus generating 10 times wider lanes. In the sequence of HF readings

0.45 — 1.45 — 2.45 — ... — 9.45

the closest to 7.3 is 7.45, and this is the reading adopted.

Calibration consequences

It is obvious that rather great calibration corrections are to be applied to the readings of the HF and LF receivers. Unfortunately a careful calibration of the LF chain was not made. The constant part of the calibration corrections of this chain has however been determined, and this, for the North pattern, is \( \Delta \Phi_0 \).

The corrections to be applied to the indications given by the lane identification box have then been determined on the basis of two different assumptions:

— First assumption: correction of land paths are the same for both chains; therefore they cancel out by subtraction. The correction of the indications of the lane identification box will therefore be reduced to the constant:

\[ 10 (\Delta \Phi_0 - \Delta \Phi'_0) \]

— Second assumption: it is more grounded to assume that velocity over land \( V_t \) is the same for both the HF and LF waves. If the correction for the North HF pattern is of the following form

\[ [- (BM)_t + (BS)_t] p - \Delta \Phi_0 , \]

(*) IHB Note: Mod. 1 means: to within an integer.
for the North LF pattern it will be as follows

\[ - (BM) + (BS) \] \( p' - \Delta \Phi_0 \).

\( V_t \) being common to the two superposed chains, we can see easily that

\[ p' = 0.9 \]

and the correction to be applied to the indication of the lane identification box is:

\[ - (BM) + (BS) \] \( p + 10 (\Delta \Phi_0 - \Delta \Phi'_0) = \{ - (BM) + (BS) \} \( p + \Delta \Phi_0 \}

\[ + (9 \Delta \Phi_0 - 10 \Delta \Phi'_0) \]

This equals the calibration correction for the HF chain, to within a constant. More accurately, to within a constant, it varies within the limits of 2 to 3 tenths of a lane. These are narrow, although not negligible limits. This correction differs little from the one obtained in the first assumption. It is finer and is easy to apply as soon as the calibration chart is available.

**Results obtained:** The operators' attention having been centred on the accuracy trials more than on those for lane identification, the indication given by the lane identification box was registered for 48 positions randomly chosen on both patterns, i.e. equivalent to 100 positions on one pattern.

For convenience' sake, the difference between the theoretical box reading and the observed and corrected reading has been called "divergence", the correction being determined for each one of the assumptions mentioned above.

Let us note that:

— the average of observed divergences can be zero only in the case when the constant parts \((\Delta \Phi_0, \Delta \Phi'_0)\) of the calibration corrections have been perfectly determined;

— this point being established, the divergence should not exceed 0.4 in absolute value. A divergence of 0.6 or more reveals an error of one lane; a divergence of 0.5 leads to uncertainty.

The rather small number of observations was ill-suited to a statistical study. The results obtained are, however, summed up in the percentages given below, although it may be noted that these are not highly significant. The percentage of uncertainties or errors in the readings given by the lane identification box is, according to whether or not a calibration correction is applied to the readings, as follows.

<table>
<thead>
<tr>
<th>Corrections applied</th>
<th>Percentage of errors or of uncertainties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 pattern only</td>
</tr>
<tr>
<td>zero (uncorrected readings)</td>
<td>10 %</td>
</tr>
<tr>
<td>constant (1st assumption)</td>
<td>2 %</td>
</tr>
<tr>
<td>fine (2nd assumption)</td>
<td>0 %</td>
</tr>
</tbody>
</table>
Let us point out that in the second assumption the observed divergence varied from $-0.4$ to $+0.4$. With a greater number of observations the divergences would perhaps have been larger in absolute value, which in consequence would have been the cause of uncertainties (0.5), or even of errors (0.6 and more).

Nevertheless it remains true that the results obtained are of great interest, considering the unfavourable location of the chain antennae. Above all it is important to recall that, in the light of this study, any utilization or critical study of a lane identification process depends closely on a careful calibration.