CONSTRUCTION OF SEXTANT ANGLE GRAPHS WITH THE AID OF A COMPUTER

by F. P. Mawdsley
Extra Master, Hydrographic Surveyor
and R. W. Drozdiak, B. Sc.
Computer Programmer of the Mersey Docks and Harbour Board

In its simplest form the construction of a sextant angle graph may be achieved without calculations provided that the three or four survey marks and the survey area can be plotted on the same plotting sheet. When this is not possible, then the lattice must be prepared by calculation. The method used by the Mersey Docks and Harbour Board for preparing a lattice was by choosing a grid framework; the boundary northings and eastings of which embraced the survey area. Three or four suitably disposed survey marks were chosen, the coordinates of which were known, and the points along the grid boundary where the bearings from one pair of marks intersected were calculated for every half degree change in bearing. Rays were drawn across the framework between points of equal bearing. The differences of bearing between the two marks were given at every intersection, and arcs were then drawn joining points of equal difference of bearing (equal subtended angle, figure 1). The process was then repeated for the second pair of survey marks, thus giving two families of curves. The calculations involved were simple but tedious.

It was decided to see if the computer could be used to reduce the time and effort in producing sextant angle graphs (as a large proportion of the regular surveys in the River Mersey are made using station pointer fixes). If we could mark on the plotting sheets positions which facilitate the drawing of the position circles without first having to draw the rays of bearings, then time would be saved, and the plotting sheet would look far less confusing. The computer therefore would have to print out coordinates for a particular subtended angle to give sufficient control to draw the arcs.

In figure 2, A and B are the survey marks, the coordinates of which are known. P is any point in the framework, and we wish to evolve a formula to obtain the difference of northing between any point P and one of the survey marks. Similarly, a formula is required to find the difference of easting. A, being the northern survey mark, was chosen as the reference point from which to calculate these differences. The different positions of P were chosen to give the maximum number of different cases which can be found in practice, and which might result in different formulae for the required difference of northings and difference of eastings. All the calculations were made assuming that the limits of the distances involved are such that the earth may be assumed to be flat. This limits the programme to maximum distances of about 12 miles, and this may reduce its usefulness. For our own requirements, maximum distances have been less than about 5 miles. This limitation avoids the necessity of having to consider particular projections, and change in scale factor, etc.,

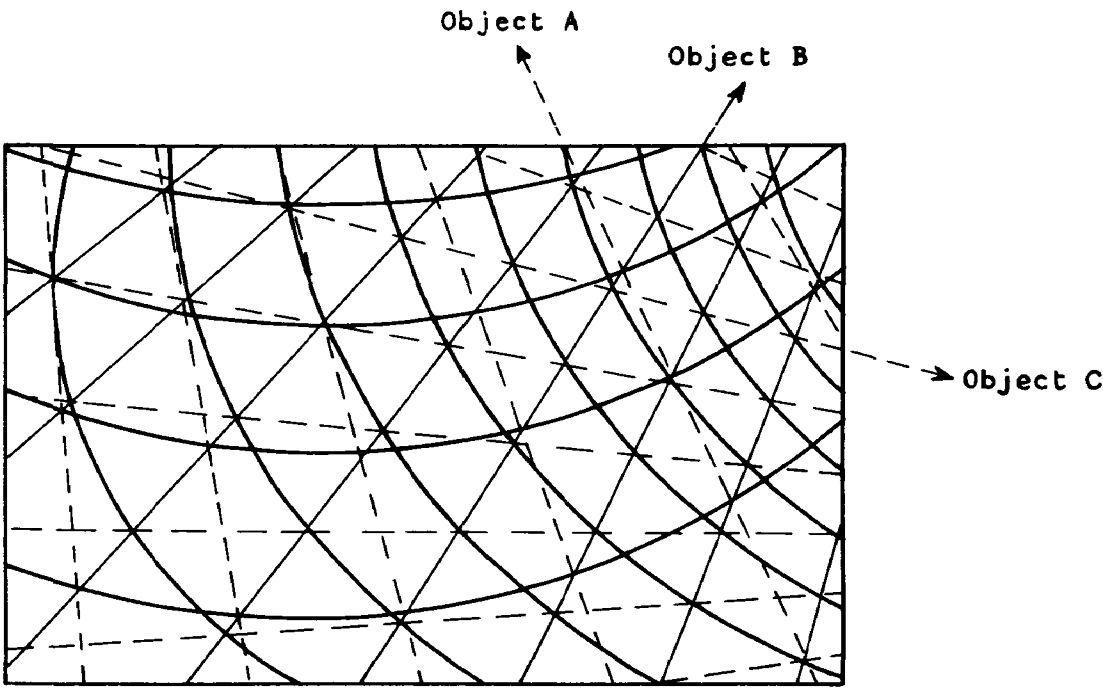

FIGURE 1

Let the difference of northing A — B = z
» » A — P = PW
» » B — P = W

Let the difference of easting A — B = DE
» » A — P = x
» » B — P = y

With the help of geometry the following equation may be obtained :

$$PW^2 - PW(\text{CotP}(x - y) - z) + x(y - z \,.\, \text{CotP}) = 0$$

This equation is valid for all cases, provided the following sign convention is introduced.

Differences of northing will be reckoned algebraically along cartesian axes — positive towards the North, and differences of easting positive towards the East. When P is East of AB the + angle P is used, when P is West of AB the — angle P is used. Let $R = \text{Cot}\,P\,(x - y) - z$, and let $Q = y - z \cdot \text{Cot}\,P$, then $PW^2 - R \cdot PW + x \cdot Q = 0$.

Therefore $PW = \frac{1}{2} R \pm \sqrt{\left(\frac{1}{2} R\right)^2 - Q \,.\, x}$ and A(northing) + PW = P(northing).

We now wish to obtain a formula for calculating the easting for any given northing and subtended angle. This was evolved in a similar manner :

$$x = \frac{1}{2} \cdot S \pm \sqrt{\left(\frac{1}{2} \cdot S\right)^2 - T \,.\, PW}$$

where $S = \text{Cot}P(W - PW) - DE$ and $T = De \,.\, \text{Cot}P + W$

Therefore Easting $A + x =$ Easting of P.

For any subtended angle and easting, there are two solutions for the corresponding northing, that is, the line of easting intersects the position circle in two places. Similarly, for any given subtended angle and northing, there are two solutions for the corresponding easting. This fact is evident from the 2nd order equations obtained above.

We now have expressions for finding an easting for a given northing and subtended angle, and a northing for any given easting and subtended angle. Because angle P changes its sign when P lies to the west of the line AB we have to decide how the signs behave when both A and B lie on the same northing. The procedure for finding PW and x was repeated with A and B on the same northing and it was found that if the western survey mark is chosen as A, when P lies to the North of the line AB, then angle P is positive, and when P is south of the line, then angle P is negative.

In programming these results, the instructions were reasonably straightforward until the time came to instruct the computer which sign angle P was to take. Two or three methods were tried but the only satisfactory way was to determine visually on which side of the line the survey area lay and to print this instruction on the data card. This means that the survey area cannot lie across the line AB, or AB extended. In considering the practical aspect of sextant angles the angles would either increase to 180 degrees or decrease to zero if this were the case. In both these situations another pair of objects would, in practice, be chosen. From the same considerations, angles below 15 degrees and above 165 degrees were rejected. These limits, of course, may be altered.

Summarizing to date, we have a method of determining northings (eastings) where the position circles cut a given easting (northing). Thus if a number of eastings (northings) are chosen conveniently spaced, the northings (eastings) can be plotted giving sufficient control to draw the arcs. After programming this far, the computer would print columns giving the subtended angles, northings and eastings and the information could be plotted on the plotting sheet.

The next step was to convert these co-ordinates to gird distances. Hence : On a chosen easting, P (northing) — northing (south limit) = D_1.

Then grid distance = $D_1 \times 100 \times$ natural scale (in centimetres) (assuming metres as units of measurement).

Also, chosen easting — easting (western limit) = D_2.

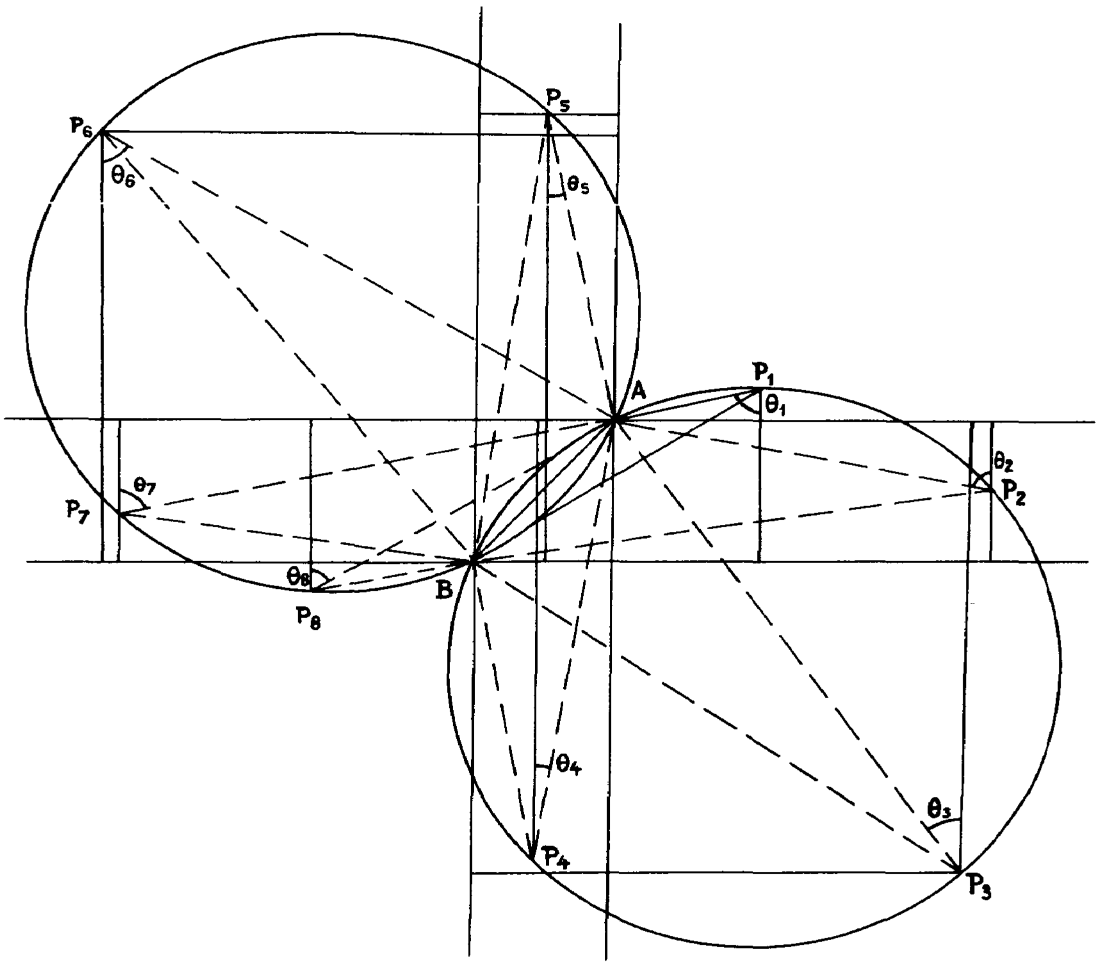

FIGURE 2

Then grid distance = $D_2 \times 100 \times$ natural scale (in centimetres). At the Mersey Docks and Harbour Board two grid systems are used : the National Grid system and the Dock Board system. The unit of measurement of the former is the metre, whereas the latter uses feet. Therefore, when using the Dock Board system a conversion factor is used so that grid distances may be measured in centimetres. Because the distances involved are small and the local scale factor is practically unity for these purposes, no allowances have been made for them in the calculations. Ground and grid distances are therefore considered to be the same.

The computer can now be programmed to print :

Angle	Easting	Northing	Grid distances from S.W. boundary	
			North	East

(See Figure 3 for computer output)

Plotting was carried out using a plastic foil printed with a millimetric grid. A point is chosen as the S. W. boundary of the framework and the

SEXTANT ANGLE GRAPH

REFFRENCE MARKS A= 149500.00, 348000.00   B= 149000.00, 349000.00

LIMITS   N= 151000.00, S= 149500.00, W= 348500.00, E= 349500.00

TEST SEXANG   AREA EAST OF LINE (BC)

NATIONAL GRID   NATURAL SCALE = 1/ 1000.00   CHART DISTANCES

ANGLE	EASTING	NORTHING	EASTING	NORTHING
35.0	348500.00	150870.94	0.0	137.09
40.0	348500.00	150662.93	0.0	116.29
45.0	348500.00	150500.00	0.0	100.00
50.0	348500.00	150368.49	0.0	86.85
55.0	348500.00	150259.70	0.0	75.97
60.0	348500.00	150167.63	0.0	66.78
65.0	348500.00	150088.84	0.0	58.88
70.0	348500.00	150019.88	0.0	51.99
75.0	348500.00	149958.82	0.0	45.88
80.0	348500.00	149904.09	0.0	40.41
85.0	348500.00	149854.47	0.0	35.45
90.0	348500.00	149809.02	0.0	30.90
95.0	348500.00	149766.98	0.0	26.70
100.0	348500.00	149727.76	0.0	22.78
105.0	348500.00	149690.87	0.0	19.09
110.0	348500.00	149655.91	0.0	15.59
115.0	348500.00	149622.54	0.0	12.25
120.0	348500.00	149590.48	0.0	9.05
125.0	348500.00	149559.50	0.0	5.95
130.0	348500.00	149529.39	0.0	2.94
135.0	348500.00	149500.00	0.0	0.0
35.0	349000.00	150928.15	50.00	142.81
40.0	349000.00	150691.75	50.00	119.18
45.0	349000.00	150500.00	50.00	100.00
50.0	349000.00	150339.10	50.00	83.91
55.0	349000.00	150200.21	50.00	70.02
60.0	349000.00	150077.35	50.00	57.74
65.0	349000.00	149966.31	50.00	46.63
70.0	349000.00	149863.97	50.00	36.40
75.0	349000.00	149767.95	50.00	26.79
80.0	349000.00	149676.33	50.00	17.63
85.0	349000.00	149587.49	50.00	8.75
90.0	349000.00	149500.00	50.00	0.0
35.0	349500.00	150696.52	100.00	119.65
40.0	349500.00	150359.15	100.00	85.91
45.0	349500.00	150000.00	100.00	50.00

FIGURE 3

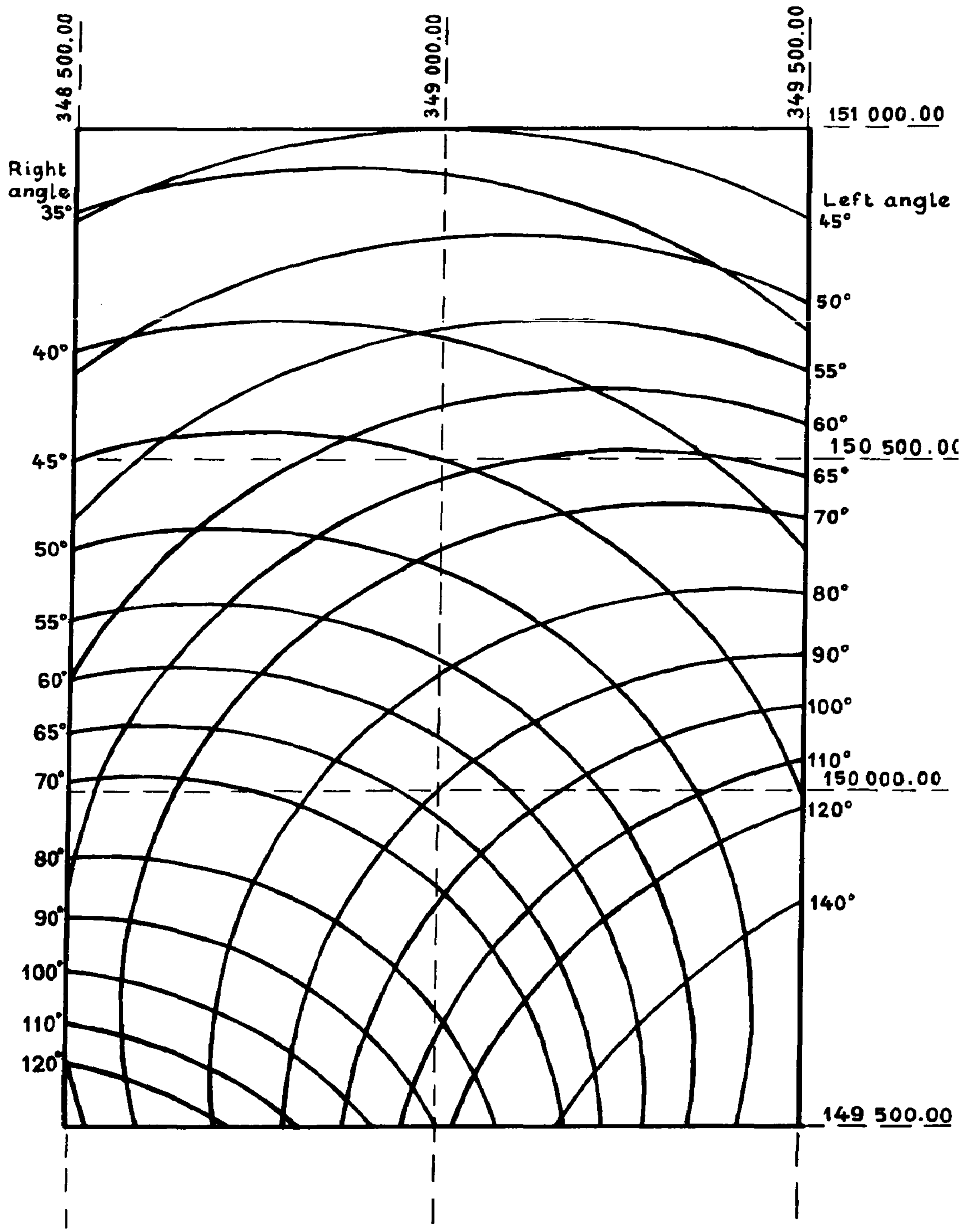
348 500.00
349 000.00
349 500.00
151 000.00
150 500.00
150 000.00
149 500.00
Right angle
35°
40°
45°
50°
55°
60°
65°
70°
80°
90°
100°
110°
120°
Left angle
45°
50°
55°
60°
65°
70°
80°
90°
100°
110°
120°
140°

FIGURE 4

grid distances are measured from this point without the use of beam compasses. For the natural scales used in port surveys, this method is considered sufficiently accurate, and vastly reduces the time taken to produce a lattice. A chart of the survey area is prepared on stable transparent material, and the lattice is transferred to it. For regular surveys, dye line prints of the working sheets are made as necessary.

The information to be given to the computer is first printed on a Key Punch Data Sheet. One data sheet is used for each set of required position circles. Instructions for completing the sheet are printed on it.

The following is an example which, it is hoped, will make the procedure clear :

— Co-ordinates of the three survey marks :

1) 348 000.00 E 149 500.00 N
2) 349 000.00 E 149 000.00 N
3) 350 000.00 E 150 000.00 N

— Co-ordinates of limits of survey area :

348 500.00 E — 349 500.00 E 149 500.00 N — 151 000.00 N

Natural scale : 1/1 000.

Ground distance : 500 metres (i.e. ground distances between chosen northings (eastings) for which the eastings (northings) are to be calculated).

Angle increment : 5 degrees (This means that the computer will print out results for every 5-degree alteration in subtended angle).

This information is printed on the Data sheet .

(As the Mersey Docks and Harbour Board use two grid systems, the computer is instructed accordingly, in order to calculate the chart distances.)

One of the sheets of the computer output from which the lattice was drawn (figure 4) is reproduced above (figure 3).

CONCLUSION

In practice this method of producing a lattice has proved both very satisfactory and much faster than the previous method. The limitations which at first appear to restrict its use are not as narrow as the limitations of the sextant. The accuracy of the computer output depends only on the accuracy of the information supplied on the data sheet. In drawing the curves, the accuracy depends on the amount of control obtained. If a family of curves lie almost tangential to the eastings covering the survey area, then the number of points calculated on the eastings where the curves cross will be very few, and it is unlikely that sufficient control will be obtained to draw the curves. But for the same curves, the points calculated on the northings where the curves cross will give excellent control. It is for this reason that the computer has been programmed to print out results for both eastings and northings.

The system used by the Dock Board of plotting the results on a millimetric grid and then tracing these onto a transparent chart has proved very satisfactory. It was found that if all the information obtained from the computer was plotted, more control than necessary was gained and so time was saved by selecting sufficient points on certain northings and eastings that gave the best control, instead of plotting the whole output. Being able to produce a graph quickly has meant that graphs have been made for certain areas in the River Mersey that had been previously surveyed either by using station pointer fixes or by transits and distance wire. It is probable that the method would be useful to other conservancy authorities.

Editor's Note : Additional details on the punch card format and the computer programme may be obtained by application to The Marine Surveyor and Water Bailiff, *Mersey Docks and Harbour Board, Dock Office, Liverpool 3., England.*