NAVIGATION USING THE ALTITUDE AND AZIMUTH OF AN ARTIFICIAL SATELLITE

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1. — INTRODUCTION

When a ship has a radio sextant and can measure the altitude and azimuth of an artificial satellite she can find her position by one observation. The ship does not need to radiate any electro-magnetic waves. The characteristics of a radio-wave from a satellite are simple, and any satellite whose position is accurately known can be used for this purpose. Navigation by distance measurement gives the maximum error on the line connecting two subsatellite points. Navigation by Doppler shift also gives the maximum error on the subsatellite track. Navigation by altitude and azimuth, however, gives a good position if the ship is at the subsatellite point. Therefore this method can also be used to complement the other two methods.

2. — CALCULATION OF THE POSITION

The position fixing is carried out using the differences $\Delta\alpha$ between the observed and the estimated altitudes, and ΔZ between the observed and the estimated azimuths, α and Z being the observed altitude and the observed azimuth of the satellite. One observation, therefore, yields a pair of values, $\Delta\alpha$ and ΔZ.

The difference of altitude $\Delta\alpha$ arises from the difference of latitude Δl and the difference of longitude Δh between the true position and the dead reckoning position. This relation is given by :

$$\Delta\alpha = \left(\frac{d\alpha}{d\theta} \cdot \frac{\partial\theta}{\partial l}\right) \Delta l + \left(\frac{d\alpha}{d\theta} \cdot \frac{\partial\theta}{\partial h}\right) \Delta h \tag{1}$$

θ being the geocentric zenith distance, l the latitude of the ship, and h the difference of longitude between the satellite and the ship. Between α and θ there is the following relation

$$\frac{\cos\alpha}{R + H} = \frac{\cos(\alpha + \theta)}{R} \tag{2}$$

R being the radius of the earth, and H the height of the satellite. Differentiating this equation, we obtain :

$$-\frac{d\alpha}{d\theta} = 1 + \frac{\mathrm{R} \sin \alpha/(\mathrm{R} + \mathrm{H})}{\sin(\alpha + \theta) - \mathrm{R} \sin \alpha/(\mathrm{R} + \mathrm{H})} \tag{3}$$

Between θ, l and h there are the following relations

$$\frac{\partial \theta}{\partial l} = \cos \mathrm{Z}$$

$$\frac{\partial \theta}{\partial h} = \cos l \sin \mathrm{Z} \tag{4}$$

where Z is the azimuth. Inserting values of R, H, α, Z and l based on the dead reckoning position into equations (3) and (4), we are able to calculate the coefficients of equation (1).

ΔZ, the difference of azimuth, is calculated in a similar way. In this case we obtain

$$\Delta \mathrm{Z} = \cot \theta \sin \mathrm{Z}\, \Delta l - \left(\frac{\sin l}{\sin^2 \theta} - \frac{\cos \theta}{\sin^2 \theta} \sin d\right) \Delta h \tag{5}$$

where d is the declination of the satellite.

Thus we have two equations, (1) and (5), with two unknown values Δl and Δh. We can solve this system of simultaneous equations and obtain values for Δl and Δh which will be used for correcting the dead reckoning position.

3. — TWO OR MORE OBSERVATIONS

If we can see and use two satellites at the same time, we can have two pairs of values for $\Delta\alpha$ and ΔZ. Or else, provided that the satellite moves and that the observation can be made in a short length of time (perhaps ten minutes or less), we may repeat the observations and get a number of pairs of values for $\Delta\alpha$ and ΔZ. Thus, we shall have more than two equations for the two unknowns, and will be able to use the least squares method to determine Δl and Δh more accurately.

It is also possible to solve more complex problems. As concerns the altitude, we must take into account the fact that a radio wave suffers refraction when passing through the ionosphere and the troposphere. The value for this refraction can be approximated since it is proportional to cot α, except at very low altitudes. The coefficient of proportionality depends on the total number of electrons in the ionosphere as well as on the index of refraction of the ground. As a rule this value cannot be determined by observation from an ordinary ship. We may therefore consider this coefficient of proportionality as the third unknown, which we shall designate β, and can add the term β cot α to equation (1).

$$\Delta\alpha = \left(\frac{d\alpha}{d\theta}\frac{\partial\theta}{\partial l}\right)\Delta l + \left(\frac{d\alpha}{d\theta}\frac{\partial\theta}{\partial h}\right)\Delta h + \beta \cot \alpha \tag{6}$$

There are many causes of azimuth error, but the error in the north reference of the gyrocompass itself is considered to be the most important. However, since this error can retain the same value for a while it is considered as the fourth constant unknown, and denoted as C, which is then added to equation (5). There are now four unknowns Δl, Δh, β and C, for the determination of which we must have at least two pairs of values for $\Delta\alpha$ and ΔZ.

4. — THE ERROR IN POSITION LINES OBTAINED BY ALTITUDE MEASUREMENTS

In this and the following sections we shall treat the case of a single satellite. The position line obtained by altitude α is a small circle, the centre of which is the subsatellite point, and whose radius is θ. According to equation (3) an altitude error $\delta\alpha$ introduces an error $\delta\theta$ in position line, and this is given by

$$\delta\theta = \left\{ \frac{R}{R + H} \frac{\sin\alpha}{\sin(\alpha + \theta)} - 1 \right\} \delta\alpha \tag{7}$$

$\delta\theta$ denoting the displacement of the position line either towards the satellite or away from it.

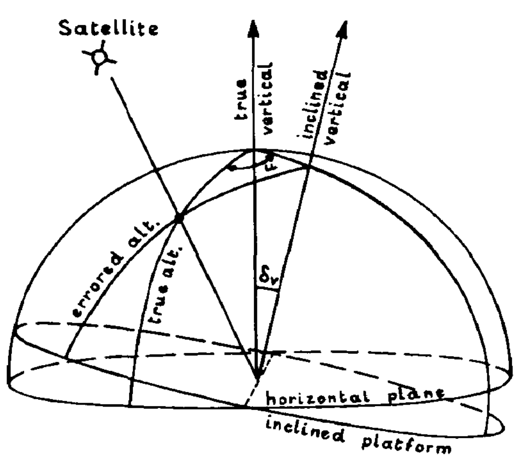

FIG. 1.

The causes of error in the altitude measurement are considered to be the directional sensitivity σ_A of the aerial, and the vertical sensitivity σ_V of the platform. The altitude error caused by the vertical sensitivity is

$$\delta\alpha = \sigma_V \cos u \tag{8}$$

u being the angle between the direction of the satellite and the vertical plane containing the " inclined " vertical axis (figure 1). As the angle u can extend over the whole 360°, the " effective " (*) value of the altitude error due to this effect will be $(1/\sqrt{2})\ \sigma_V$. Similarly, the altitude error due to the aerial sensitivity is $(1/\sqrt{2})\ \sigma_A$. These two errors are thought to be independent of each other, and the resultant of the two effects will therefore be :

$$\delta\alpha = \left\{ \frac{1}{2}\,\sigma_V^2 + \frac{1}{2}\,\sigma_A^2 \right\}^{\frac{1}{2}} \tag{9}$$

This effect arises solely from the sensitivity of the instruments and is independent of the altitude.

In equation (7) the coefficient of $\delta\alpha$ becomes $-H/(R+H)$ at the subsatellite point. At the limit of visibility where the angle α has become zero, the coefficient becomes -1, and its absolute value is maximum. In general, the coefficient is between -1 and $-H/(R+H)$. In this connection, the lower the height of the satellite, the more accurate are the position lines. However, a low satellite can only be seen from relatively small areas. The most unfavourable value, -1, is the same as the one obtained in conventional astronomical navigation using the natural celestial body. In the case of a synchronous satellite, $H/(R+H)$ has a value of 0.8487. In any event when both the height H and the instrument sensitivity are known, the error in position line is dependent solely on θ.

5. — ERROR IN POSITION LINES FROM AZIMUTH MEASUREMENTS

The position lines obtained by azimuth are the well-known special lines radiating from the subsatellite point. Due to an azimuth measurement error δZ, the position line is displaced by a quantity δS

$$\delta S = \frac{\cos l \sin \theta}{\cos d \sqrt{1 - \cos^2 l \sin^2 h}}\ \delta Z \tag{10}$$

where δS is expressed in nautical miles if δZ is counted in minutes. When the declination d is given, the observer in the higher latitudes can get a better value at the same distance than the one in a lower latitude. At the same latitude the error is approximately proportional to $\sin\theta$. If we can freely choose the declination of the satellite it would be best to choose a lower declination. In the very extreme case, the measurement of the azimuth of the Polestar does not give us the position line : in fact all we get is a check on the azimuth measurement. In this respect the most favourable satellite is one remaining always on an equatorial orbit.

The azimuth measurement error is composed of the errors arising from the sensitivity of the aerial, the vertical sensitivity and the north reference sensitivity. The aerial sensitivity effect is :

$$\frac{1}{\sqrt{2}}\,\sigma_A \sec\alpha \tag{11}$$

(*) $\delta\alpha$ varies in fact according to the cosine curve whose average quadratic value is $1/\sqrt{2}$. This concept of " effective "value is used in electricity in the study of alternating current.

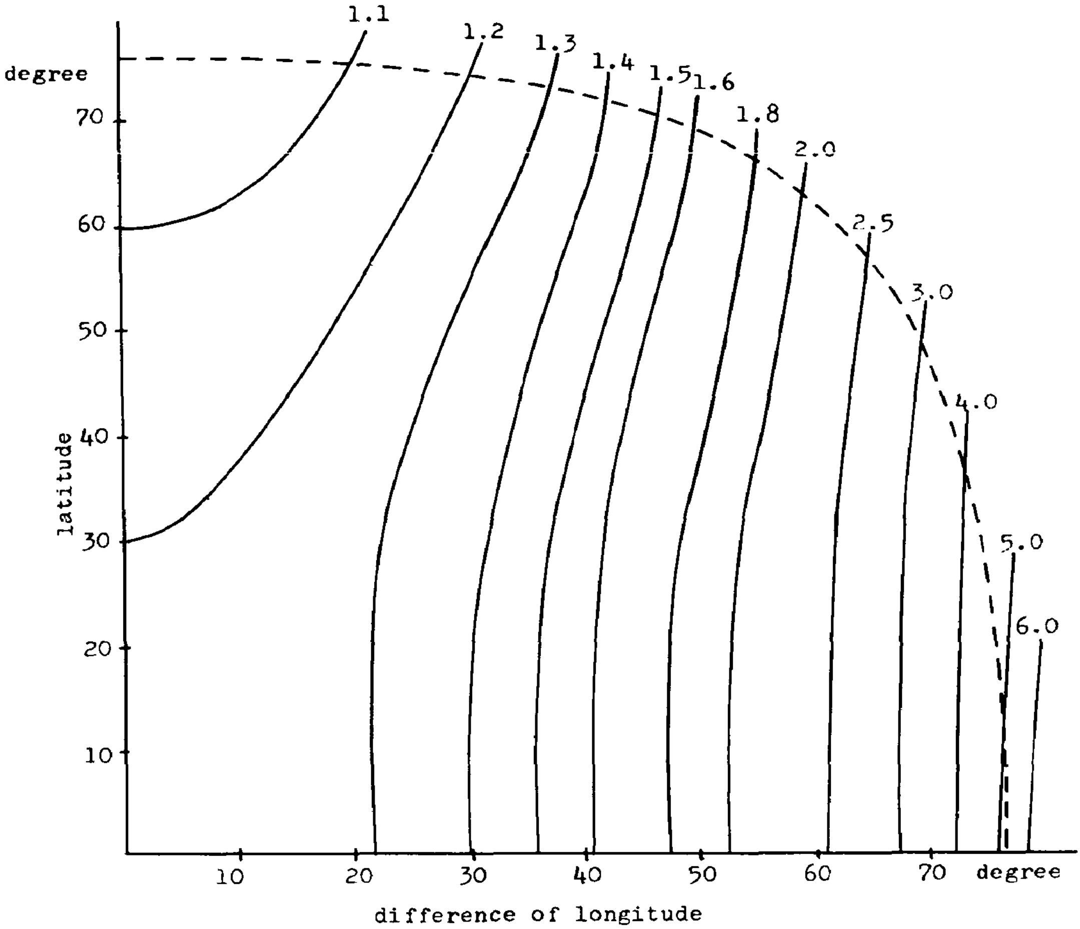

FIG. 2. — The error in the fix, in nautical miles, Case (i) : $\sigma_A = 1'$, $\sigma_V = 1'$, $\sigma_N = 1'$. Dotted line shows the limit of altitude 5°.

the vertical sensitivity effect being :

$$\frac{1}{\sqrt{2}}\,\sigma_V \tan\alpha \tag{12}$$

The north reference error σ_N has a direct effect on the azimuth error. As all these three are independent of each other, the resultant error is :

$$\delta Z = \left\{\frac{1}{2}\,\sigma_A^2 \sec^2\alpha + \frac{1}{2}\,\sigma_V^2 \tan^2\alpha + \sigma_N^2\right\}^{\frac{1}{2}} \tag{13}$$

At low altitudes, only σ_A and σ_N have to be taken into account. Conversely, in the vicinity of the subsatellite point, altitude α becomes nearly 90°, and the terms σ_A and σ_V become larger than σ_N. Thus we can make the following approximation :

$$\delta Z_{\alpha \to 90^\circ} \doteq \left\{\frac{1}{2}\,\sigma_A^2 + \frac{1}{2}\,\sigma_V^2\right\}^{\frac{1}{2}} \frac{1}{\cos\alpha} \tag{14}$$

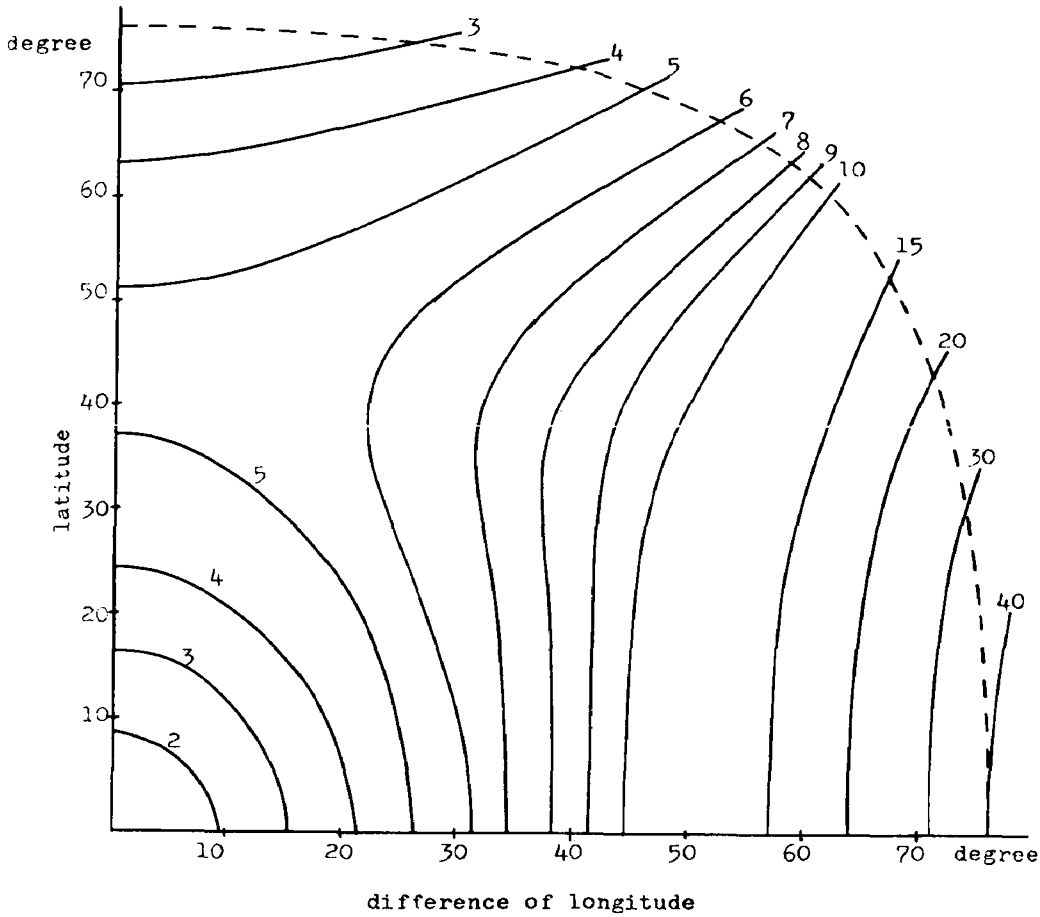

FIG. 3. — The error in the fix, in nautical miles, Case (ii) : $\sigma_A = 1'$, $\sigma_V = 1'$, $\sigma_N = 10'$. Dotted line shows the limit of altitude 5°.

In addition, if the value of θ is close to zero equation (2) becomes :

$$\cos \alpha = \frac{R + H}{H} \sin \theta \tag{15}$$

Inserting equation (15) into (14), we have :

$$\delta Z_{\theta \to 0} \doteqdot \left\{ \frac{1}{2} \sigma_A^2 + \frac{1}{2} \sigma_V^2 \right\}^{\frac{1}{2}} \frac{H}{R + H} \frac{1}{\sin \theta} \tag{16}$$

Inserting equation (16) into (10), and making $l = d$, and $h = 0$, we obtain the following relation :

$$\delta S_{\theta \to 0} = \left\{ \frac{1}{2} \sigma_A^2 + \frac{1}{2} \sigma_V^2 \right\}^{\frac{1}{2}} \frac{H}{R + H} \tag{17}$$

This is the same result as we obtain for the case of altitude measurement at the subsatellite point. For in fact when the radio sextant is pointing vertically upwards we know that the ship is at the subsatellite point, to within the error due to aerial and to vertical sensitivity. In this case, the north reference error has no influence on the position error.

For a synchronous satellite ($H = 3.578 \times 10^7$ m) the calculation of position line errors has been made for all the serviceable areas.

We have taken both σ_A, the directional sensitivity of the radio sextant, and σ_V, the vertical sensitivity, with a value of 1′. For the north reference error we have taken two cases : (i) $\sigma_N = 1'$ and (ii) $\sigma_N = 10'$.

In case (i) the displacement of the position line is less than 1 mile in about half of the areas in which the altitude of the satellite is more than 5°. In high latitudes, the position line error is small. The worst conditions are those at low latitudes and at greatest differences of longitude East and West. In case (ii) the error is the same as for case (i) at the subsatellite point, but it increases with θ up to about $\theta = 30°$. At the limit of visibility the error is nearly ten times greater than in case (i).

6. — THE ERROR IN THE FIX

Given that the error on the altitude position line is $\delta\theta$, the error on the azimuth position line δS, and the angle between the two position lines γ, the position is determined as :

$$\{(\delta\theta)^2 + (\delta S)^2\}^{\frac{1}{2}} \operatorname{cosec} \gamma \qquad (18)$$

Since the position line by altitude is at right angles to the azimuth, the angle between the azimuth and the position line by azimuth is $(90° - \gamma)$. The direction K of the position line by azimuth is given by

$$\tan K = \tan q \cos \theta \qquad (19)$$

q being the parallactic angle, i.e. the azimuth of the ship as seen from the satellite.

The position error is calculated using the same assumptions as in the preceding section. The results are shown in figures 2 and 3. In case (i) the error is nearly constant in the areas where $\theta < 20°$, and this constant value is

$$(\sigma_A^2 + \sigma_V^2)^{\frac{1}{2}} \frac{H}{(R + H)} \cdot$$

It amounts to 1.2 mile at the subsatellite point. In areas where the difference of longitude is larger, the error increases. On the same longitude the higher latitudes give a better position.

In case (ii), the error is the same as in case (i) at the subsatellite point, but increases with θ up to around 30°. In this case also, large differences of longitude give rise to large errors. In latitudes higher than 45° the error decreases.

7. — CONCLUSION

a) In navigation by altitude and azimuth measurement it is desirable that the ship be situated at a higher latitude than the satellite. Therefore the most appropriate satellite is a synchronous satellite with an orbit always lying on the equator.

b) At the subsatellite point the position error is dependent on aerial sensitivity and vertical sensitivity only. North reference sensitivity does not enter into this particular error. Thus, when the north reference error is greater than either the aerial sensitivity or the vertical sensitivity the position error at the subsatellite point is smaller than it would be anywhere else. This is the remarkable merit of this method in contrast to other methods of navigation by satellite, such as the distance measurement or the transit methods.

c) On the contrary, at the limit of visibility where the altitude is very low it is the north reference error which has the prime influence on the position error.

d) When a synchronous satellite is used, the position error is smaller in the high latitudes. This makes it possible for us to use this method in the higher latitudes, notwithstanding the fact that gyrocompasses usually offer poor indications in such higher latitudes.

e) Because the position error is maximum when the difference of longitude is large, if several synchronous satellites can be arranged at suitable intervals the conditions of maximum error can be eliminated. When such satellites are arranged at every 60° of longitude we can avoid the disadvantage of having to observe differences of longitude of more than 30°. Furthermore, where the difference of longitude approaches 30° we are able to see two satellites — one east and one west — and can eliminate both the constant error in azimuth and the refraction error affecting the altitude.

Notations

α : Altitude of the satellite
θ : Geocentric zenith distance of the satellite
l : Latitude of the ship
h : Difference of longitude between the satellite and the ship
R : Radius of the earth
H : Height of the satellite above the surface of the earth
Z : Azimuth of the satellite
d : Declination of the satellite
Δl : Difference of latitude between the true position and the dead reckoning position of the ship
Δh : Difference of longitude between the true position and the dead reckoning position of the ship

$\Delta\alpha$: Difference between the observed and the estimated altitude of the satellite

$\Delta\theta$: Difference between the true geocentric zenith distance of the satellite and its zenith distance computed from its dead reckoning position

ΔZ : Difference in azimuth of the satellite between observed and estimated azimuth

β : Coefficient of proportionality of refraction to cot

C : Constant north reference error

σ_A : Directional sensitivity of the aerial

σ_V : Vertical sensitivity of the platform

δ : Prefix to α, θ, Z or S to express the error in the altitude, zenith distance, and azimuth or position line by azimuth

u : Angle made by the direction of the satellite with the vertical plane containing the inclined vertical axis

σ_N : North reference sensitivity

γ : Angle between the altitude position line and the azimuth position line

S : Lateral displacement of the position line by azimuth

K : Direction of the position line by azimuth

q : Parallactic angle