MEAN SOUNDING VELOCITY.
A BRIEF REVIEW

by George A. Maul
Environmental Science Services Administration,
Atlantic Oceanographic and Meteorological Laboratories,
Miami, Florida

and James C. Bishop, Jr.
Environmental Science Services Administration,
Coast and Geodetic Survey,
Seattle, Washington.

INTRODUCTION

In the three decades since the publication of J.D. Matthews' *Tables of the Velocity of Sound in Pure Water and Sea Water for use in Echo-Sounding and Echo-Ranging* (1939), numerous improvements have been made in the techniques of deep water echo-sounding: stabilized, narrow-beam transducer systems capable of resolving ±1 fathom in 4 000 fathoms are operational (Hickley, 1965); velocimeters able to measure acoustic velocity *in situ* with a repeatability of ±0.02 metre/second have been developed (Bissett-Berman Corp.); the equation for the velocity of sound in sea water has been significantly improved (Wilson, 1960); and high-speed computer technology has been applied to the problems of collection, reduction, and assimilation of bathymetric data (Karo, 1963; Bernstein, 1966; Maul, 1969).

Present day requirements for precise depth determinations are rapidly approaching third-order accuracy (1/5 000): Luskin *et al.* (1954) proposed one part in 3 000 for bathymetric mapping; marine geodesists computing the Bouguer gravity anomaly introduce more than 0.1 milligal error for each fathom of depth error; slant range echo-sounders capable of sounding wide swaths of the sea floor require knowledge of variations in sound speed behavior over long oblique distances; and deep sea taut-wire mooring installations require precise depth information for accurate deployment of sensors.

In view of modern capabilities and requirements, it appears profitable to review from time to time certain simplifying assumptions which can become a significant source of error. Incorrect methods of calculating the "mean sounding velocity", (also referred to in the literature as "sounding velocity", "mean sound velocity", or "mean vertical sound velocity") are in this category.

MATHEMATICAL DEVELOPMENT

Historically, echo soundings have been made by assuming a conventional value for the velocity of sound (1463 or 1500 metres/second) and then correcting for variations of this assumed value in the actual water column (Matthews, 1939; Jeffers, 1960). It must be recognized that echo-sounders are time measuring devices, and the problem is to convert half of the round-trip travel time of a sound pulse to depth by multiplying by a suitable mean velocity.

Required is the vector quantity, the mean sounding velocity (MSV), which by definition is the quotient of the distance (Z) a sound wave travels in the vertical and the length of the associated travel time interval (T) :

$$\mathrm{MSV} = \frac{Z}{T} \tag{1}$$

This definition does not say "mean velocities from the surface to the stated depth" (Sverdrup, *et al.*, 1942) nor "mean values for velocity of sound through the vertical water column" (Baker, *et al.*, 1966).

In correcting for sound velocity variations, the water column is considered to be divided into a series of layers (figure 1), each with thickness ΔZ_i and an associated velocity V_i. The time interval ΔT_i for the sound wave to pass vertically through the i^{th} layer is :

$$\Delta T_i = \frac{\Delta Z_i}{V_i}$$

Summing the time intervals of all n layers in the water column from the surface ($Z = 0$) to depth (Z),

$$T = \sum_{i=1}^{n} \Delta T_i = \sum_{i=1}^{n} \frac{\Delta Z_i}{V_i}$$

For a continuous function V(Z), in the limit as ΔZ approaches zero,

$$T = \int_0^Z \frac{dZ}{V} \tag{2}$$

Substituting into equation (1) and rearranging terms we have the correct expression for the mean sounding velocity :

$$\mathrm{MSV} = \left[\frac{1}{Z} \int_0^Z \frac{dZ}{V} \right]^{-1} \tag{3}$$

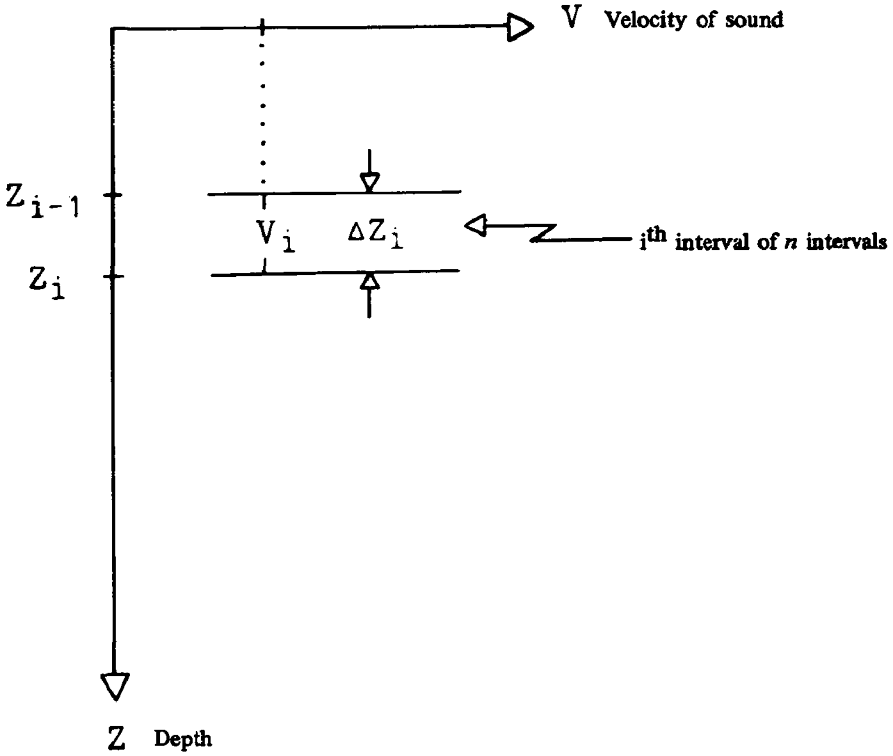

FIG. 1. — The ocean is considered to be composed of a series of finite layers ΔZ_i with an associated V_i, which in the limit are the infinitesimal layers dz.

Equation (3) is the integral form of the harmonic mean, that is, the reciprocal of the mean of the reciprocals. The equation used by MATTHEWS to prepare his tables, as discussed by DIETRICH (1963), is

$$Sv = \frac{1}{Z} \int_0^Z V\,dZ \tag{4}$$

where Sv is the "sounding velocity". Equation (4) is the integral form of the arithmetic mean which introduces a small error because the mean velocity required is not the thickness weighted average of the speeds, but is the total distance traveled divided by the total travel time. Because small velocities contribute more strongly, the harmonic mean will always be less than the arithmetic mean except when the velocity is constant, in which case the two types of mean value are equal.

To illustrate the concept, consider a two layer ocean with layer thickness and associated velocity values as shown on figure 2. From equations (3) and (4) :

$$MSV = \left[\frac{\sum_{i=1}^{n} \frac{\Delta Z_i}{V_i}}{\sum_{i=1}^{n} \Delta Z_i}\right]^{-1} = \left[\frac{\frac{1\text{ km}}{2\text{ km/s}} + \frac{2\text{ km}}{1\text{ km/s}}}{1\text{ km} + 2\text{ km}}\right]^{-1} = \frac{3}{2.5}\text{km/s}$$

$$S\nu = \frac{\sum_{i=1}^{n} V_i \Delta Z_i}{\sum_{i=1}^{n} \Delta Z_i} = \frac{(2\ \text{km/s})\,(1\ \text{km}) + (1\ \text{km/s})\,(2\ \text{km})}{1\ \text{km} + 2\ \text{km}} = \frac{4}{3}\ \text{km/s}$$

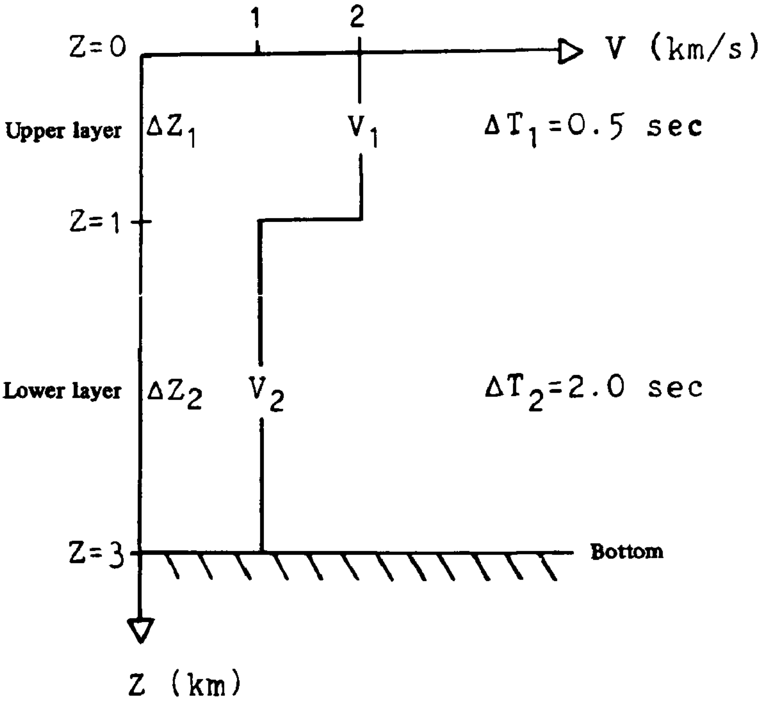

FIG. 2. — Two layer model of the ocean.
The layer thicknesses are ΔZ_1 and ΔZ_2, the associated velocities V_1 and V_2, and the travel times ΔT_1 and ΔT_2, where the subscripts 1 and 2 refer to the upper and lower layers respectively.

The travel time of the sound wave is 2½ seconds. From equation (1) :

$$Z\ (\text{using MSV}) = \frac{3}{2.5}\ \text{km/s} \times 2.5\ \text{s} = 3\ \text{km}$$

$$Z\ (\text{using } S\nu) = \frac{4}{3}\ \text{km/s} \times 2.5\ \text{s} = 3\ 1/3\ \text{km}$$

Although this is a very hypothetical case and not representative of actual oceanic conditions, it clearly demonstrates the validity of equation (3), since the correct depth is 3 km by design.

In the "standard ocean" ($35^0/_{00}$, 0°C), which is approximated in polar regions where there is a near linear increase of sound speed with depth, the magnitude of this error is approximately 0.1%. This is significant in comparison to the 1 part in 4000 (0.025%) capability of modern echo-sounders. On the average, however, the error is anticipated to be less than 0.1%, because in middle and low latitude typical profiles of the speed of sound with depth have a mid-depth minimum which reduces the range of variation and hence the difference between harmonic and arithmetic means.

DISCUSSION

As is well known, the velocity of sound is a slowly varying function of depth; for this reason the error introduced by the use of equation (4) has, until recently, been acceptable (MATTHEWS; OFFICER, 1958; and CREASE *et al.*, 1964). Other authors (cf. GABLER, 1961; DIETRICH, 1963; MAUL, 1969) have ignored the subtlety entirely.

Computer integration of velocimeter output is easily accomplished. The analog output of this instrument can be digitized at a high enough rate that the integral in equation (3) can be very closely approximated by finite differences.

If classical sampling techniques are employed, the integral can be evaluated by assuming a linear change of the velocity of sound between sample points, by the trapezoidal rule. In figure 3 for example, the velocity

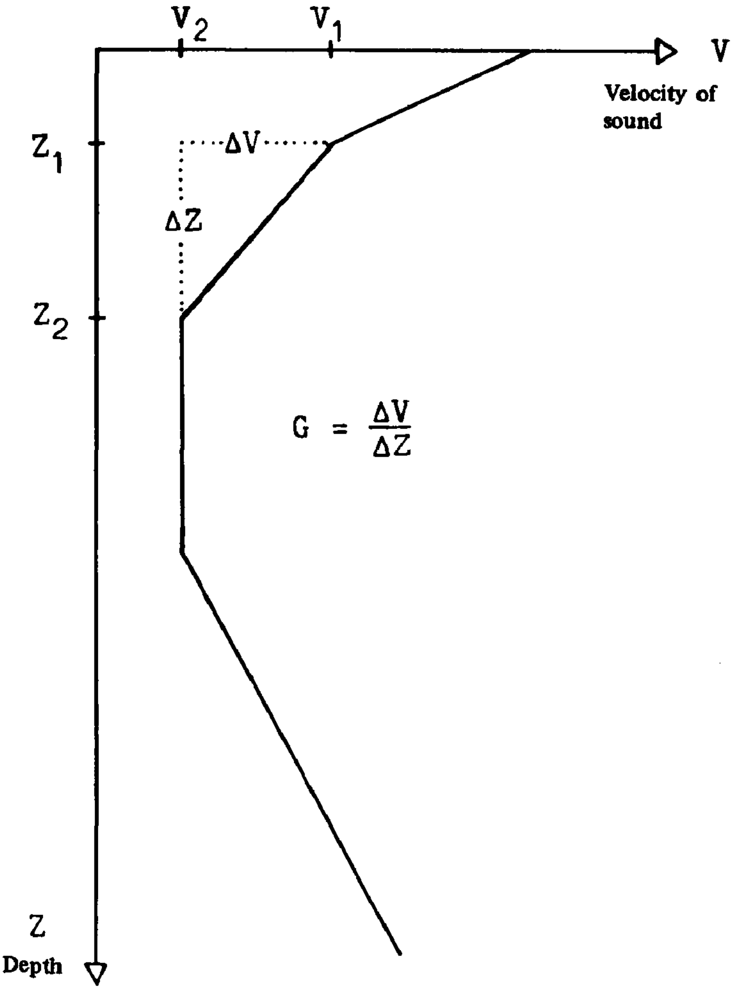

FIG. 3. — Linear approximation of the change of the velocity of sound between standard oceanographic depths.

of sound V_1, V_2 is computed at the standard oceanographic depths Z_1, Z_2 respectively, from temperature and salinity observations at these points. The time for the sound to travel the distance ΔZ is given by equation (2) integrated from Z_1 to Z_2,

$$T = \int_{Z_1}^{Z_2} \frac{dZ}{V}$$

The equation for the straight line connecting the points (V_1, Z_1) and (V_2, Z_2) is $V = V_1 + GZ^*$ where $G\,(= \Delta V/\Delta Z)$ is the velocity gradient, and $Z^* = Z_1 - Z$ in the interval $(Z_1 \leqslant Z \leqslant Z_2)$. Substituting

$$T = \int_{Z_1}^{Z_2} \frac{dZ}{V_1 + GZ^*} = \frac{1}{G} \ln (V_1 + GZ^*) \Bigg|_{Z_1}^{Z_2}$$

Since $V_1 + GZ^* = V_1$ at $Z = Z_1$ and $V_1 + GZ^* = V_2$ at $Z = Z_2$,

$$T = \frac{1}{G} (\ln V_2 - \ln V_1) = \frac{1}{G} \ln \frac{V_2}{V_1} \tag{5}$$

which is in agreement with RYAN and GRIM (1968). Equation (3) can be evaluated by Simpson's Rule. This requires the assumption that the velocity of sound varies from point to point in a smooth manner which can be approximated by a parabola. Observations in the ocean (cf. WOODS, 1967) indicate that a layered structure exists which implies that the variables essentially change like a step-function. With this uncertainty, the reasonable approximation seems to be by the trapezoidal method.

In MATTHEWS' method, as adopted by the U.S. Coast and Geodetic Survey (JEFFERS), the temperatures and salinities at Z_1 and Z_2 are averaged, and a velocity is calculated for the mid-depth of the layer; it is assumed that this "mean velocity" is applicable over the entire layer. This introduces a small error, because the velocity of sound is a *non-linear* function of temperature, salinity and pressure.

If Matthews Tables are used, as in DISHON and HEEZEN (1968), errors of 12 metres in 5000 metres of water depth are introduced because Matthews' equation for sound speed is used instead of Wilson's equation (CAPURRO, 1963). Furthermore, if true depth is used as the entering argument as in MATTHEWS or JEFFERS, instead of echo-sounder depth (also called "recorded depth"; KRAUSE, 1962) an error of the order of 2.5% is introduced in the independent variable. This source of error is easily avoided in machine processing by using time as the independent variable and true depth as the dependent variable.

CONCLUSION

It is recommended that hydrographers re-evaluate mean sounding velocity corrections in the light of new instrumentation and more stringent requirements on bathymetry for economic and scientific activities. The abundant sources of errors in marine surveys make it imprudent to add others that require no additional labor to eliminate. Electronic sampling and calculating techniques can remove many of the uncertainties imposed by the point source data of classical oceanography. The work of MATTHEWS should be updated to reflect our vastly increased knowledge of the distribution of the oceanic variables and how they behave.

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