THE APPLICATION OF TORAN TO SEA TRIALS(*)

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SUMMARY

For the first time Toran has been used for a ship's sea trials.

After recalling the principles of this radio system and giving a brief description of the installations used in these trials, the present article supplies information on the procedures to be used and the performances to be expected from this system.

Examples concerning speed, turning and crash stop trials show up the very good accuracy obtainable. Thanks to this an elaborate analysis of the kinematic properties of the tracks is made possible.

INTRODUCTION

The manœuvrability trials which the *Magdala* carried out in areas of varying depths for studying the influence of the ocean bottom on the manœuvrability of large tankers are described in other articles.

For many of these trials the ship's position had to be determined with accuracy at a given moment in order to know either the distance run by the ship in a given time so that its speed might be deduced, or else the ship's path to determine the typical range of a turn, a stopway, or merely the path itself, in certain manœuvrability trials, as for example astern.

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An accurate positioning system had to be found, one which could be used offshore, out of sight of land; only a radio-electric system met these requirements.

The performances of the existing radio navigation Decca chains with a frequency of the order of 100 kHz are excellent for the requirements of usual navigation and for trials in certain suitable areas. They were however insufficient, in particular in the Seine Bay, for the accurate measurements we wished to make.

A system using higher frequencies, of the order of 1.5 to 2 MHz — the Toran system — was adopted for these trials. This system was first developed by the Compagnie Générale de Géophysique for the requirements of oil prospectors, and then used for hydrographic surveys, aeronautical applications, port dredging and sounding [1].

Magdala's trials at sea were the first application of Toran for accurate determination of certain performances of a ship. The first part of this article will give a brief description of the installations and equipment used, after having recalled the principles of the system. In the second part we shall explain how the Toran chains installed were operated, how measurements were made, and some information will be given on the accuracy of the method and on the processing of data. Finally, part III examines Toran's application to trials, particularly speed trials, pointing out certain strange phenomena that are emphasized by the system's accuracy. The possibilities of further extending the use of Toran for ship trials at sea are considered in the conclusion.

I. — PRINCIPLES AND DESCRIPTION OF THE INSTALLATION

1. Principles of Toran

Toran is a radio-electric system for hyperbolic position-fixing. The general principles of such systems are well known. Only the main characteristics will be recalled, and the typical Toran features pointed out.

A locus of the mobile's position is determined by the difference between the distances that separate the mobile from two fixed stations. Two pairs of fixed stations then allow two hyperbolae to be defined, at whose intersection the mobile is found.

Each of the two sets of points determining the mobile's position is characterized by the phase shift referred to a particular frequency, called "comparison frequency", that exists between two signals received direct from the transmitters. This phase shift is the hyperbolic coordinate of the mobile.

At each of the two fixed stations making up a pair there is a transmitter. In the Toran system each transmitter radiates an unmodulated wave; both transmitters are simply free oscillators, with crystal controlled frequency; they are neither synchronized nor locked in phase or frequency.

In order to eliminate the instability of these free transmitters, an additional fixed receiver — called the compensation receiver — is required, as well as an additional transmitter, that is also fixed, called the reference transmitter, this last being placed close to the compensation receiver.

With both these systems, it is possible to supply the ship with a phase proportional to the differences of the fixed distances covered by the waves from the two transmitter foci. This phase is used as a reference, and because it also contains the instabilities mentioned above, these are eliminated.

A few additional technical details on the Toran principles and on the role of the compensation receiver and the reference transmitter are given as an appendix.

2. Toran chains used for the *Magdala* trials

Two trial areas were chosen for the *Magdala* trials [2]. In each of these areas a temporary Toran chain giving complete coverage in the relevant area was set up.

a) *South Brittany chain*

The chain set up in South Brittany was composed of two pairs of transmitters. The transmitters of the first pair (AA′) were sited at Pointe de Penmarc'h and at Saint-Gildas-de-Rhuys; those of the second pair (BB′) at Saint-Gildas-de-Rhuys and Notre-Dame des Monts. The compensation receivers and the reference transmitter in both cases were installed on Noirmoutier Island.

Transmitter power was 60 W.

The chain's frequency characteristics are given in table I.

TABLE I

	Comparison frequency	Reference transmitter
Pair AA′	1 887 Kc	1 610 Kc
Pair BB′	1 851 Kc	1 784 Kc

The lane width on the baseline joining the transmitting foci of a pair was 79.395 m for pair AA′ and 80.939 m for pair BB′.

Figure 1 shows the siting of this chain.

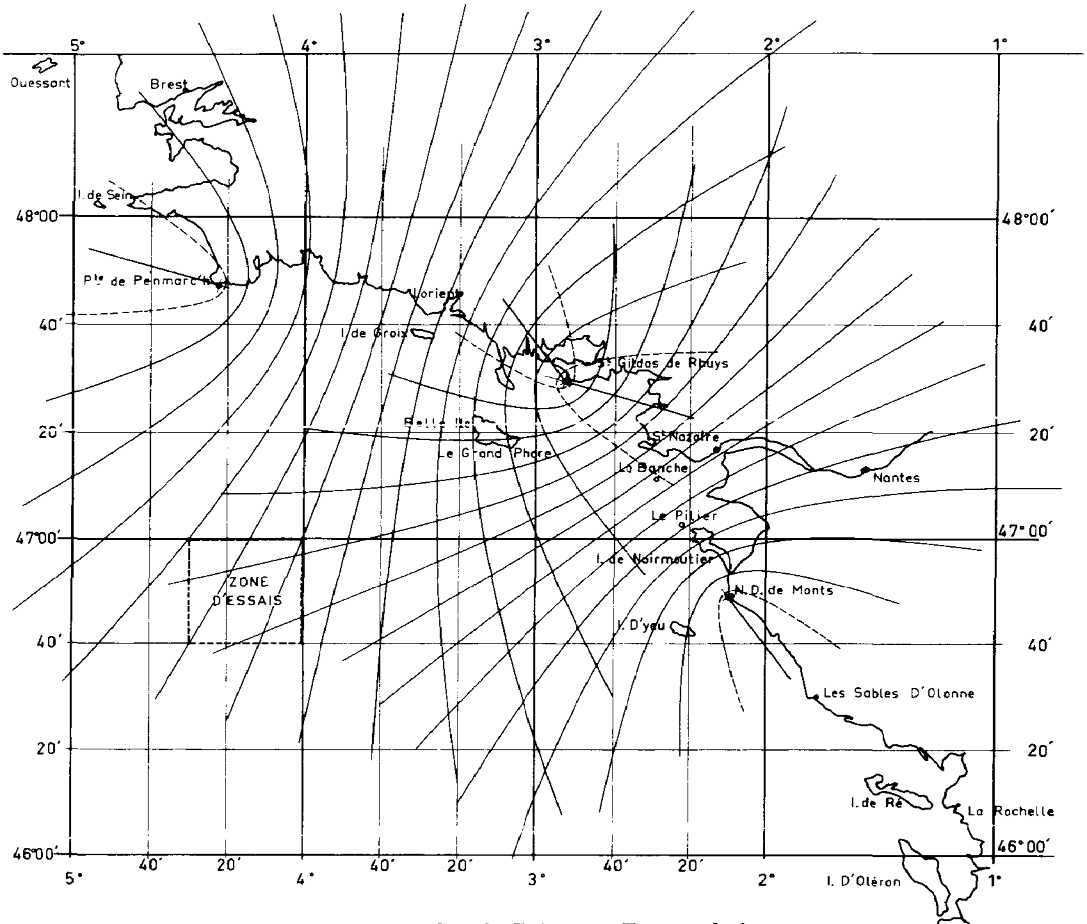

FIG. 1. — South Brittany Toran chain.
Zone d'essais = Area of trials.

b) *Seine Bay chain*

This chain was also made up of two pairs of transmitters; those of the first pair (AA′) were set up near Barfleur and near Courseulles, those of the second pair (BB′) at Heuqueville (close to Cap d'Antifer) and at Pointe du Hoc.

Since the required range was low, the transmitter power was only of the order of 10 W. The comparison frequency for the pairs was :

1 884 Kc for pair AA′
1 816 Kc for pair BB′

which, on the baseline, corresponded to lane widths of respectively 79.522 m and 82.499 m.

For this chain, which is called type Toran X, it should be noted that, following a recent method, the role of reference transmitter for a pair was played by one of the transmitters of the other pair.

This particular method offers certain advantages to the user: it reduces the quantity of material to be set up as well as the number of stations to be tended, thus decreasing the expenses, and also only two radio electric frequencies are used instead of four.

3. Cartography

For both the trial areas 1/25 000-scale charts showing the two lattices of hyperbolae were used. Such charts may be used onboard, either for navigational needs, or else to supply approximate values of the results of the trials quickly and on the spot. The scale chosen is a compromise between the pursuit of accuracy and a desire to maintain charting costs within reasonable limits. If it is thought possible to plot a fix onboard on a chart with a sharp pencil with an accuracy of the order of 0.2 mm on each of its coordinates then the absolute error on the distance between the two points is less than 0.6 mm, which to scale represents 15 m. If the distance to be measured is of the order of 5 000 m — which for instance may be the case for a speed trial — the relative error on the distance is of the order of 15/5 000 or 3/1 000, and for a speed of the order of 16 knots the resulting error will be less than 5/100 knot, on the generally accepted hypothesis that the length of time is known with much greater precision.

4. Equipment onboard

Equipment taken onboard *Magdala* firstly included a Toran receiver, with its antenna, on whose phasemeters the hyperbolic coordinates could be read. In order to obtain a record of the successive Toran fixes a device for digitizing information was installed and was connected to a digital recorder called a "printer". At each recording, which may be either automatic or pedal controlled mechanically, were displayed the time in round seconds and both hyperbolic coordinates in 1/100 of lane.

A timepiece was also available, for totalling the pips delivered at each second by the clock onboard and for releasing the recording system automatically at every tenth second. Finally, an automatic track plotter comprising ten x and y scales — only one of which was actually used during the trials — allowed a record of the successive evolutions of the ship to be kept.

Receiver, track plotter and printer may be seen from left to right on figure 2.

For the whole equipment the required power was available on batteries. A set of filters protected the devices against the ship's own transmissions. In addition, a VHF transmitter-receiver was placed onboard for ensuring liaison with the phase-carrier plane, to which we shall return later.

It is of interest to note that for the preliminary ship trials a magnetic tape recorder was also set up onboard, on which the time factor and the hyperbolic coordinates could be recorded. In fact, it turned out that the printer was sufficient for these particular data, and thus it was decided not to use the magnetic tape recorder.

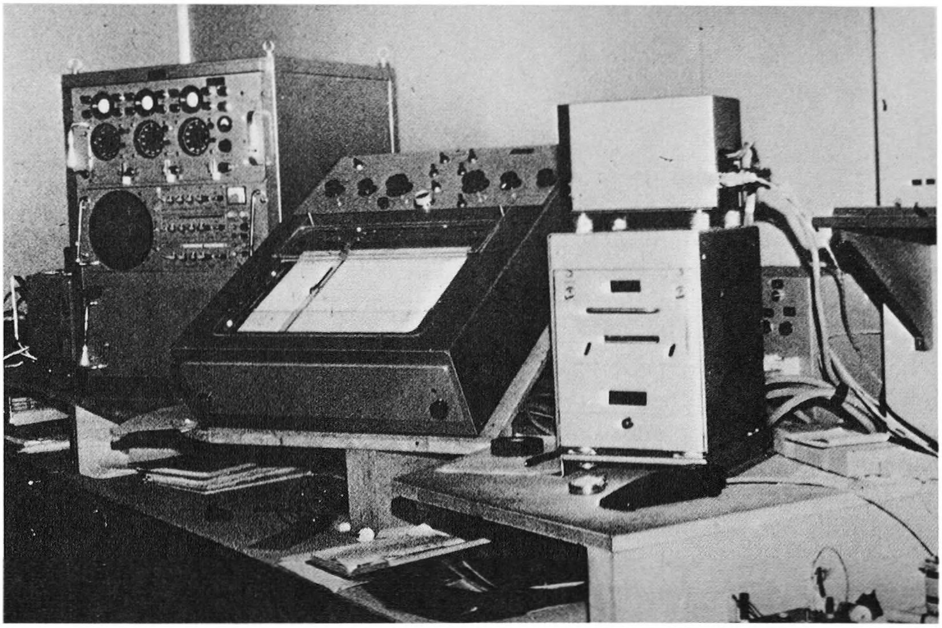

FIG. 2. — Toran equipment on the *Magdala*.

II. — TORAN OPERATION

1. Checking the pattern — Elimination of ambiguities

a) *Phase-carrier plane*

The measured phase difference is known only within a multiple of 2 π. Consequently, phase measurements give rise to position ambiguities, for the phase shifts observed at a given moment vary periodically in space. Totalizers of the number of lanes crossed by the moving ship and connected to phasemeters do not of course obviate the initial ambiguity. One of the methods frequently used to partially eliminate this ambiguity is to superpose upon the basic pattern a homofocal pattern of lower frequency.

In the case of temporary chains to be used for a short time, such an elaborate method for eliminating ambiguities may be avoided and a plane, called the "phase carrier" plane, may be used. This is simply a plane equipped with a Toran receiver.

Figure 3 shows this Toran receiver in the cabin of the helicopter used in the Seine Bay chain.

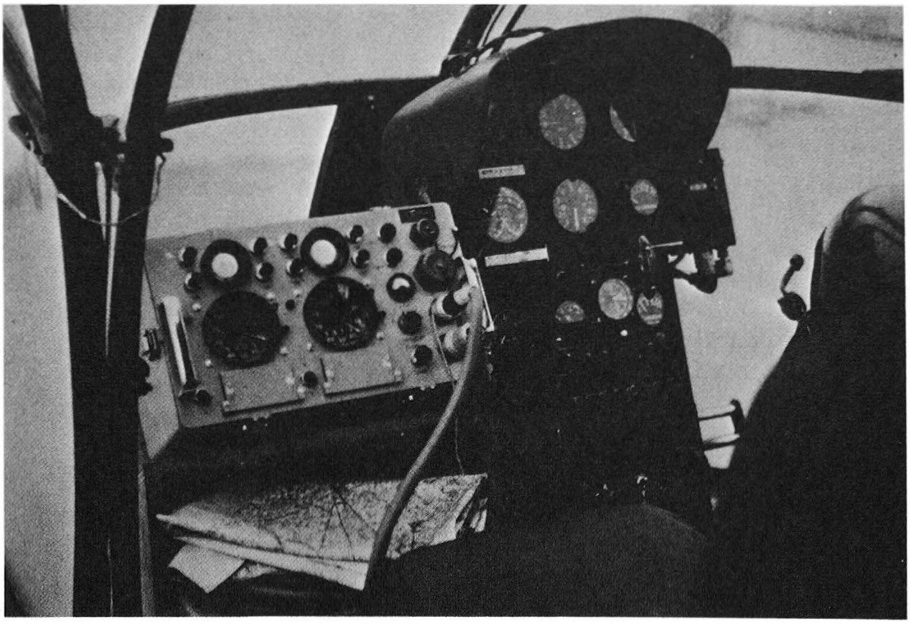

FIG. 3. — Toran receiver on a helicopter.

b) *Checking the chain*

The first role assigned to the phase-carrier plane is checking the chain as it goes into service. This is easily done: the hyperbolic coordinates for certain known conspicuous points ashore are determined by computation, then checked with Toran. As an example, we may mention that the adjustment points used for the South Brittany chain were four in number: the axis of the major N.E. lighthouse on Groix Island, the axis of the Banche lighthouse, a Toran antenna (*) on the Pilier lighthouse, and the Poulain lighthouse in the north-east of Belle-Ile.

c) *Adjustment of the ship's receiver — Elimination of ambiguities*

The initial adjustment of the pattern once made, the adjustment of the ship's receiver may then be carried out. This is done when the plane after having passed vertically over the ship gives the coordinates read on its receiver. These coordinates obviously only need to be known with an accuracy of the order of the lane itself, i.e. about 50 metres.

This initial adjustment is made before the ship sails or, if the ship is coming in from the open sea, when it enters the area covered by the Toran chain. There is no need of re-adjustment as long as the receiver has not been stopped, or as long as there has been no very lengthy interruption in the transmission. Actually, the use of the track plotter allows any random hyperbola jumping due to interference or to radio-

(*) This antenna is the one belonging to the Toran transmitter for the chain covering the Saint-Nazaire — Donges — Nantes approaches.

electric disturbances to be detected immediately. In such a case the receiver can, therefore, be at once re-adjusted.

For both the preliminary and the final trials of the *Magdala* the initial adjustment of the onboard receiver was verified by the phase-carrier plane, or helicopter, and found to be correct.

Random hyperbola jumpings were very few. In the case of the South Brittany chain these were near the coastline and occurred at the beginning of the night and were due to interference caused by the night effect to which all radio electric detection systems are sensitive. A jumping was also observed once in the Seine Bay, this being due to jamming by a radiotelephone transmitter onboard which was wrongly using a frequency very close to the Toran frequency. The jumping was, however, immediately detected, and corrected by the track plotter.

2. Measurements

A fix is obtained simply by reading the phasemeters. To make a series of measurements, the printer has merely to be started and then the recording of information is automatically made every ten or twenty seconds according to the pre-set timing for the time-piece adopted.

As already mentioned, it is possible at any time to start the recording in order to mark either the beginning or the end of a trial, or else to ensure synchronism with other measurement equipment.

In order to allow recorded tapes to be processed, the printing of information required for identification — i.e. date, time, nature of trial, etc. — must not be forgotten.

3. Measurement accuracy

We will not enter into a theoretical discussion on the various parameters affecting the accuracy of measurements, but will confine ourselves to giving practical details which may prove useful both to those in charge of the installation of a provisional mobile chain and to users.

Whenever possible, a long baseline is recommended, since it entails a smaller increase of the lane width when moving away from baselines.

It is not possible to make good use of a highly accurate radio location system unless the propagation velocity of the waves is constant. This is generally the case when the wave paths are entirely above homogeneous ground, for instance the sea in the case of marine applications. The stations should be set up so as to meet this condition. This was done for the *Magdala* trials.

According to specifications supplied by the equipment manufacturers the instrumental error on the phase does not exceed ± 0.5/100 of a hyperbolic lane. The signal/noise ratio may also yield an error of ± 0.5/100 of hyperbolic lane. Topography errors and errors in the propagation velocity may also entail errors of the order of ± 0.5 to ± 1/100 of lane. In good conditions, that is when transmitters are situated directly on the coastline, the overall quadratic error does not exceed ± 1/100 hyperbolic lane for one pattern. For two patterns the resultant error may be approximately and fairly easily obtained at any point in the coverage by measuring the length of the "diagonal" of the curvilinear quadrilateral formed by the intersection of two hyperbolic lanes at the place under consideration. In other words, the greatest absolute position error of a fix should not exceed 6.50 m in the case of the Atlantic chain in South Brittany, and in the Seine Bay this same error would probably be less than 1.50 m. For a length measurement the error could be double this, i.e. respectively 13 m and 3 m.

It is worth noting that the actual accuracy obtainable when determining the length of a measured base is much higher than these figures since these relate to the most unfavourable case where the direction of the base is parallel to the major diagonal of the parallelogram made up by the two lattices of hyperbolae.

By carefully following a heading perpendicular to the closest lattice of hyperbolae, the distance error, estimated under the same conditions, did not exceed 3.50 m in the South Brittany trial, and 1.70 m in the Seine Bay area.

The gain in accuracy obtained by a careful selection of the direction of the measured base is greater for the South Brittany trial area than it is for the Seine Bay. This is simply due to the fact that the ratio between the length of the major diagonal of the parallelogram formed by the hyperbolae and the width of the narrowest lane is nearly 4 in the one case whilst it does not amount to 2 in the second case.

It may be of interest to complete the above information by giving certain results of the verification of the South Brittany chain with the phase-carrier plane.

Control of the radio-electric length of the base has given the results shown in table II.

From this table we see that the discrepancy on the length of the base is 3.2 m for pair AA′ whilst it is 25.1 m for pair BB′.

The first of these two figures is low; the second seems high considering the topographic work on the spot, the accuracy of which is probably better than one metre, and considering the accuracy of the frequency which is probably better than 10^{-6}. The second figure can be explained by a discrepancy in the propagation velocity of the waves when their path is tangent to the coastline; in this case the discrepancy is lower offshore.

Whatever the origin of this radiotopographic error concerning pair BB′ it represents a relative discrepancy of

$$\frac{25}{89\,300} = 2.8.10^{-4}$$

on the length of the base. In the worst assumption that it would also apply to wave paths offshore — which seems unlikely — it would entail an expansion of the hyperbolae of lattice BB′ of $3 \cdot 10^{-4}$ which, under the worst measurement conditions, would entail an error on velocity of the same order, that is, less than 0.5/100 knot for a speed of 16 knots. This being so, it was not thought necessary to correct this error whose practical influence on measurements was entirely negligible.

TABLE II

	Nominal value of phase	Observed value of phase
Pair AA′		
Penmarc'h	4 259.61	4 259.61
Saint Gildas	5 740.39	5 740.43
Electric length of baseline	1 480.78	1 480.82
Length in metres	117 566.5	117 569.7
Pair BB′		
Saint Gildas	4 448.35	4 448.49
Notre Dame des Monts	5 551.65	5 551.48
Electric length of baseline	1 103.30	1 102.99
Length in metres	89 300.00	89 274.90

The accuracy of the South Brittany pattern was, moreover, confirmed by the checking carried out with the phase-carrier plane flying straight over the various lighthouses used as adjustment points. Finally, it may be noted that the value of the phase observed when the ship receiver was crossing the Penmarc'h shadow verifies to within one hundredth — i.e. approximately 80 cm — the value observed with the phase-carrier plane.

4. Processing of measurements

a) *Quick processing onboard*

A rapid first processing of data can be made onboard so as to obtain approximate values during the course of the work. The successive positions of the ship are therefore plotted on Toran charts, and the length of a speed trial base, a crash stop distance and the diameter of a turning circle are measured thereon.

As already seen, the accuracy of the results obtainable with this method is limited by the chart scale used.

If in the course of the work a greater accuracy is desired than is obtainable with the chart, in the first place the trials will have to be carried out within an area for which certain computations have been made in advance, and in the second place, simple calculations will have to be made onboard. In the latter case, the procedure will be as follows.

If α and β are the hyperbolic coordinates of any point M on the chart, and x and y its rectangular cartesian coordinates, x and y are functions of the two variables α and β.

If we assume that the two hyperbolic coordinates α_0 and β_0 and the two rectangular coordinates x_0 and y_0 of a point M_0 are known, and if, in addition, the hyperbolic cordinates α and β of a point M fairly near to M_0 have been measured with the Toran phasemeters, then the cartesian coordinates x and y of point M may be calculated approximately but with a very good approximation, by means of the conventional formulas:

$$x - x_0 = \left(\frac{\partial x}{\partial \alpha}\right)_0 (\alpha - \alpha_o) + \left(\frac{\partial x}{\partial \beta}\right)_0 (\beta - \beta_o)$$

$$y - y_0 = \left(\frac{\partial y}{\partial \alpha}\right)_0 (\alpha - \alpha_o) + \left(\frac{\partial y}{\partial \beta}\right)_0 (\beta - \beta_o)$$

The object of the pre-calculation is to yield the two hyperbolic coordinates α_0 and β_0 as well as the values of the four coefficients :

$$\left(\frac{\partial x}{\partial \alpha}\right)_0 \left(\frac{\partial x}{\partial \beta}\right)_0 \left(\frac{\partial y}{\partial \alpha}\right)_0 \left(\frac{\partial y}{\partial \beta}\right)_0$$

for a certain number of reference points, such as $M_0(x_0\ y_0)$, suitably spread over the area selected for the trials.

This method may be used onboard efficaciously if the number of points to be computed is rather limited; such as is, for example, the case for a speed trial where the computation has only to be made for both ends of each run, that is, for a total of six or eight fixes per trial for respectively three or four runs.

b) *Complete processing*

The drawing of a track may be carried out by plotting fixes on a very large scale chart, at 1/5 000 for example, upon which hyperbolic lattices have already been traced. This is a laborious task which may be suitable for a single trial, but involves an enormous amount of work if several trials have to be processed. The best is to resort to an electronic computer which converts the hyperbolic coordinates to rectangular cartesian coordinates. Plotting the points on millimetric plotting paper is then immediate, whereas determining the distance between two points whose rectangular cartesian coordinates are known entails elementary computation.

The computer may be used for additional calculations. In the case of the *Magdala* the processing programme with the computer yielded not only the x and y cartesian coordinates, but also the successive variations of x and y, as well as the distance the ship covered between two successive fixes. The kinematic analysis of the speed, turning and crash stop trials of the *Magdala* — a report on which is given elsewhere, [2] and [3] — was made from the information supplied by the computer.

III. — TORAN APPLICATIONS TO PARTICULAR TRIALS

1. General

It should be borne in mind that for any given moment Toran provides an accurate position for the antenna installed onboard referred to the bottom, i.e. to the land. When analysing the various trials, methods must be used that either directly eliminate the influence of the current from the result, or else determine the current as an auxiliary parameter, thus allowing the track in relation to the bottom (called here the ground track) to be later converted into the track in relation to the water mass (called here the surface track). This necessity is not peculiar to Toran, as is well known, for it is also the case whenever a fix is referred to a fixed mark, whether by means of a radio-electric system or by sightings on landmarks.

These processing methods are well known and we shall not study them in detail. We shall stress here how the excellent accuracy of Toran allows a very detailed exploitation and a fine analysis of the measurements obtained. We shall also mention some strange results that the *Magdala* trials brought into evidence.

2. Speed trials

Two, three or four runs in alternate directions are required to eliminate the current, according as to whether the current is constant or whether it varies following a linear or parabolic law.

The mean ground speed for one run is obtained by dividing the distance between the two terminal points of the run by the time interval. If a head perpendicular to the closest hyperbolae lattice has been carefully followed, then the error on the distance is minimum, as has been pointed out in paragraph 3 of Part II.

In the case of the South Brittany area, this would entail an absolute error of 3.50 m, or a relative error of 0.7/1 000, for a distance covered of the order of 5 000 m. For a speed of the order of 16 knots this would lead to an absolute error on speed of the order of 1/100 knot, since the timing error is much smaller when the timepiece pulses are controlled by a stabilized crystal clock, as was the case on the *Magdala*.

For the Seine Bay chain where accuracy is twice that of the South Brittany area the error under the same conditions would be reduced by approximately half.

If so desired, it would be easy to know the possible variation of speed during a run. Since recordings are obtained every ten seconds, or in some cases every twenty seconds, it will be possible to determine the mean speed for any time interval that is a multiple of ten (or twenty) seconds.

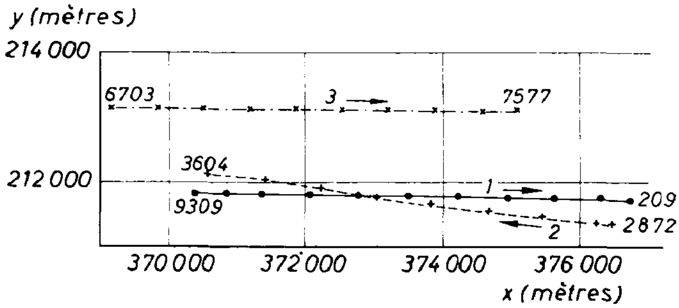

FIG. 4. — Seine Bay. Speed trial in 30-m depths. Tracks.
The beginning and ending times of each run are indicated in seconds.

To illustrate the above, figure 4 shows the diagram of the three runs in the speed trials carried out with the *Magdala* in depths of 30 m; the terminal fixes selected for computing the mean velocity for each of the runs have been plotted, and the successive fixes of the ship every 100 s have also been plotted in between. It will be noted that the time interval between a terminal fix and its next intermediary fix is not equal to 100 s, since the initial and final moments expressed in seconds are not whole multiples of 10.

In figure 5 are shown, for each of the three runs, successive mean speeds for a period of 700 s, over intervals of 100 s. Speed was practically

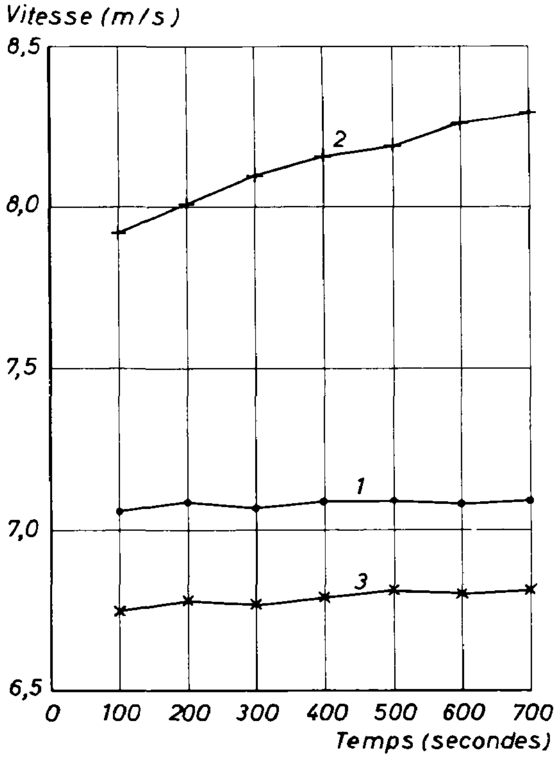

FIG. 5. — Seine Bay. Speed trial in 30-m depths.
The variation in speed during each run.

constant during the first and the last runs; on the other hand, for the middle run, the ship was still accelerating after a complete turn.

A finer analysis of the speed variation during the first run led to figures 6 and 7. Figure 6 shows the fluctuation of the mean speed computed over 10 s all along the first of these runs. The average of the 90 values considered is 7.081 m/s. The deviation either side of this mean speed rarely exceeds 0.1 m/s and is due partly to an actual fluctuation in speed and partly to measurement errors.

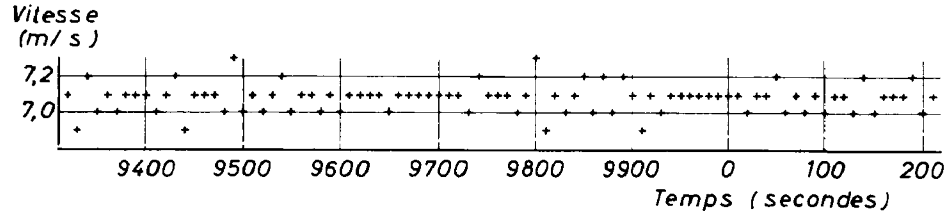

Fig. 6. — Seine Bay. Speed trial in 30-m depths.
First run. Fluctuations in mean speed over 10 s.

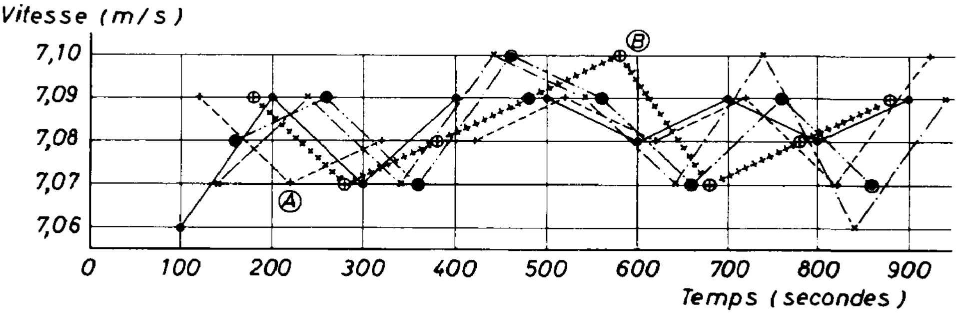

Fig. 7. — Seine Bay. Speed trial in 30-m depths. First run.
Mean speed over 100 s.

Figure 7 shows five graphs of the variation of average of speeds taken over 100 s, the initial times being at intervals of 20 s.

If scattered points, such as A and B, are not taken into account, a certain fluctuation in speed can be observed, the period of which will be of the order of 400 s, and for whose origin we have no explanation.

It is worth noting that the average of the 90 speed values taken over 10 s was found to be rigorously equal to the mean speed computed by taking the same total interval of time and the distance between the corresponding two terminal points. This seems to confirm the excellent accuracy of the measurements and shows clearly that for this run the track of the Toran antenna was almost a straight line.

3. Turning trials

The successive fixes permit us to trace very accurately the ground track followed by the Toran antenna. When two whole turning circles have been made, the current during the trial can be calculated with great accuracy, and thus the track through the water can be determined. This method allows all parameters determining the ship's movement along the turning circles to be obtained.

Figures 8 and 9 illustrate the excellent measurements that Toran's accuracy has made possible.

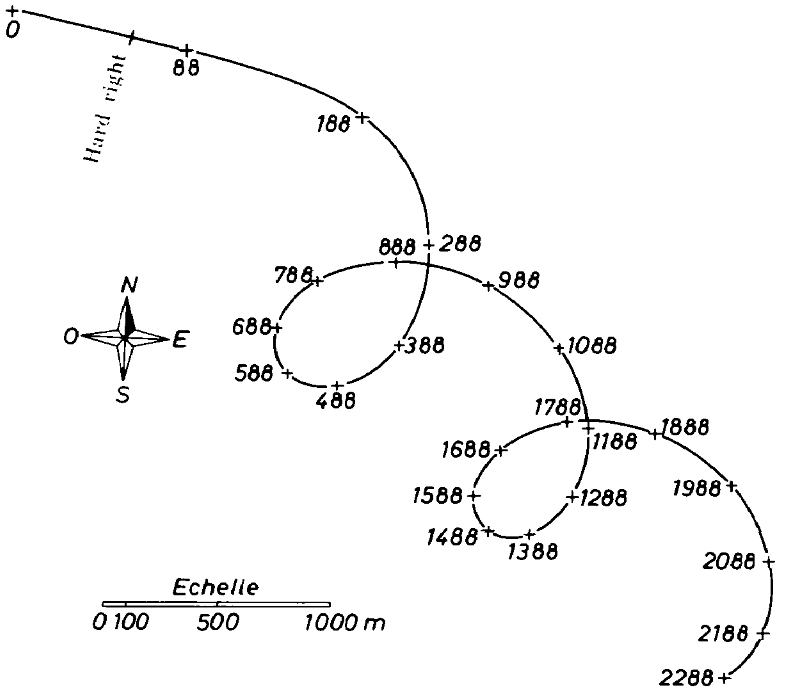

FIG. 8. — Seine Bay. Turning in 38-m depths. Ground track. The figures show the time count in seconds, reckoned from the start of the trial.

4. Crash stop trials

The ground track may be plotted immediately. For determining the track at the surface the speed and the current direction during the trial must be known.

In order to follow the variation of speed easily, graphs giving the variations Δx and Δy over equal intervals of time are plotted. To within a numerical factor, these are graphs of the components of the mean speed computed over equal successive intervals of time.

Figure 10 is an illustration of the above. It shows the values of Δx and Δy for 15 s intervals.

When the ship is stopped at sea and drifts owing to the current, the ultimate values of Δx and Δy represent the components of the current speed, and thus an estimate of the current's speed and direction is possible.

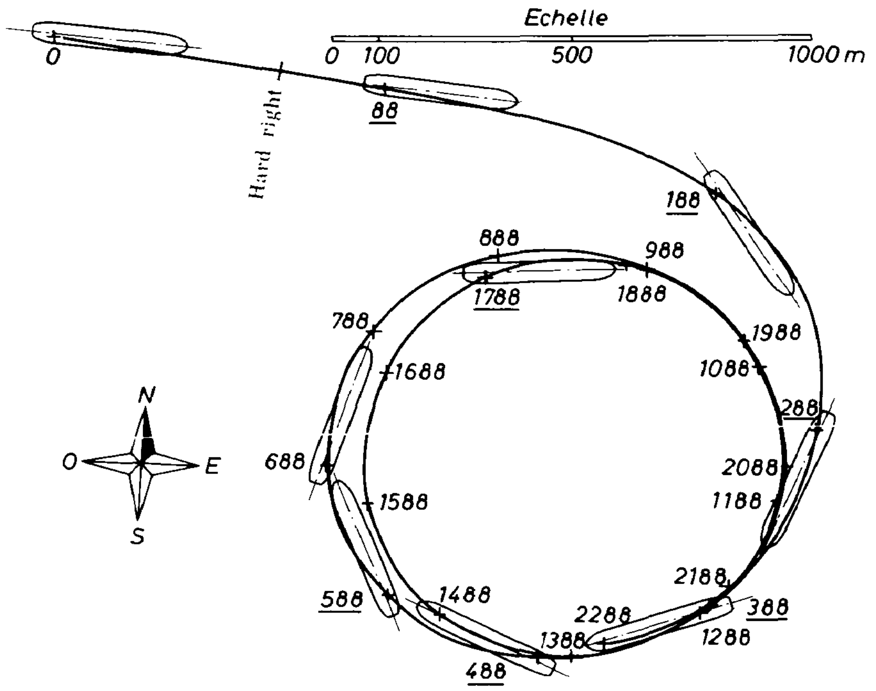

FIG. 9. — Seine Bay. Turning in 38-m depths. Surface track.
At the underlined times the ship is shown in relation to the Toran antenna (+) track.

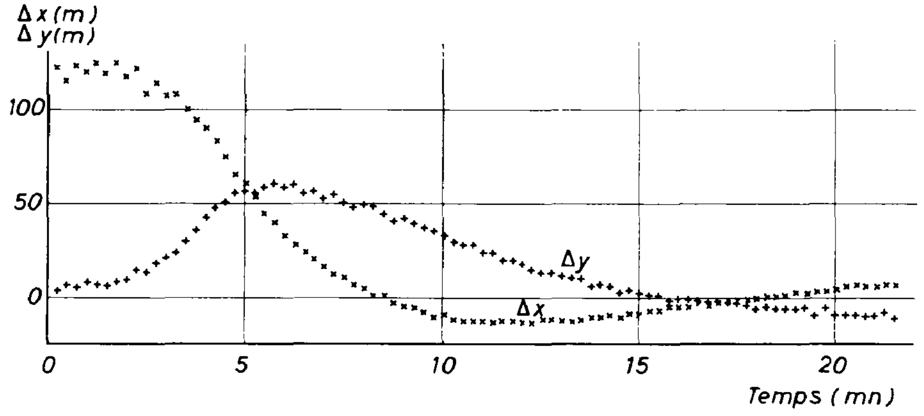

FIG. 10. — Seine Bay. Stopping trial. Variations of Δx and Δy in function of time.
Interval between points : 15 s.

IV. — CONCLUSIONS

In conclusion, the present article, after describing a Toran chain, and using the sea trials of the tanker *Magdala* as an example, emphasizes the service that the Toran system renders for all trials necessitating a very accurate knowledge of the successive positions of the ship at any moment. Several examples have been given, showing the great amount of information obtainable from speed, turning and crash stop trials. Toran's very good performances allow an excellent fine kinematic analysis of the trials. This is the essential point.

It will be recalled that the equipment is easy to operate, and that the onboard processing of measurements gives approximate results very quickly, pending their complete processing by computer.

Finally, the author expresses the hope that the fixed Toran chains at present being installed on the coasts of France, together with the equipment specially designed for trials onboard, will allow the further development of a technique whose use during the *Magdala* trials has proved particularly successful.

APPENDIX

Note on the operation of a pair of Toran transmitters

Some details of the principles of operation of Toran are given in this appendix. Our thanks are due to the Société Sercel for supplying us with the necessary information.

The notations here used are shown in figure 11.

Fixed transmitters A and B transmit on frequencies F*a* and F*b* that are close to one another and which may have any independent phases φ_a and φ_b.

The onboard receiver M receives the two signals transmitted by A and B at the end of periods MA/V and MB/V, V being the velocity of the wave propagation.

Expressed in cycles, the respective phases of the waves received at receiver M are :

$$\varphi_a + F_a \frac{MA}{V} \quad \text{and} \quad \varphi_b + F_b \frac{MB}{V}$$

Receiver M detects the beat signal of these two waves; this beat signal has a frequency $f = F_a - F_b$, and a phase

$$\varphi_m = \varphi_a - \varphi_b + F_a \frac{MA}{V} - F_b \frac{MB}{V} + k_m$$

k_m being a constant phase shift of the beat signal within the receiver's low frequency circuits.

Introducing the beat frequency f, φ_m may be written :

$$\varphi_m = \varphi_a - \varphi_b + \frac{F_a}{V}(MA - MB) + f\frac{MB}{V} + k_m$$

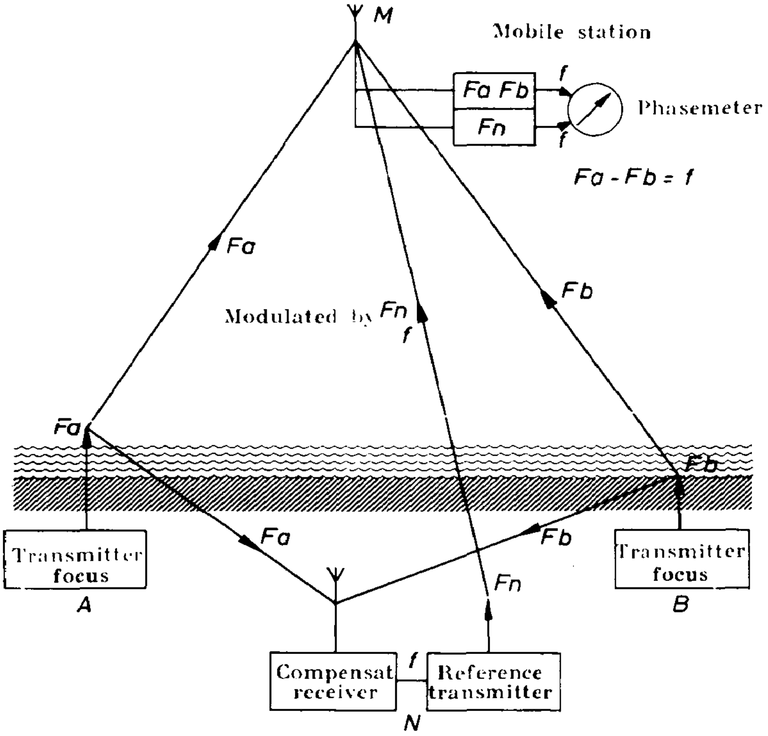

FIG. 11. — Scheme of Toran.

Signals transmitted by A and B are also received by the compensation receiver N which is fixed and identical to the onboard receiver M.

The low frequency beat phase detected by the compensation receiver N is expressed in the same way as for the ship's receiver M :

$$\varphi_n = \varphi_a - \varphi_b + \frac{F_a}{V}(NA - NB) + f\frac{NB}{V} + k_n$$

where k_n is a constant which plays for N the same role as k_m does for M.

The reference transmitter emits to M a wave modulated by the beat signal detected by N. M receives this signal on a second channel and detects the low frequency beat modulating the signal. The phase of this beat signal is then :

$$\varphi_{nm} = \varphi_a - \varphi_b + \frac{F_a}{V}(NA - NB) + f\frac{NB}{V} + f\frac{NM}{V} + k_n + k_{nm}$$

k_{nm} being a constant similar to k_m and k_n.

A phasemeter on the receiver M indicates the phase difference

$$\varphi = \frac{F_a}{V}(MA - MB) + \frac{f}{V}(MB - MN) - \frac{f_a}{V}(NA - NB) - \frac{f}{V}NB + k_m - k_n - k_{nm}$$

In this equation, the unknown terms φ_a and φ_b have been eliminated as they are compensated.

Term F_a/V (MA — MB) is the hyperbolic term that is to be measured.

Terms F_a/V (NA — NB) and f/V BN are constant, since they depend only on A, B and N which are fixed; they are globally eliminated once and for all at the same time as are constants k_m, k_n and k_{nm} when the initial adjustment of the pattern is made during the setting up of the chain.

Term f/V (BM — MN) depends on M, and since f is extremely small compared with

$$F_a \left(\frac{f}{F_a} \neq \frac{1}{20\,000}\right).$$

this term is practically always negligible. It can, however, be taken into account if necessary and introduced as a correction in the computation of the hyperbolic lattices.

Finally, the readings on the phasemeter M thus represent the phase $\varphi = F_a/V$ (MA — MB) evaluated in phasemeter cycles.

Footnote : It will be recalled that in the Toran X method it is one of the second pair of transmitters generating the chain's second hyperbolic pattern that is used as reference transmitter. The number of stations then involved is only four, and the independence of the pairs — an essential factor of the geometric accuracy — is retained, the number of radio-electric frequencies required being reduced to two.

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