SOME TRIGONOMETRIC FORMULAE FOR THE INTERPRETATION OF CONTINUOUS SEISMIC PROFILES

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ABSTRACT

Methods are discussed for the calculation of depth and dip of sub-bottom reflectors in seismic profiles. An example is given of a convenient form of interpretation table based on the formulae of CURRY *et al.* (1965), for use when the horizontal separation of acoustic source and hydrophone array is significant. For the case where the horizontal separation of source and receiver is negligible, simple formulae are presented which facilitate the calculation of reflector dip. Following a discussion of the calculation of apparent dip along the profile direction, methods are outlined for the computation of true dip at profile intersection points.

INTRODUCTION

Continuous seismic profiling (CSP) has become a routine surveying method in offshore geological surveys. As yet no comprehensive account of CSP surveying techniques and interpretation methods has been published although there are numerous papers describing the results of specific surveys. A valuable review of many aspects of seismic profiling has recently been published by LEENHARDT (1969). The present paper may be regarded as complementary to this earlier contribution, for it deals with different aspects of interpretation to those emphasized by LEENHARDT.

Sophisticated field procedures involving multiple acoustic sources, multiple hydrophone arrays or variable separations are in use, but the vast majority of CSP applications utilize simple equipments with a single acoustic source and hydrophone array at a fixed separation. With this type of surveying, much interpretation can be carried out using the following basic concepts.

CALCULATION OF APPARENT DIP

In shallow water surveys, the separation of acoustic source and hydrophone array often has to be taken into consideration in order to obtain accurate sub-bottom information. CURRY *et al.* (1965) derived the following simultaneous equations, in h and s, for calculating the depth h to a horizontal reflector using travel times taken from a CSP record, and which take account of the separation of source and receiver :

$$h = (V_0^2/2V_1)(1 - n^2s^2)^{1/2}\,[T_D/s - ((T_0^2 - T_D^2)/(1 - s^2))^{1/2}] \qquad (1)$$

and

$$h = (V_1/2)(1 - n^2s^2)^{1/2}\,[T_1 - ((T_0^2 - T_D^2)/(1 - s^2))^{1/2}] \qquad (2)$$

where
- s : sine of the angle of incidence on the sea bed of the sub-bottom reflected ray (see fig. 1);
- T_0 : two-way travel time of the bottom (sea bed) reflection;
- T_D : travel time of the direct ray;
- T_1 : two-way travel time of the sub-bottom reflection;
- V_0 : velocity of the water layer;
- V_1 : velocity of the layer overlying the reflector;

and n : V_1/V_0.

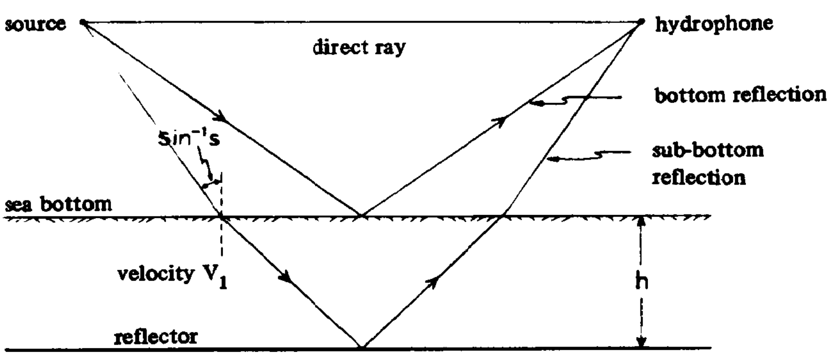

FIG. 1. — Elastic ray paths between acoustic source and hydrophone array.

An ICL 1905 computer has been used to produce tabulated solutions of equations (1) and (2) for particular cases of T_0, T_D, T_1 and V_1. An example of these interpretation tables, which have been produced by M. S. THOMPSON (Swansea), is given in Table 1.

Further details of the construction of these tables may be obtained from Mr. THOMPSON or the author.

Equations (1) and (2) assume no dip, but for true dips less than 10° the resultant error in depth estimation would normally be less than 3 % (*op. cit.*, p. 324). Consequently, in areas of shallow dip the above method can be used to compute the apparent dip of a reflector along a seismic profile by the calculation of its depth below two adjacent points.

TABLE 1

Part of an interpretation table based on the formulae of CURRY et al. (1965)

$V = 2\,250$ m/s.

$T_D = 20$ ms (T_D will normally be constant for a particular survey).

Depth, h (m)

	$d^* = 4$ m $T_0 = 20.7$ ms	$d = 8$ m $T_0 = 22{,}7$ ms	$d = 12$ m $T_0 = 25.6$ ms	$d = 16$ m $T_0 = 29.2$ ms	$d = 20$ m $T_0 = 33.3$ ms	$d = 24$ m $T_0 = 37.6$ ms	$d = 28$ m $T_0 = 42.2$ ms
T_1 (ms)							
18	5.2	–	–	–	–	–	–
20	9.1	–	–	–	–	–	–
22	12.5	4.0	–	–	–	–	–
24	15.6	8.2	–	–	–	–	–
26	18.5	11.5	2.7	–	–	–	–
28	21.2	14.3	6.9	–	–	–	–
30	23.8	17.1	10.4	2.7	–	–	–
32	26.3	19.8	13.2	6.1	–	–	–
34	28.8	22.5	16.0	9.0	–	–	–
36	31.3	25.0	18.6	12.0	5.0	–	–
38	33.8	27.5	21.2	14.7	7.9	0.7	–
40	36.2	30.0	23.7	17.3	10.7	3.8	–
42	38.6	32.4	26.2	19.8	13.3	6.7	–
44	41.0	34.8	28.6	22.3	15.9	9.5	–
46	43.4	37.2	31.0	24.8	18.5	12.1	5.5
48	45.7	39.6	33.4	27.2	20.9	14.6	8.2
50	48.1	42.0	35.8	29.6	23.4	17.1	10.7
52	50.4	44.3	38.2	32.0	25.8	19.6	13.3
54	52.8	46.7	40.5	34.4	28.2	22.0	15.7
56	55.1	49.0	42.9	36.8	30.6	24.4	18.2
58	57.4	51.3	45.2	39.1	33.0	26.8	20.6
60	59.7	53.6	47.6	41.5	35.3	29.2	23.0

d^* is water depth in metres.

If the horizontal separation of acoustic source and hydrophone array is small compared with the depth to reflecting horizons, sufficiently accurate interpretations may be obtainable without taking this separation into account. In this case, outgoing and incoming rays are assumed to follow identical paths as shown in fig. 2.

The difference in travel path of the reflected ray from a planar reflector AB, at two points X and Y a distance x apart, is $2\,x \sin \alpha'$ where α' is the apparent dip of the reflector along the profile direction. The difference in travel time of the reflected ray on a CSP record would be :

$$t = 2\,x \sin \alpha'/V_1$$

where V_1 = seismic velocity of the layer overlying the reflector. Hence the gradient ($= t/x$) of a reflection on a CSP record is given by :

$$m = 2x \sin \alpha'/V_1 \,.\, x$$

Therefore :

$$\alpha' = \arcsin\,(m . V_1/2) \qquad (3)$$

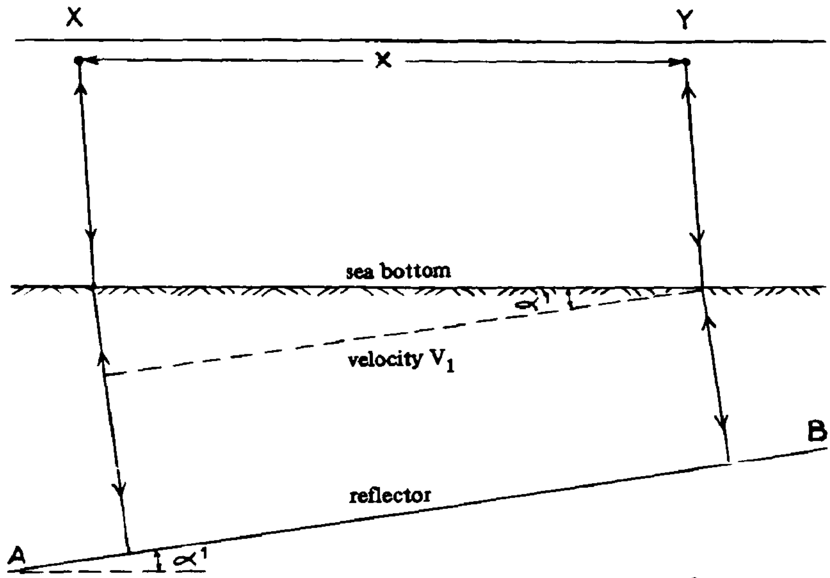

FIG. 2. — Reflected ray paths from a dipping interface.

Thus the apparent dip along the profile direction can readily be determined by using a graphical solution of equation (3) as shown in fig. 3 or by means of a graticule. In any application it is worth comparing the above simple approach with that of CURRY *et al.*, to determine whether any significant differences of apparent dip result.

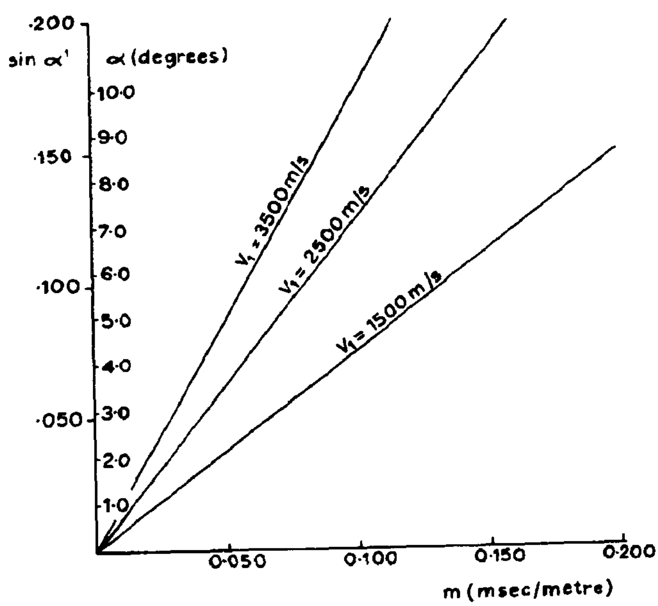

FIG. 3. — Graphical representation of apparent dip formula [equation (3)].

It may be noted that in fig. 2 the sea bed is horizontal. With a sloping sea bed, m can still be measured for a sub-bottom reflector, usually with negligible error, giving a dip value relative to sea-bed rather than to the horizontal.

It should be noted that, in general, the dip value obtained is not strictly the apparent dip along the profile direction XY. As shown in fig. 4, the recorded reflection passing through X travels along the path XZ′ normal to the reflecting interface and does not lie in the vertical plane XYZ containing the profile direction.

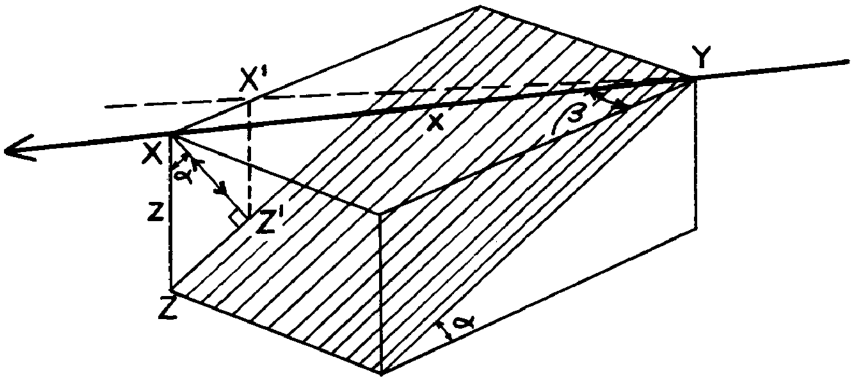

Fig. 4. — Path of a reflected ray below a profile running obliquely to the true dip direction.

By using equation (3), we assume that the apparent dip α' is given to a sufficiently close approximation by :

$$\alpha' = \arcsin (z \cos \alpha / x) ,$$

whereas it is actually given by :

$$\begin{aligned} \alpha' &= \arcsin (z \cos \alpha' / x) \\ &= \arcsin [z/x \, (\tan^2 \alpha \cos^2 \beta + 1)^{-\frac{1}{2}}] \end{aligned} \qquad (4)$$

where

α : true dip of the reflector;

β : angle in the horizontal plane between the true dip direction and the profile direction (see fig. 4);

and

z : difference in vertical depth to reflector between X and Y.

As an example of the size of the resultant error in α', consider a profile at 60° to the direction of true dip of a reflector dipping at 30° : the apparent dip along the profile direction is 16.1°, but the value derived from a CSP record by the above method would be 14.5°, giving an error of -10%. Normally CSP surveys are concerned with dips below 15°, in which case this "dip error" will be insignificantly small.

CALCULATION OF TRUE DIP

As is well known, pairs of apparent dip values at intersection points can be used to derive the direction and amount of true dip at those points.

Many graphical solutions to this general problem have been published, and solutions can also be derived by means of stereograms. But following an extensive CSP survey, involving numerous profile intersection points, it may be desirable to process results by digital computer. This is particularly appropriate when dip values are to be further processed after having been computed. It is a simple matter to program a computer to calculate true dip and azimuth, given (i) the gradients of reflections (i.e., m-values) taken directly from CSP records and (ii) profile bearings.

Referring to fig. 5, we assume that apparent dips α' and α'' are known along directions WX and XY, respectively β' and β'' degrees from the true dip direction in the horizontal plane.

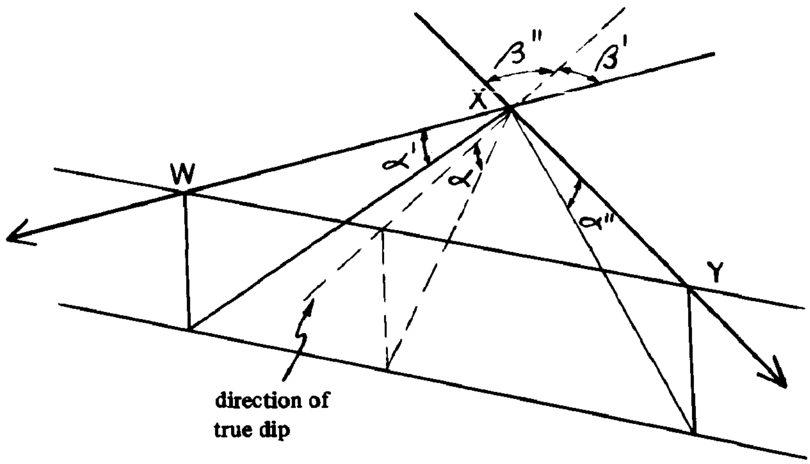

FIG. 5. — Apparent dips along two intersecting profiles.

If true dip $= \alpha$, then :

$$\tan \alpha' = \tan \alpha \cos \beta' \quad (5)$$

and

$$\tan \alpha'' = \tan \alpha \cos \beta'' \quad (6)$$

Let

$$t = \tan \alpha' / \tan \alpha'' = \cos \beta' / \cos \beta''$$

and

$$p = \cos (\beta' + \beta'')$$

Then,

$$p = \cos^2 \beta'/t - [(1 - \cos^2 \beta')(1 - (\cos^2 \beta'/t^2))]^{1/2}$$

from which,

$$\cos \beta' = t[(1 - p^2)/(1 - 2pt + t^2)]^{1/2} \quad (7)$$

and hence :

$$\cos \beta'' = [(1 - p^2)/(1 - 2pt + t^2)]^{1/2} \quad (8)$$

Note that p is simply the cosine of the angular separation of the two profile directions and that t is simply the ratio of the tangent of the two apparent dip values.

Equations (7) and (8) give the *direction* of true dip in relation to the profile directions. By inserting one of these values into equation (5) or (6) we obtain the *amount* of true dip at the profile intersection point X. If necessary, the original apparent dip values can be corrected on the basis of equation (4) and the above procedure repeated to obtain more accurate values of α, β' and β''. Details of an interpretation program (in Fortran IV) based on the above formulae are obtainable from the author.

TRIPLE INTERSECTION POINTS

If three profiles, A, B and C intersect in a point, giving three values of apparent dip, it is theoretically possible to compute true dip and velocity at the intersection point. The simplest method is to take the three pairs of apparent dip values, A + B, B + C and A + C, and for each pair to compute the amount and direction of true dip for a range of velocity values, using equations (5) to (8). The three dip results should be consistent only for a single velocity, which would be the indicated true velocity. The method is exemplified in table 2 which is based on a true dip of 50° and a velocity of 2439 m/s (8000 fm/s). Three estimates of true dip are obtained (one for each pair of profiles) and two estimates of the horizontal angle between each profile direction and the direction of true dip. The various results are consistent only when the correct velocity value is used.

TABLE 2

Estimation of true velocity using three pairs of apparent dip results

Assumed Velocity (fm/s)	Computed Dip, (degrees)	Discrepancies in results(*) (degrees) $\Delta\alpha$	$\Delta\beta_A$	$\Delta\beta_B$	$\Delta\beta_C$
5 000	29.6	1.6	1.6	4.6	2.7
6 000	36.0	1.2	1.1	2.9	1.8
7 000	42.7	0.7	0.6	1.5	0.9
8 000	50.0	0.0	0.0	0.0	0.0
9 000	58.2	0.4	0.4	0.8	0.4
10 000	68.6	1.3	1.6	3.0	1.4

(*) $\Delta\alpha$ is the range covered by the three estimates of dip.
$\Delta\beta_A$, $\Delta\beta_B$ and $\Delta\beta_C$ are the differences between the two estimates of the horizontal angle between true dip and profiles A, B and C respectively.

In practice, with low dips a significant error in velocity does not generate significant discrepancies in the resulting sets of true dip and azimuth values : i.e. an erroneous velocity produces erroneous dip values which nonetheless agree, within the limits of experimental error, between the three pairs of results. For example, results obtained over a reflector dipping at 5° and with V_1 = 2 286 m/s (7 500 fm/s) are consistent over the velocity range from 1 524 m/s (5 000 fm/s) to 4 267 m/s (14 000 fm/s), though actual dip values are, of course, increasingly inaccurate the greater the error in the assumed velocity. Nevertheless, the above procedure using triple intersection points may in some cases distinguish between areas of greatly differing top layer velocities. The method is also useful in checking the accuracy of apparent dip estimates derived from poor CSP records. If no consistency is obtainable with any reasonable velocity value it must be assumed that erroneous information has been taken from the CSP record.

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