

THE RESOLUTION OF TIDAL CONSTITUENTS

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Abstract

The effect of the background noise on the analyzed values of the tidal constituents can be evaluated if it is assumed to be of purely random character. The noise effectively prohibits the resolution of very close constituents while its influence is virtually negligible on well separated constituents. In the latter instance, the interference of the unanalyzable constituents becomes the major source of errors in short analyses.

INTRODUCTION

From experience it is known that the successful resolution of tidal constituents depends on :

- 1) the duration of the observations;
- 2) the size of the constituents;
- 3) the presence of neighbouring constituents;
- 4) the amount of errors or disturbances present in the observations.

These facts may be put on a firm mathematical basis and expressed quantitatively. We shall see that :

- 1) the expected error on the value of any constituent is independent of its amplitude and frequency (LECOLAZET, 1956);
- 2) that it grows drastically when one attempts to separate two or more closely lying constituents (VAN ETTE and SCHOEMAKER, 1966);
- 3) that in ordinary sequences of observation, the expected error for well separated constituents is really negligible compared to the interference of the unanalyzed constituents;
- 4) that an improved resolution of neighbouring constituents depends almost exclusively on extending the interval of observations.

1. The analysis

Let us consider a sequence of $2N + 1$ observations, Δt units of time apart :

$$\{x'(j)\} \quad j = -N, \dots, -1, 0, 1, \dots, N \quad (1)$$

x' may be considered as the superposition of two quantities :

- a) the tidal signal $x(j)$;
- b) the noise $n(j)$ which reflects rounding errors, mistakes in reading, gauge malfunctions, storm surges, tsunamis, etc.

We therefore write :

$$x'(j) = x(j) + n(j) \quad (2)$$

We assume that $n(j)$, in the long run, is a purely random quantity and behaves as white noise :

$$E[n(j)] \simeq \frac{1}{2N + 1} \sum_{j=-N}^N n(j) = 0 \quad (3)$$

$$E[n(j)n^*(j')] \simeq \frac{1}{2N + 1} \sum_{j=-N}^N n(j)n^*(j') = v^2 \delta_{jj'} \quad (4)$$

where

$$\delta_{jj'} = \begin{cases} 0 & j \neq j' \\ 1 & j = j' \end{cases}$$

$E[\]$ means "the expected value of". v^2 is the variance of the noise.

We notice that the tidal constituents do satisfy (3) but on the other hand they are correlated and they do not satisfy (4) for $j \neq j'$.

Whatever the technique of analysis used, the end result will be the representation in the frequency space (σ) of the function (1). In tidal practice we know that the representation of (1) in σ space is of importance only over very narrow frequency bands, especially the diurnal and semi-diurnal bands over which the constituents are very closely packed. In the low frequency band which extends from 0 cycle/day to 1 cycle/day exclusively, the tidal signal becomes very weak. Although the background noise does not increase significantly in this band, it does become of the same magnitude as the tidal signal and normally masks it. We shall focus our interest on the higher frequency bands over which the bulk of the tidal energy is concentrated.

$x(j)$, the tidal signal, can be written as

$$x(j) = \sum_{k=-n}^n a_k e^{2\pi i \sigma_k j \Delta t} \quad (5)$$

a_k is the complex amplitude of the k th tidal constituent and it has to be evaluated; σ_k is its frequency which is known very accurately a priori.

The a_k 's may be considered as the basis of a vector space, while from this point of view, the $\exp [2 \pi i \sigma_k j \Delta t]$'s are operators transforming vectors from one space to another, namely from the frequency space to the time space and vice versa. Thus

$$x = L a \tag{6}$$

is an abstract representation of (5) where L is the linear operator represented by the $\exp [2 \pi i \sigma_k j \Delta t]$'s. The inverse mapping of (6) is

$$a = L^{-1} x \tag{7}$$

which in our case can be written out as

$$a_k \delta(0) = \sum_{j=-\infty}^{\infty} e^{-2 \pi i \sigma_k j \Delta t} x(j) \tag{8}$$

$\delta(0)$ is the value of the delta function $\delta(x)$ at $x = 0$, namely ∞ .

On account of the limited span of observations, (8) has to be replaced by

$$\sum_{m=-n}^n A_{km} a_m = \sum_{j=-N}^N e^{-2 \pi i \sigma_k j \Delta t} x(j) \equiv c_k \tag{9}$$

where

$$A_{km} \equiv \sum_{j=-N}^N e^{-2 \pi i (\sigma_k - \sigma_m) j \Delta t} \tag{10}$$

and therefore tends to $\delta(\sigma_k - \sigma_m)$ for $j \rightarrow \infty$.

The solution for the a_m 's is

$$a_k = \sum_{m=-n}^n A_{km}^{-1} c_m \tag{11}$$

which is the end result of a standard analysis on a finite segment of observations. For the choice (10), (11) gives a *least square fit to the observations*. A_{km}^{-1} are the elements of the inverse matrix A^{-1} .

For an actual set of tidal observations, such as given by (1), the end result of an analysis is

$$a'_k = \sum_{m=-n}^n A_{km}^{-1} c'_m = \sum_{m=-n}^n \sum_{j=-N}^N e^{-2 \pi i \sigma_m j \Delta t} x'(j) A_{km}^{-1} \tag{12}$$

a'_k is only an estimate of a_k .

2. The expected error

From (1) and (12), we see that the error on a_k is

$$\Delta_k = \sum_{m=-n}^n \sum_{j=-N}^N A_{km}^{-1} e^{-2 \pi i \sigma_m j \Delta t} n(j) \tag{13}$$

Δ_k is a complex quantity like a_k . It is easier to consider its absolute value square :

$$|\Delta_k|^2 = \sum_m \sum_{m'} \sum_j \sum_{j'} A_{km}^{-1} A_{km'}^{-1} e^{(\sigma_m j - \sigma_{m'} j') 2\pi i \Delta t} n(j) n^*(j')$$

Assuming $n(j)$ behaves as white noise, (3) and (4) yield

$$E[|\Delta_k|^2] = \frac{\nu^2}{|A|^2} \sum_m \sum_{m'} \sum_j (-)^{m+m'+2k} \alpha_{km} \alpha_{km'} \alpha_{kk}$$

since

$$A_{km}^{-1} = \alpha_{km} / |A|$$

α_{km} being the minor of A_{km} , and $|A|$ the determinant of A . The expression can be compressed further by noting that

$$\sum_m (-)^{m+k} A_{mm} \alpha_{km} = \delta_{mmm} |A|$$

is the development of A into its minors. Therefore

$$E[|\Delta_k|^2] = \frac{\nu^2 \alpha_{kk}}{|A|} \quad (14)$$

a concise expression for the expected absolute square error on the amplitude of the constituent a_k .

Now we will evaluate what this error amounts to for the three cases of well separated constituents, of doublets and of triplets.

a) *Well separated constituents*

In the case of good separation

$$A_{km} = \delta_{km} (2N + 1) + \epsilon_{km} \quad (15)$$

where the ϵ_{km} 's are quantities an order of magnitude smaller than $2N + 1$; they will therefore be ignored from our subsequent considerations.

(14) becomes

$$E[|\Delta_k|^2] = \nu^2 / (2N + 1) \quad (16)$$

in which ν^2 can be estimated from :

$$\nu^2 \simeq \frac{1}{2N + 1} \sum_{j=-N}^N |x'(j) - \sum_k a'_k e^{2\pi i \sigma_k j \Delta t}|^2 \quad (17)$$

(16) indicates that the expected square error is independent of frequency, of the amplitude a_k of the constituent and that it is inversely proportional to $2N + 1$, which measures the duration of the observations.

To 95% probability, the amplitude of a_k will fall between the limits

$$\pm 2\nu / (2N + 1)^{1/2} \quad (18)$$

In the physical notation of a constituent as $A_k \cos(\sigma_k t - \zeta_k)$ which is the equivalent of the $a_k \exp[2\pi i \sigma_k j \Delta t] + a_{-k} \exp[-2\pi i \sigma_k j \Delta t]$ in our formulas with $a_{-k} = a_k^*$ for real observations, A_k being the real amplitude and ζ_k , the phase lag; $|a_k|$ is equivalent to $(1/2)A_k$ and ζ_k is the phase lag

of a_k . The limits of confidence on A_k, ζ_k are therefore

$$\Delta A_k = \pm 4\nu/(2N + 1)^{+1/2} \tag{19}$$

$$\Delta \zeta_k \simeq \pm 4\nu/A_k(2N + 1)^{+1/2} \text{ radians} \tag{20}$$

to 95% confidence (LECOLAZET, 1956).

We can introduce some numbers into (18) in order to obtain an idea of its magnitude. In ordinary observations on the vertical tide, a value of $\nu = 3$ cm is about of the usual order of magnitude. For this value, and

$$2N + 1 = 25 \text{ hours(1 day), } 181 (\sim 1 \text{ week}), 361 (\sim 15 \text{ days}), \\ 8000 (\sim 1 \text{ year}) .$$

$$\pm 2\nu/(2N + 1)^{+1/2} = \pm 1.2, 0.45, 0.32, 0.07 \text{ cm.}$$

The major tidal constituents such as M_2, K_2, S_2, N_2, K_1 and O_1 are of the order of 30 cm, so that with a background noise of 9 cm² their value should be affected by a relatively slight uncertainty if they happen to be well separated. On the other hand, the margin of error gives the minimum value of the separated constituents which can be considered as meaningful from a given analysis.

b) *A Doublet*

From short sequences of observations, it may happen that two constituents of importance such as M_2 and S_2, M_2 and N_2, K_1 and O_1, K_1 and P_1 , etc., are not well separated. Mathematically this means that in (15) the ϵ_{km} 's relating to the pair are no longer negligible with respect to $2N+1$ and that the matrix A loses its quasidiagonal character.

In this case we return to (12) and (14) and rewrite the matrix as

$$\begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & \\ 0 & & A_R \end{pmatrix} \tag{21}$$

in which the two close constituents have been labelled 1 and 2. The remainder matrix A_R can have any form.

In (21) :

$$A_{11} = A_{22} = 2N + 1$$

$$A_{21} = A_{12} = \frac{\sin[(2N + 1)\pi\Delta t(\sigma_1 - \sigma_2)]}{\sin[\pi\Delta t(\sigma_1 - \sigma_2)]}$$

From (14) the expected error is :

$$E[|\Delta_1|^2] = E[|\Delta_2|^2] = \frac{\nu^2 A_{11} |A_R|}{\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} |A_R|} = \frac{\nu^2 A_{11}}{\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}} \simeq \\ \simeq \frac{3\nu^2}{\pi^2 (2N + 1)^3 \Delta t(\sigma_1 - \sigma_2)^2} \tag{22}$$

The approximation holds when $(2N+1)\pi\Delta t(\sigma_1-\sigma_2)$ is a small quantity; this depends on the frequency difference $\sigma_1-\sigma_2$ being so small that in spite of its multiplication by $2N+1$, there results still a small quantity.

(22) may be rewritten as

$$E[|\Delta_1|^2] = E[|\Delta_2|^2] = \frac{12\nu^2}{(2N+1)(\Delta\phi)^2} \quad (23)$$

where

$$\Delta\phi \equiv 2\pi(2N+1)\Delta t(\sigma_1-\sigma_2) \text{ radians} \quad (24)$$

$\Delta\phi$ measures the relative phase difference between the two constituents over the interval of observations. For instance, if we consider the doublet $S_2 K_2$, over an interval of 360 hours, $\Delta\phi$ amounts to :

$$\Delta\phi = 2\pi \times 361 \times (0.0833333333 - 0.0835614924) \text{ rad} = 0.52 \text{ rad} \quad \text{or} \quad 29.65^\circ$$

Then

$$E[|\Delta_{K_2}|^2] = E[|\Delta_{S_2}|^2] = 45\nu^2/(2N+1)$$

so that the margin of error on the analyzed amplitude of the doublet considered would be increased by a factor of about 7 compared to the margin of possible error for the well separated constituents.

So by attempting to resolve a close doublet, the margin of expected error is increased by a factor of $2\sqrt{3}/\Delta\phi$ which becomes infinitely large for infinitely close constituents. Once again the expected error depends on the variance of the noise and it is independent of the amplitude and frequency of the constituents considered.

c) A triplet

Now we attempt the resolution of three very close constituents. Amongst the triplets of interest, we have $N_2 M_2 S_2$ and $T_2 S_2 K_2$. We may label the elements of the triplets as 1, 2 and 3. Then (14) yields

$$E[|\Delta_1|^2] = \frac{\nu^2 \alpha_{11}}{|A|} = \frac{\nu^2 \begin{vmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}} \quad (25)$$

with similar formulas for constituents 2 and 3. To particularize formulas such as (25) one must use some special relationships between the σ 's of the constituents. For the triplets concerned

$$\sigma_1 - \sigma_2 \approx b \quad \text{and} \quad \sigma_2 - \sigma_3 \approx 2b$$

where $b = 0.54^\circ/\text{hour}$ for $N_2 M_2 S_2$ and $b = 0.04^\circ/\text{hour}$ for $T_2 S_2 K_2$. Under these special circumstances, we may write

$$E[|\Delta_{N_2}|^2] = E[|\Delta_{T_2}|^2] = \frac{80\nu^2}{(2N+1)(\Delta\phi)^4} \quad (26)$$

$$E[|\Delta_{M_2}|^2] = E[|\Delta_{S_2}|^2] = \frac{180\nu^2}{(2N+1)(\Delta\phi)^4} \quad (27)$$

$$E[|\Delta_{S_2}|^2] = E[|\Delta_{K_2}|^2] = \frac{20\nu^2}{(2N+1)(\Delta\phi)^4} \quad (28)$$

where

$$\Delta\phi = (2N+1)2\pi\Delta t b .$$

The equalities between the expected values of the triplets are only formal as the b 's are different. We note from (26) to (28) that the margin of probable error increases by a factor of $(\Delta\phi)^{-2}$ compared to a factor of $(\Delta\phi)^{-1}$ for a doublet, at least for the special set considered here and that the constituent in the middle suffers the most. If we take $\Delta\phi \sim 1/2$ radian, the margin of error is increased by factors of 35, 54 and 18 for the elements of the triplets considered with the margin of probable error of a single well separated constituent.

Therefore there is a drastic increase in the probability of making an error in the resolution of two or more close constituents. The risk of error can be eliminated simply by not attempting to resolve close lying constituents.

3. Rayleigh's criterion

We now have to define precisely what we mean by well separated or by close constituents or better, what we mean by separable and non-separable constituents. This may be done by considering the matrix which depends on the relative phase difference between the pairs of constituents analyzed from a set of observations.

In the instance of a doublet

$$A_{21} = A_{12} \simeq \frac{1}{\pi\Delta t(\sigma_1 - \sigma_2)}$$

while :

$$A_{11} = A_{22} = 2N + 1 \quad (29)$$

A matrix with elements like (29) is a source of numerical instability during the course of the inversion of the matrix A and its use should be avoided. The criterion for the off diagonal elements to be smaller than the diagonal elements is

$$\frac{1}{\pi\Delta t(\sigma_1 - \sigma_2)} < 2N + 1$$

or

$$(2N + 1)\pi\Delta t(\sigma_1 - \sigma_2) > 1$$

or

$$\Delta\phi > 2 \text{ radians or } 115^\circ \quad (30)$$

Those familiar with the practice of the analysis of tides know that the criterion used for the separability of constituents is the so called Rayleigh's criterion which states that two constituents are separable if their relative phase difference over the interval of observation equals or exceeds 2π radians :

$$\Delta\varphi \geq 2\pi \text{ radians or 1 cycle} \quad (31)$$

Rayleigh's criterion is derived from the laboratory practice of optics where it makes physical sense. It is a rule, not a mathematical theorem. (30) indicates that in tidal analysis, it is *overstringent*. For constituent phase differences less than the one given (31), the resolution is still possible with a margin of probable error which can be calculated with the help of (4). (14) does indicate that the error risks becoming very great for close lying constituents. For constituents a fair distance apart, although less than the one indicated by (31), the probable error may remain within tolerable limits.

In our personal practice of tidal analysis we resolve constituents which lie as close as 0.8 cycle apart. This rule may be considered as our definition of separable and non-separable constituents.

$$\Delta\varphi \geq 0.8 \text{ cycle or } 1.6 \pi \text{ radians or } 288^\circ \quad (32)$$

4. The relative unimportance of the background noise compared to the influence of the unresolved constituents

For two relatively close constituents resolved according to criterion (32), the margin of probable error is approximately given by

$$\frac{\pm 2\gamma}{(2N+1)^{1/2} \frac{(1 - \sin^2(1/2)\Delta\varphi)^{1/2}}{(1/4)\Delta\varphi^2}} < \frac{\pm 2\gamma}{(2N+1)^{1/2} (1 - 4\Delta\varphi^{-2})^{1/2}} \approx \frac{\pm 2\gamma}{(2N+1)^{1/2}} \quad (33)$$

for all the pairs satisfying (32).

The criterion of separability (32) therefore allows us to use (18) as an adequate measure of the margin of error. We notice in section 2a that it amounts to a fraction of a centimetre for ordinary values of the background noise. This fact simply indicates that there is little smearing of pairs of close constituents if they satisfy at least the criterion of separation (32).

We are still left with the problem of the presence of the close (hidden, unanalyzable) constituents which could not be resolved on account of the very fact that the presence of the background noise makes their resolution virtually impossible or at least highly untrustworthy. These hidden constituents turn out to be the major problem of tidal analysis and not really the presence of the unavoidable background noise.

To trace the effect of these hidden constituents a series of twenty-four consecutive monthly analyses were made of the vertical tide at Victoria, B.C. The results were compared with those of a global two-year analysis

which could be considered as giving nearly definitive values for the major constituents. In the monthly analyses, K_1 in particular exhibited oscillations which had to be attributed to some close unanalyzed constituents and not to the background noise. Table 1 shows the observed fluctuations in K_1 .

TABLE 1

Deviations in the amplitude and phase of K_1 as observed in a series of twenty-four monthly analyses at Victoria, British Columbia, Canada

Deviations Found in Monthly Analyses					
Month Number	A cm	g degrees	Month Number	A cm	g degrees
1	2.1	0.1	13	0.6	0.7
2	0	1.1	14	-2.1	0.4
3	-2.7	0.4	15	-2.7	0.5
4	1.5	1.5	16	-0.6	1.1
5	-0.6	0.3	17	-0.3	1.9
6	-0.6	1.6	18	-1.5	-0.2
7	-0.6	-1.2	19	0.6	-1.8
8	1.2	-2.1	20	2.1	-1.1
9	1.8	-1.0	21	3.1	-0.6
10	1.2	0.0	22	1.8	-0.2
11	0.6	0	23	3.4	-0.5
12	2.4	-1.3	24	2.4	0.6

Amplitude and phase of K_1 from a two-year analysis : 63.1 cm 149.0°.

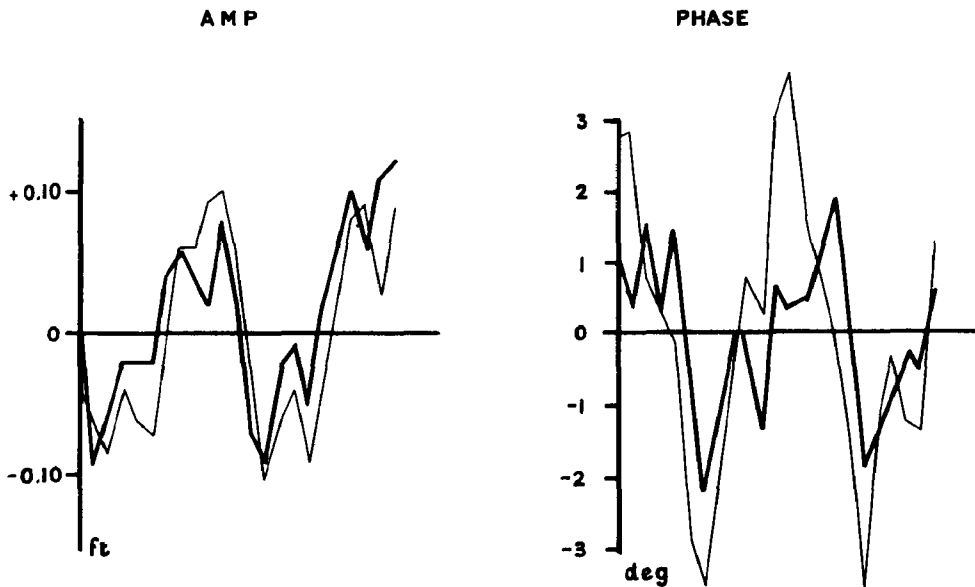


FIG. 1. — Variations in the amplitude and phase of the constituent K_1 found in monthly analyses compared to their "true" values.

Thick line : Observed values.

Thin line : The contribution of the constituents π_3 , ϕ_1 and S_1 .

The amplitude is in feet, a unit still in use in Canada.

The background noise could account for oscillations of ± 0.5 cm. In Table 1 the scatter exceeds this value and it exhibits marked periodicities. The contribution of P_1 to K_1 has been removed in the table so the perturbations must be attributed mainly to π_1 , S_1 , and ϕ_1 , other close neighbours of K_1 . The two-year analysis revealed the values of these constituents and so their monthly contribution to K_1 could be calculated: figure 1 shows that they account for the bulk of the deviations observed in the monthly values of K_1 .

Naturally if the modulation of the main constituents obtained from short analyses can be traced to their close satellites and not really to the background noise, it does not imply in turn that one learns the exact value of the predominant constituents from short analyses: they are automatically modulated by their satellites whose amplitude, frequency and even their identity are unknown at times.

The theory of error just discussed shows that it is highly perilous to look for these hidden constituents but it does not suggest any other technique for their search.

One may write formally the expression for the amplitude of the major constituent perturbed by its satellite:

$$a'_k \approx a_k \pm n + \frac{1}{2N+1} \sum_{k'=-N}^N a_{k'} \frac{\sin[(2N+1)\pi\Delta t(\sigma_{k'} - \sigma_k)]}{\sin[\pi\Delta t(\sigma_{k'} - \sigma_k)]} \quad (34)$$

where a_k = the true amplitude of the constituent analyzed;

n = the contribution of the background noise (to be effectively neglected);

$a_{k'}$ = the amplitude of the m close constituents lying in the vicinity of the k th constituent.

There is no problem in evaluating the ratio of the sine functions if the frequencies of the disturbing constituents are known, but this is as far as we can go with the help of (34).

Empirically it is known that the relative phase and amplitude of close constituents of astronomical origin have pretty constant values and may be used to get a better estimate of the value of a_k in (34). However, these relative phases and amplitudes do vary appreciably from one locality to another from their mean values. In addition, some constituents of shallow water origin do disturb appreciably the main constituents, but their probable relative phase and amplitude are really unpredictable. In tidal practice the effect of P_1 on K_1 , K_2 and T_2 on S_2 , v_2 on N_2 in short analyses is taken into consideration and the analyzed values of the main constituents are corrected with the help of an approximation to (34). But as we have seen in the case of K_1 , some modulation still remains.

In the M_2 group, HORN (1960) has noticed that two constituents very close to M_2 are quite significant. They are characterized by the Doodson Numbers (2 0 -1 0 0 1) and (2 0 1 0 0 -1). Also in the same group, KO_2 , which has exactly the same frequency as M_2 does also disturb M_2 . Its effect can be ascertained from the careful study of a succession of long analyses.

The margin of variability of the analyzed constituents may be calculat-

ed grossly by identifying some of the constituents which do disturb them, picking out of them those of astronomical origin whose relative amplitude can be calculated from Doodson's development of the tidal potential (Doodson, 1954) and by modifying their contribution by the sine factor encountered in (34) which measures their contribution to the σ_k frequency. For example, in the K_1 group, 67 % of π_1 , 84 % of P_1 , 96 % of S and 84 % of ϕ_1 do contribute to the analyzed values of K_1 in a monthly analysis. The tidal development of the tidal potential indicates that their relative amplitude should be of the order of 2 %, 32 %, 1 % and 1 % of the amplitude of K_1 . Leaving aside the contribution of P_1 which is taken into account in routine calculation, we are left with a possible 4 % of possible variation in K_1 in a monthly analysis.

It is now well established that the major source of variation in the resolution of constituents from short analyses is the presence of hidden constituents. The intensity of the modulation will depend on:

- 1) the amplitude of the analysed constituent itself;
- 2) the relative phase and amplitude of the hidden neighbouring constituents;
- 3) the interval of observations.

5. Short tidal current observations

Current observations very often are carried out only during a short interval of time on account of the difficulties of observation.

From one day of observations, the semidiurnal band can be separated from the diurnal band and surprisingly well from the point of view of the background noise. If the variance is something like 0.15 knot, a sensible value, the margin of the expected error is ± 0.06 knot. These limits spread over barely 0.1 knot while the amplitude of the major tidal constituents is of the order of 1 knot. The resolution of the constituents within the diurnal and semidiurnal bands is not achieved through this process though. It is the practice to attempt this resolution by using the relative phase and amplitude of the main constituents obtained from observations on the vertical tide at some neighbouring harbour. Naturally no new knowledge is obtained by this procedure. Genuine new information can only be obtained by prolonging the observations on the tidal currents.

The solution of the equations of hydrodynamics for the basin under consideration may yield more sensible information about the tidal streams present than some haphazard current observations, at least when non-linear interaction is not predominant. On the other hand, the theoretical values of the tidal streams may be checked with the help of a few well chosen series of current observations.

GENERAL CONCLUSIONS

The effect of the background noise on well separated constituents can be actually disregarded even if the interval of observations extends only over twenty-four hours. However, the noise does disturb drastically the resolution of close constituents and actually prohibits the attempt of resolving constituents whose relative phase difference is less than a given minimum value. This minimum relative phase difference is given approximately by Rayleigh's criterion which however is slightly overstringent in the problem under consideration. The choice of analyzable constituents by the use of such a criterion does not minimize in any way the interference of the hidden constituents. This interference can be reduced only by extending the interval of observations and by identifying and isolating the interfering constituents.

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