THE PHYSICS OF PNEUMATIC TIDE GAUGES

by D. T. PUGH
Institute of Coastal Oceanography and Tides (UK)

INTRODUCTION

As a part of its programme to monitor sea level around the British Isles, the Institute of Coastal Oceanography and Tides has a requirement for a recording system which may be quickly and easily installed, and which gives results accurate to 0.01 metre. The traditional stilling-well installation, apart from being costly and difficult to install on a temporary basis, has a number of fundamental disadvantages which limit its ability to record true water levels accurately (LENNON, 1971). Furthermore, stilling-wells may only be installed where a harbour wall, pier or other vertical structure exists, and such structures are seldom located in positions which are representative of open sea conditions. Pneumatic pressure tide gauges have a number of advantages : not only may they be easily installed on vertical structures, but they may also be used on open coast lines, along beaches, or wherever a light connecting tube can be run from an underwater gas outlet to a recorder ashore. By comparison with gauges which rely on submerged electrical transducers connected to a shore-based recorder by conducting cable, the vulnerable underwater parts of pneumatic gauges are both cheap and easily repaired.

Before adopting the pneumatic tide gauge principle for routine measurements it was necessary to investigate the physics of such systems, so that sources of error could be eliminated or subject to correction. This report contains details of these investigations; their application to coastal sea level measurements is described in more detail elsewhere (PUGH, 1971). Details of the bubbler-type gauge which uses a continuous flow of air through a connecting tube to an underwater outlet are given in the first section. However, an extension of the bubbler-gauge theory shows that a flow of air is not strictly necessary, and details of a non-bubbling system are given in the second section. This latter type of gauge is similar to gauges which use a partially inflated bag, but it has the advantage that gauge datum is defined to the same accuracy as a bubbler-gauge datum. The third section of this paper shows how these results may be applied to the design of actual pneumatic tide gauge systems.

THE BUBBLER GAUGE SYSTEM

The bubbler system is attractively simple. It consists of a supply of compressed gas, one or two pressure reducing valves, a length of tubing which leads underwater to the gas outlet, and a pressure measuring device (for example, an aneroid capsule or a mercury manometer) which records the pressure necessary to force gas out through the underwater end of the tubing. At the underwater outlet, for low rates of gas escape, the gas pressure is equal to the water pressure. This pressure may be used to calculate the head of water above the outlet using the elementary hydrostatic relationship :

$$P_m = \rho g \zeta_i + P_A \tag{1}$$

where P_m is the measured pressure, P_A is the atmospheric pressure at the water surface, g is the gravitational acceleration, and ζ_i and ρ are the water level and water density above the gas outlet. The steady flow of gas along the pressure tube connecting the gas supply and the pressure measuring device to the pressure outlet will be driven by a pressure gradient along the pipe. Thus the measured pressure is higher than the true water pressure by an amount which depends upon the tube dimensions and the rate of gas flow. Because of this, and to conserve the supply of compressed gas, the bubbling rate should be kept low. However, if the rate of supply of gas is too low, the pressure in the system may be unable to increase as rapidly as pressure changes at the outlet due to increasing water depth. Consequently water will be forced into the system until the pressures balance, and the recorded pressure will not be a measure of the water head above the gas outlet.

There are two main frequency bands within which changes of sea level occur, namely, tidal water level changes with typical maximum values of two metres in an hour, and surface waves with typical maximum values of two metres in ten seconds. In practice the procedure adopted in the design of bubbler gauges is to set the bubbling rate sufficiently high to follow tidal changes of water level, and to make corrections for the wave effects. For waves, the system effectively contains a constant mass of gas. Gas escapes at the trough of a wave, but for the remainder of the wave cycle water is forced into the system. It will be shown that the correction necessary for this is reduced if the main volume of the gas system is at the underwater end of the tubing, where it acts as a buffer for fluctuating water pressures.

The main parameters of a pneumatic tide gauge system are summarised in figure 1.

1. The minimum rate of gas flow for good data

For the measured pressure to be directly related to the water head above the gauge datum level, gas must escape continuously through the underwater outlet. This may be expressed as :

$$\left(\frac{\partial V}{\partial t}\right) > 0 \tag{2}$$

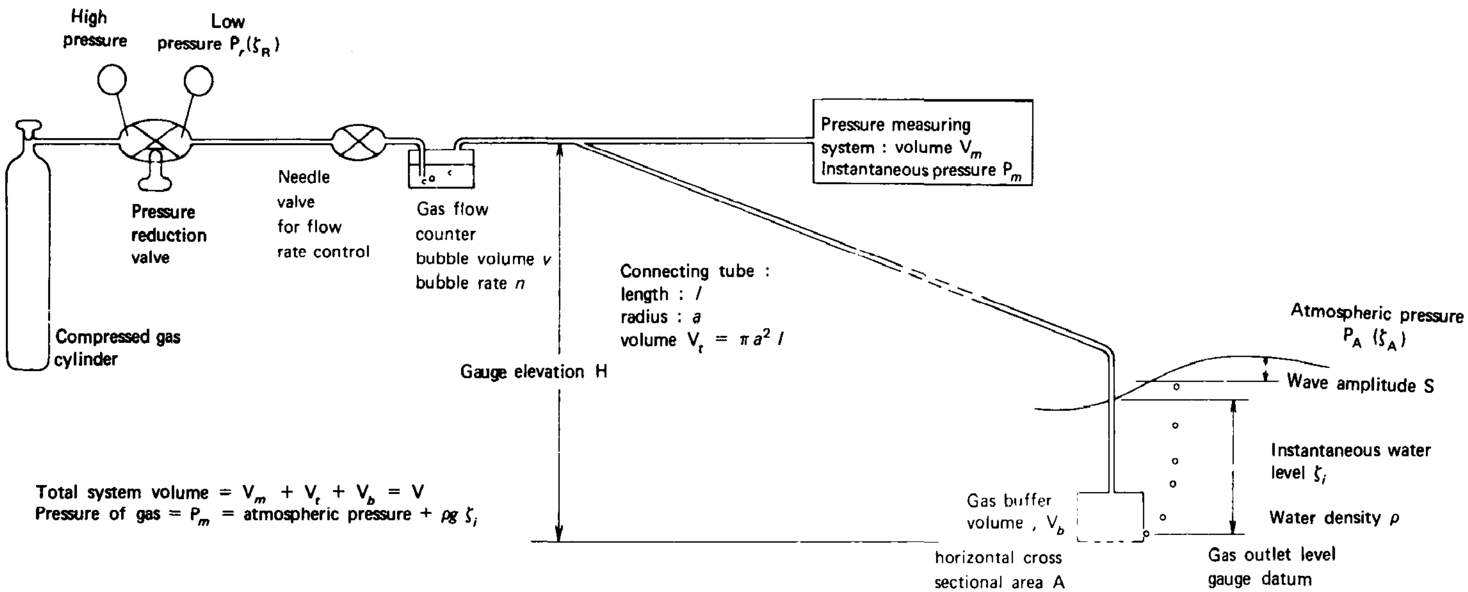

FIG. 1. — Schematic diagram of the pneumatic tide gauge and its principal system parameters.

where V is the total volume of gas in the system, and t is the time variable. But from the basic equation for an ideal gas :

$$V = \frac{R\,T\,M}{P_m} \tag{3}$$

where R is the universal gas constant, and M and T are the mass of the gas in the system and its absolute temperature, respectively.

Hence :

$$\left(\frac{\partial V}{\partial t}\right) = RT\left[\frac{1}{P_m}\left(\frac{\partial M}{\partial t}\right)_P - \frac{M}{P_m^2}\left(\frac{\partial P_m}{\partial t}\right)_M\right]$$

where the suffixes P and M denote partial differentiation at constant pressure and at constant mass respectively. To satisfy condition (2) :

$$\frac{1}{M}\left(\frac{\partial M}{\partial t}\right)_P > \frac{1}{P_m}\left(\frac{\partial P_m}{\partial t}\right)_M$$

or, in unit time :

$$\frac{\Delta M}{M} > \frac{\Delta P_m}{P_m} \tag{4}$$

Usually the rate of flow of gas into the system is measured by counting the bubbling rate through a fixed orifice submerged in liquid in a sight glass at the outlet from the pressurised gas supply (figure 1). In unit time the mass of gas entering the system is, from equation (3) :

$$\Delta M = \frac{P_m}{RT}\, n\, v$$

where v is the volume of each bubble and n is the number passing through the sight chamber. Substituting in (4) we have, after some rearrangement :

$$\frac{n\,v}{V} > \frac{\Delta P_m}{P_m}$$

or, if the pressures are expressed in terms of water head (*) :

$$\frac{n\,v}{V} > \frac{\Delta \zeta_i}{\zeta_i + \zeta_A} \tag{5}$$

where $\Delta\zeta_i$ is the change in water level and ζ_A is the water head equivalent of atmospheric pressure. For these discussions it is always adequate to take ζ_A as 10 metres.

In most systems the compressed gas from the cylinder passes through a reduction valve where the pressure is reduced typically to between 3 and 5 bars (P_R). The rate of flow of gas into the system is then controlled by a needle valve (figure 1), but the bubbling rate, n, normally depends on the pressure drop across this valve ($P_R - P_m$). As P_m increases the bubbling

(*) Pneumatic gauges measure pressure and so their theory must be developed in terms of pressure. However, as the measured water heights are in units of length, the results are often better expressed in units of water head equivalent.

1 metre of sea water head is almost exactly 10^4 N/M^2

10^5 N/M^2 = 1 bar

rate falls. If the water head equivalent of P_R is ζ_R, and if the bubbling rate is directly proportional to the pressure difference across the valve, then :

$$\frac{n}{n_0} = \frac{\zeta_R - (\zeta_A + \zeta_i)}{(\zeta_R - \zeta_A)}$$

where n_0 is the bubbling rate when the system is at atmospheric pressure,

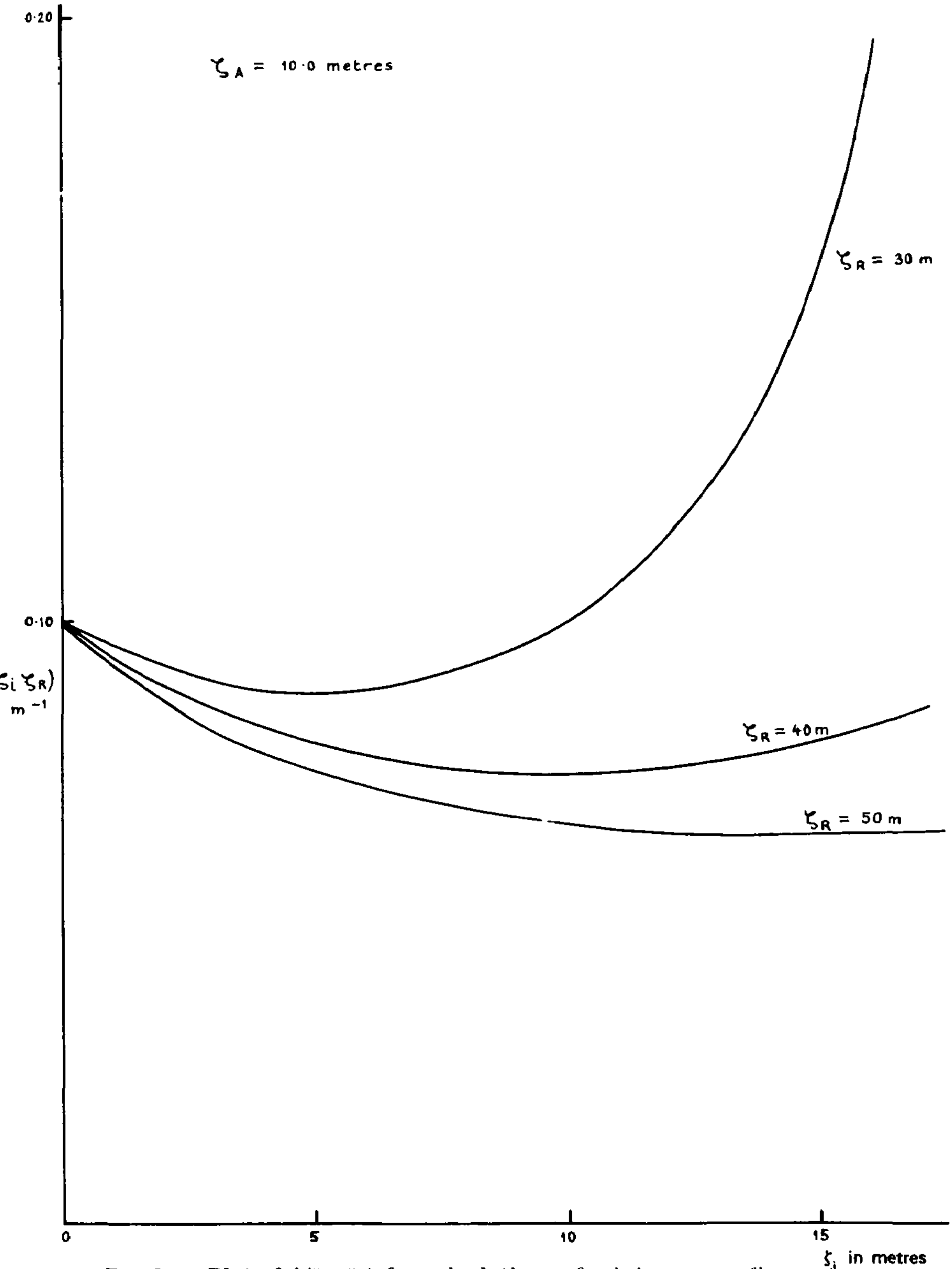

FIG. 2. — Plot of $f(\zeta_i, \zeta_R)$ for calculations of minimum gas flow rates.
In practice set ζ_R so that $f(\zeta_i, \zeta_R)$ does not exceed 0.1 m for the range to be measured.

for example, with the connecting tube disconnected from the gas supply and pressure recorder. With some rearranging condition (5) becomes :

$$n_0 > \frac{V}{\nu} \Delta\zeta_i \left[\frac{(\zeta_R - \zeta_A)}{(\zeta_i + \zeta_A)(\zeta_R - (\zeta_i + \zeta_A))} \right] \qquad (6)$$

Figure 2 shows plots of the bracketed function for three typical values of ζ_R. Note that there is a minimum value when :

$$\zeta_i = \left(\frac{\zeta_R}{2} - \zeta_A \right)$$

In practice the value of n_0 should be set sufficiently high to satisfy condition (6) for all expected values of $\Delta\zeta_i$ with the bracketed function at 0.1 metre^{-1}. The value of ζ_R is then chosen sufficiently high so that the function never exceeds this value within the range to be measured. For ranges up to 10 metres ζ_R may be set at 30 metres (3 bars), but for ranges up to 20 metres, 40 metres (4 bars) is necessary. The final expression may then be rewritten :

$$n_0 > \frac{V}{10\,\nu} \left(\frac{\partial \zeta_i}{\partial t} \right)_{max} \qquad (7)$$

2. Pressure gradients along the connecting tube

The pressure at the recorder, P_m, will not be exactly equal to the water head pressure at the underwater outlet because of pressure gradients in the connecting tube. Firstly there are the static pressure gradients due to gas pressure heads in the tube and in the atmosphere. Secondly, there are the dynamic effects due both to the steady flow of gas to the underwater outlet, and to the adjustment of gas within the system in response to changes in the pressure being measured.

a) *Static pressure heads.*

Developing equation (1) we have :

$$P_m = \rho g \zeta_i + P_A - \rho_g g H$$

where ρ_g is the density of the gas in the tube and H is the elevation of the measuring instrument above the underwater outlet. If the effects of temperature variations are negligible, then the gas density is proportional to the pressure :

$$P_m = \rho g \zeta_i + P_A - \rho_{gA} \left(\frac{\zeta_i + \zeta_A}{\zeta_A} \right) g H \qquad (8)$$

where ρ_{gA} is the gas density at atmospheric pressure. As the system is designed to measure the part of P_m due to water head, the measuring instrument normally senses the pressure difference between the tube gas and the atmosphere at the recorder level. In practice this is easily arranged by having one side of the sensor open to the atmosphere. The atmospheric pressure at the sensor will be :

$$P_A - \rho_{aA}\, g (H - \zeta_i) \qquad (9)$$

where ρ_{aA} is the air density at atmospheric pressure. The recorded differential pressure is therefore the difference between (8) and (9); if compressed air is used as the source gas a considerable simplification is possible. The measured pressure becomes :

$$\rho g \zeta_i - \rho_{aA}\, g \left(\frac{H}{\zeta_A} + 1\right) \zeta_i$$

$$= \left(\rho - \rho_{aA}\left(\frac{H}{\zeta_A} + 1\right)\right) g\, \zeta_i \qquad (10)$$

Thus the effect of static pressure heads may be accommodated by using a slightly reduced water density. For a gauge mounted 10 metres above the outlet and using air, the water density should be reduced by 2.6 kilogrammes per cubic metre. Without this correction water levels of 10 metres would be measured low by 0.025 metre. The correction becomes particularly important where a gauge is mounted high on a cliff or an off-shore rig. Where a computer is used for correcting raw gauge data, then corrected pressures are probably better calculated directly from expressions (8) and (9).

b) *Dynamic pressure gradients*

Consider the volume of gas flowing through an element Δx of the connecting tube. In unit time this is the volume of gas entering the system from the compressed supply, less the amount of gas required to increment the pressure by ΔP_m in the system on the recorder side of the element :

$$\text{volume flow} = n\nu - \frac{\Delta P_m}{P_m}\left(V_m + \pi a^2 x\right)$$

where V_m is the volume of gas in the measuring system, a is the radius of the tube, and x is the distance of the element Δx from the recorder end of the tube. The pressure drop along this element is :

$$\frac{8\eta \Delta x}{\pi a^4}\left(n\nu - \frac{\Delta P_m}{P_m}\left(V_m + \pi a^2 x\right)\right)$$

(See, for example, NEWMANN and SEARLE, 1957, p. 221), provided that the velocity of flow is sufficiently low for it to be non-turbulent. For non-turbulent flow in narrow tubes the Reynolds number must be less than the critical value (approximately 1000) and for air at atmospheric pressure and a tube radius of 2.0 mm, the critical velocity is around 3.5 metres per second; gas flows due to tidal changes of water level will not exceed this for tube lengths less than 50 kilometres. η is the dynamic gas viscosity at the temperature of measurement. The total pressure drop along the tube is obtained by integrating along its length :

$$\Delta P = \frac{8\eta l}{\pi a^4}\left(n\nu - \frac{\Delta P_m}{P_m}\left(V_m + \frac{\pi a^2 l}{2}\right)\right) \qquad (11)$$

This expression has two features : the first term in the brackets is due to steady flow of gas along the tube while the second term is due to pressure adjustment within the tube and measuring system. Figure 3 shows the

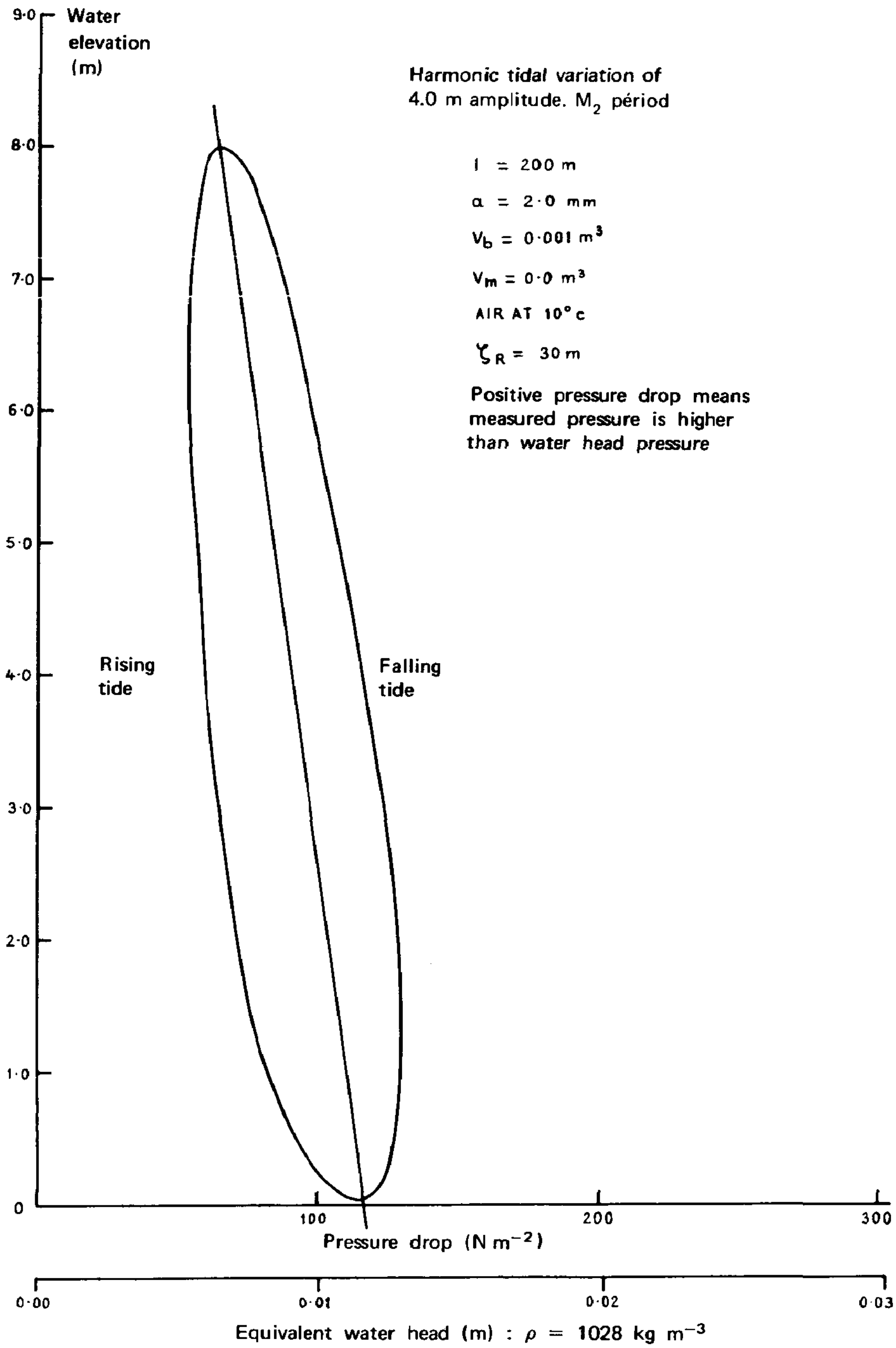

FIG. 3. — Pressure drop along a typical connecting tube as a function of water level.

theoretical pressure differences across a 200 metre length of 2.0 mm radius tubing. For a harmonic water level change of amplitude 4.0 metres and period 12.43 hours (M_2), the pressure drop is always less than 0.013 metre. However, the pressure drop will be shown later to increase as the square of the tube length, so that for longer tubes the drop becomes very large. The centre line through the curve represents the pressure drop due to the first term in (11) and is lower at high pressures because of lower rates of gas flow through the needle valve. The excursions to either side of this line are due to the second term, and are positive on the falling tide (ΔP_m negative) and negative on the rising tide (ΔP_m positive).

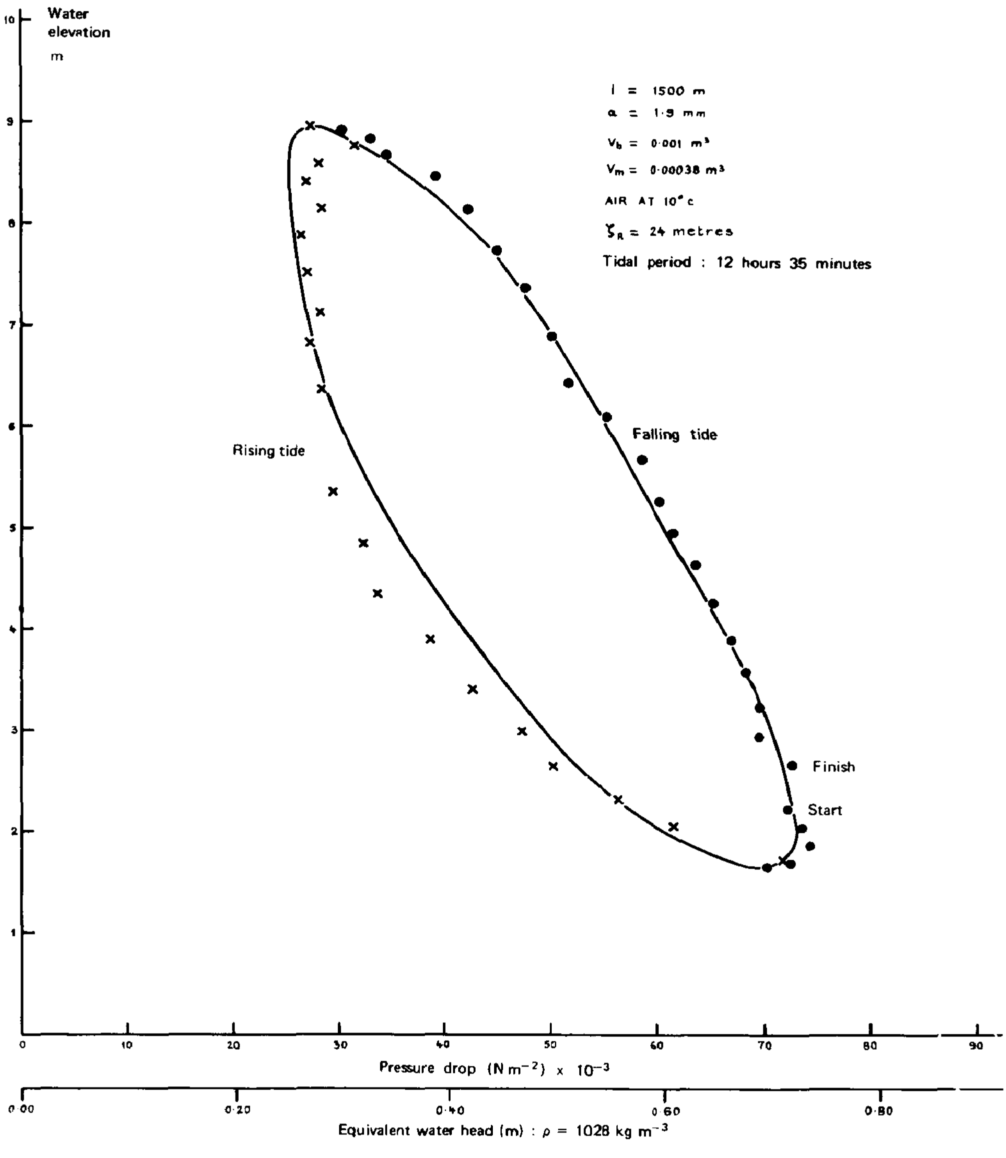

FIG. 4. — Experimental and theoretical pressure drops along a 1 500 metre length of conecting tubing. Alfred Basin stilling well, 22 Dec. 1971.

Figure 4 shows the results of an actual test on a 1500 metre length of 1.9 mm radius tubing compared with the theoretical curve of (11). The greatest divergence occurs where the rate of increase in water level is greatest because the theory does not take account of the time constant of the tube (measured as 170 seconds in this case). A more elaborate theory would be necessary for greater accuracy or when using even longer tubes, but for lengths up to 1500 metres (11) may be used for design (see section 3), and for corrections accurate to better than 0.05 metre.

3. The effects of waves

One advantage of a pneumatic system is that it may be used on open coasts and over drying areas where stilling wells are not easily installed. Although the bubbling rate may be set sufficiently high to follow tidal changes in water level (see part 1) it cannot be set sufficiently high to follow waves. Gas will escape through the underwater outlet in the trough of a wave, and the correct pressure is recorded, but for the remainder of the wave cycle water will be forced into the system until the pressures balance. The parameter to be measured is the average water level above the pressure point. While variations in pressure which reach the recorder are partly damped by the connecting tube, and while special gas filters (for example, a sintered metal membrane) may be fitted to ensure that only a mean pressure is recorded, it remains to relate this recorded mean system pressure to the average pressure at the outlet due to water head.

Suppose that water has entered the system to a height y above the gas outlet. Then we have the relationships :

$$V_i = V - Ay \tag{12}$$

$$P_m = P_i - \rho g y \tag{13}$$

where V_i is the instantaneous volume of the gas within the system, and P_i is the instantaneous water head pressure at the outlet. A is the horizontal cross-sectional area of the outlet volume. For this argument the time constant of the tubing is assumed to be zero. Because wave periods are very short compared with the tidal periods for which the gas flow rate is set, over the period of a single wave the system effectively contains a constant mass of gas. For a constant mass gas system :

$$P_m \; V_i^{\gamma} = P_0 V^{\gamma} \tag{14}$$

where P_0 is the system pressure at volume V, that is, at the trough of a wave. The relationship applies for adiabatic volume changes (those which are sufficiently rapid for no heat to enter the system); γ is the ratio between the specific heats of the gas at constant pressure and at constant volume. Eliminating y from equations (12) and (13) :

$$P_i - P_m = \frac{\rho g}{A}(V - V_i)$$

and substituting from (14) :

$$\frac{P_i - P_m}{\rho g} = \frac{V}{A}\left(1 - \left(\frac{P_0}{P_m}\right)^{1/\gamma}\right) \tag{15}$$

If the volume changes are isothermal then equation (14) is simply Boyle's Law, and the γ^{-1} term in (15) is unity. For air, or any diatomic gases, the value for γ^{-1} to be used in practice will lie between 1.0 and 0.7, and will depend on the value of A, and the wave period. Equation (15) is a convenient representation as it shows the error involved in computing a true water head from a measured pressure P_m, instead of from the true outlet pressure, P_i. The error involved in averaging P_m over a complete wave cycle may be calculated supposing :

$$P_i = P_0 + s\,(1 + \sin \omega t) \tag{16}$$

represents the wave with a pressure amplitude of s and a period $(2\pi / \omega)$. The value of P_i is P_0 in the trough of a wave, and the mean value is $(P_0 + s)$. If we suppose that the volume changes are isothermal, then the average error over a complete wave cycle is obtained by integrating (15) :

$$E = \frac{\omega}{2\pi}\int_{t=0}^{t=\frac{2\pi}{\omega}} \frac{P_i - P_m}{\rho g}\, dt = \frac{\omega}{2\pi}\int_{t=0}^{t=\frac{2\pi}{\omega}} \frac{V}{A}\left(1 - \frac{P_0}{P_m}\right) dt$$

Adopting an iterative procedure to evaluate the integral, we set $P_m = P_i$ as a first approximation :

$$E = \frac{V}{A}\,\frac{\omega}{2\pi}\int_{t=0}^{t=\frac{2\pi}{\omega}} \left(1 - \frac{P_0}{P_0 + s\,(1 + \sin \omega t)}\right) dt$$

which reduces, for $s/P_0 \ll 1$, to :

$$E = \frac{V}{A}\,\frac{s}{P_0} \tag{17}$$

For a typical installation (V/A) is 0.50 metre so that for a wave of amplitude 1 metre on water of 6 metres depth, the recorded pressure will be lower by (0.50) (1/15) = 0.033 metre, compared with the true mean water level. Expressed as a fraction of the wave amplitude the error is :

$$\frac{V}{A}\left(\frac{1}{\zeta_0 + \zeta_A}\right)$$

where ζ_0 is the water head above the outlet in the wave trough. This factor is almost always less than 0.05 so that a second iteration is not necessary for an accuracy of 0.01 metre.

For waves of large amplitude :

$$E = \frac{V}{A}\,\frac{2\pi}{\omega}\int_{t=0}^{t=\frac{2\pi}{\omega}} \left(1 - \frac{1}{(1 + \alpha + \alpha \sin \omega t)}\right) dt$$

where

$$\alpha = \frac{s}{P_0}$$

This becomes :

$$E = \frac{V}{A}\left(1 - \frac{1}{\pi}\frac{1}{(1+2\alpha)^{1/2}}\right)\left[\tan^{-1}\frac{(1+\alpha)\tan\left(\frac{\omega t}{2}+\alpha\right)}{(1+2\alpha)^{1/2}}\right]_{t=0}^{t=\frac{2\pi}{\omega}}$$

$$= \frac{V}{A}\left(1 - \frac{1}{(1+2\alpha)^{1/2}}\right)$$

The exact solution may be expanded to several terms in powers of α :

$$E = \frac{V}{A}\left(\alpha - \frac{3\alpha^2}{2} + \frac{15\alpha^3}{6} - \ldots\right) \tag{18}$$

which reduces to (17) for small values of α. As before, the average error will not exceed 0.05 of the wave amplitude in practice, so that a second iteration is not necessary. Expression (18) shows that the correction calculated from (17) should be reduced to 0.030 metre. In almost all practical applications the correction term may be calculated to sufficient accuracy using (17).

The theoretical development is only strictly true for waves with a period sufficiently long for volume changes to be isothermal and for the time constant of the tubing to be comparatively negligible. Where the time constant of the tubing is much longer than the wave period, then the effective volume in (17) is the outlet volume V_b. Although, for chart recording, the trace width generated by the waves is related to the amplitude of their fluctuations above the outlet, the relationship depends on the period of these fluctuations; for normal rates of chart progression (up to 0.02 metre per hour) individual waves are not distinguishable and so their periods cannot be determined. Theoretically the chart does not contain sufficient information for making accurate corrections, nor is it practicable to install special equipment at each gauge location for monitoring waves.

The wave correction procedure adopted at this Institute is to calculate an average tube attenuation factor from direct observations of trace width and corresponding wave heights, for typical wave periods at the installation site. These observations are made during routine gauge inspections. Subsequently the correct tidal water level is taken as the trace centre reading plus the wave correction term [expression (17)] calculated from the trace width and the average attenuation factor. P_0 is taken to be the measured pressure minus the calculated wave amplitude. The correction is minimised by designing for a small (V/A) system parameter, and the dependence of the effective volume on the wave period is kept small by having a buffer volume V_b which is large compared with the tube and measuring system volumes.

The parameter (V/A), which has the dimensions of length, is a fundamental characteristic of any pneumatic tide measuring system, as will be clear in subsquent sections. For minimum wave corrections it should be small, which requires a large cross sectional area at the outlet but a small total system volume. The use of the open end of a connecting tube as the gas outlet gives a very small cross-sectional area and large errors in the presence of waves. It has been assumed that there is no interaction between the non-turbulent steady gas flow (11) and the more rapid and often turbulent gas flows due to wave pressure fluctuation.

THE NON-BUBBLER GAUGE SYSTEM

For very long period waves such as those of tidal frequency, the changes of volume due to water entering the gas system under increasing pressure will be isothermal, and (15) becomes :

$$\frac{P_i - P_m}{\rho g} = \frac{V}{A}\left(1 - \frac{P_I}{P_m}\right) \tag{19}$$

where P_I is the pressure at which the system is fully inflated. Provided that this correction is added when calculating the water head above the outlet there is no need to keep the system fully inflated by continuously forcing air down the connecting tube. Figure 5 shows the magnitude of the correction for typical systems. Two immediate advantages of this are the saving on compressed gas supplies and the elimination of much of the tube pressure difference term so that longer lengths of tube may be used. This type of operation is similar to that of gauges where pressure equalisation with the surrounding water is effected through a sealed and partially inflated bag. However, with the non-bubbler gauge, datum is much better defined : it remains at the gas outlet level even though gas is not actually escaping to the water.

1. Bubbler gauge corrections applicable to a non-bubbler

Much of the bubbler gauge theory is applicable to non-bubbler gauges, often in a simplified form. The formula for calculating minimum rates of supply of compressed gas are not required, and the dynamic pressure gradient due to this flow disappears from (11), which now becomes :

$$\Delta P = -\frac{8\eta l}{\pi a^4}\,\frac{\Delta P_m}{P_m}\left(V_m + \frac{\pi a^2 l}{2}\right) \tag{20}$$

Where the recorder volume V_m is much less than the tube volume the pressure difference becomes :

$$\Delta P = -4\eta\left(\frac{l}{a}\right)^2 \frac{\Delta P_m}{P_m} \tag{21}$$

A static pressure head correction is still required. Although allowance should be made for the reduction in H due to water in the outlet volume this is quite unnecessary in practice. Corrections for wave effects are also unnecessary as the system adjusts to both high and low levels, although theoretically a small wave correction due to the non-linearity of non-bubbler output is indicated (see Appendix 2).

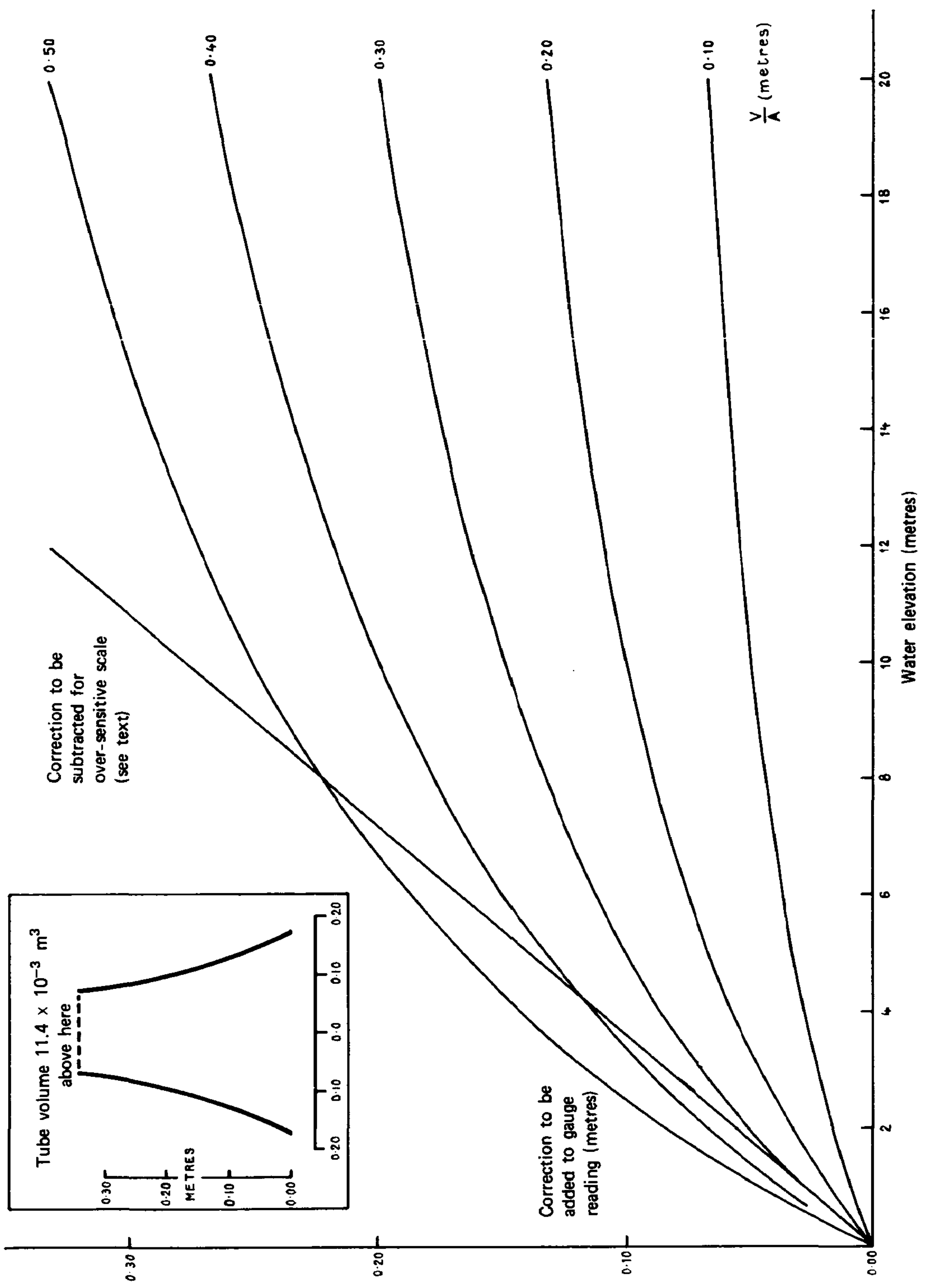

FIG. 5. — Non-bubbler correction factors for typical (V/A) gauge parameters.
Inset : buffer volume with variable (V/A) for linear scale (see text).

2. Corrections which are particular to the non-bubbler gauge

The development of the non-bubbler correction factor (19) was based on the assumption that the system contains an ideal gas at constant temperature. However over tidal periods temperatures will vary and the gas will have sufficient time to adjust for equilibrium water vapour pressures throughout the cycle.

a) *Temperature variations*

For an ideal gas, from (3), we have :

$$\frac{P_m V_i}{(T + \Delta T)} = \frac{P_I V}{T}$$

for volume changes at slowly changing temperatures. T is the temperature at which the gauge was inflated and ΔT is the subsequent change. Using this in place of (14), the non-bubbler correction term becomes :

$$\frac{V}{A}\left(1 - \frac{P_I}{P_m}\,\frac{T + \Delta T}{T}\right)$$

In the extreme case of a gauge inflated at 10 °C and atmospheric pressure measuring a 10 metre water head at a new system temperature of 20 °C, for a (V/A) factor of 0.50 metre the true correction term is 0.241 metre whereas the term calculated without temperature correction would be 0.250 metre. With an underwater tube and outlet volume containing most of the gas in the system temperature changes are likely to be much less than this so that corrections are seldom necessary.

b) *Equilibrium water vapour pressures*

In the presence of water vapour (14) becomes :

$$(P_m - w)\, V_i = (P_I - w)\, V$$

where w is the water vapour pressure. This remains constant for slow pressure changes at constant temperature. The non-bubbler correction becomes :

$$\frac{V}{A}\left(1 - \frac{P_I - w}{P_m - w}\right)$$

and because w is much less than either P_I or P_m, this simplifies to :

$$\frac{V}{A}\left(1 - \frac{P_I}{P_m}\right)\left(1 + \frac{w}{P_m}\right)$$

For the example discussed above the calculated non-bubbler correction of 0.250 metre should be increased to 0.252 metre at 10 °C, where the water vapour pressure is equivalent to 0.125 metre of water head. At 20 °C the correction becomes 0.253 metre, but no correction is necessary until temperatures exceed 30 °C when the water vapour pressures increase rapidly.

c) *Absorption of gas from the system into seawater*

If records from a non-bubbler gauge are compared against records from a standard gauge at the same site over a period of several weeks, a gradual upward movement of the non-bubbler gauge datum is observed. This is because of the gradual absorption of gas from the system into the sea water. Over a period of ten weeks at Hilbre Island in the Dee Estuary, a non-bubbler gauge with a (V/A) factor of 0.42 metre suffered a datum drift of approximately 0.15 metre (figure 6). As this drift must be minimised if useful long period measurements are to be obtained, a theory for predicting drift rates is necessary; gauges may then be designed for greater stability.

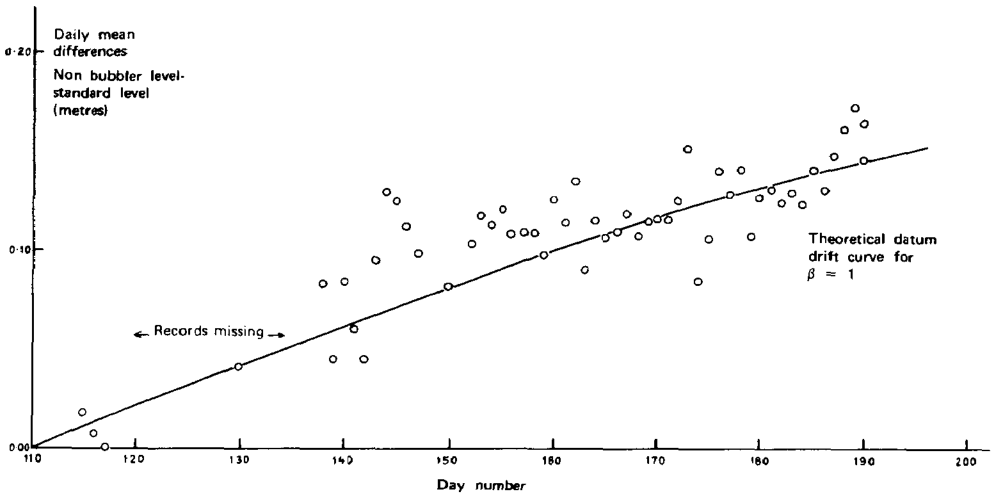

FIG. 6. — Non-bubbler datum drift at Hilbre Island, April to July, 1971.

During the course of a tidal cycle water enters the outlet volume from the sea. Even though this water may be saturated with dissolved nitrogen and oxygen at the existing temperature and at atmospheric pressure, at the higher pressures which exist within the container further absorption of gas can occur. Suppose that the volume of gas removed from the system is proportional both to the volume of water exchanged per tidal cycle, and to the degree of undersaturation which exists at the mean system pressure. Suppose also that we have an average tidal wave of pressure amplitude S and a low water pressure of P_0 (as previously, P_0 includes the atmospheric pressure, P_A). At high water the volume of water in the system is :

$$\left(\frac{V}{A}\right)\left(1 - \frac{P_I}{P_0 + 2S}\right)A$$

while at low water the volume is :

$$\left(\frac{V}{A}\right)\left(1 - \frac{P_I}{P_0}\right)A$$

The volume of water exchanged per tidal cycle is therefore :

$$\left(\frac{V}{A}\right)\left(\frac{P_I}{P_0}\right)\left(\frac{2S}{P_0 + 2S}\right)A \qquad (22)$$

Assuming that the saturation volume of dissolved gas per cubic metre of sea water is proportional to the gas pressure (Henry's Law), and that the sea water is initially saturated with both nitrogen and oxygen at atmospheric pressure, the excess gas dissolved is :

$$\beta = \text{(saturation capacity at atmospheric pressure)}\ \frac{(P_m - P_A)}{P_A} \qquad (23)$$

Although in practice the nitrogen and oxygen will be absorbed from a system containing air at slightly different rates, for simplification consider a single gas whose saturation volume when dissolved in sea water is the sum of the separate saturation volumes of nitrogen (14.6×10^{-3} m³ per m³ of sea water at 10 °C and a chlorinity of 20 ‰; see Horne, 1969) and oxygen (7.1×10^{-3} m³ per m³ at 10 °C and 20 ‰ chlorinity). The coefficient β represents the fraction of the undersaturation which is utilised for gas removal, and will vary between unity for complete absorption when the discharging sea water is completely saturated at the mean pressure, and zero when no extra gas is absorbed and the discharging water contains no more dissolved gas than during entry. We assume, also, that the average degree of undersaturation is related to the mean pressure, $P_0 + S$. Therefore the volume of gas removed in a tidal cycle is, from (22) and (23).

$$\Delta V = \left(\frac{V}{A}\right)\frac{P_I}{P_0}\ \frac{2S}{P_0 + 2S}\ A\ \beta(21.7 \times 10^{-3})\left(\frac{P_0 + S - P_A}{P_A}\right)$$

at 10 °C and atmospheric pressure. At the inflation pressure this volume is reduced by a factor (P_A/P_I), so that the datum shift in one tidal cycle is :

$$\frac{\Delta V}{A} = \left(\frac{V}{A}\right)\beta\,(21.7 \times 10^{-3})\,\frac{2S(P_0 + S - P_A)}{P_0(P_0 + 2S)}$$

Writing the pressure heads in metres of water head :
datum drift per tidal cycle equals :

$$\left(\frac{V}{A}\right)\beta\,(21.7 \times 10^{-3})\,\frac{\text{(tidal range x mean water level)}}{\text{(Low water level + 10) (High water level + 10)}}$$

which is defined as :

$$\left(\frac{V}{A}\right)(1 - \psi)$$

The drift factor, ψ, is a function of the tidal regime to be measured, of the ambient temperature, and of the efficiency of the gas absorption process. For the next tidal cycle the system volume has been slightly reduced and so the shift of datum will be slightly smaller. The new (V/A) parameter is :

$$\left(\frac{V}{A}\right) - \left(\frac{V}{A}\right)(1 - \psi) = (V/A)\,\psi$$

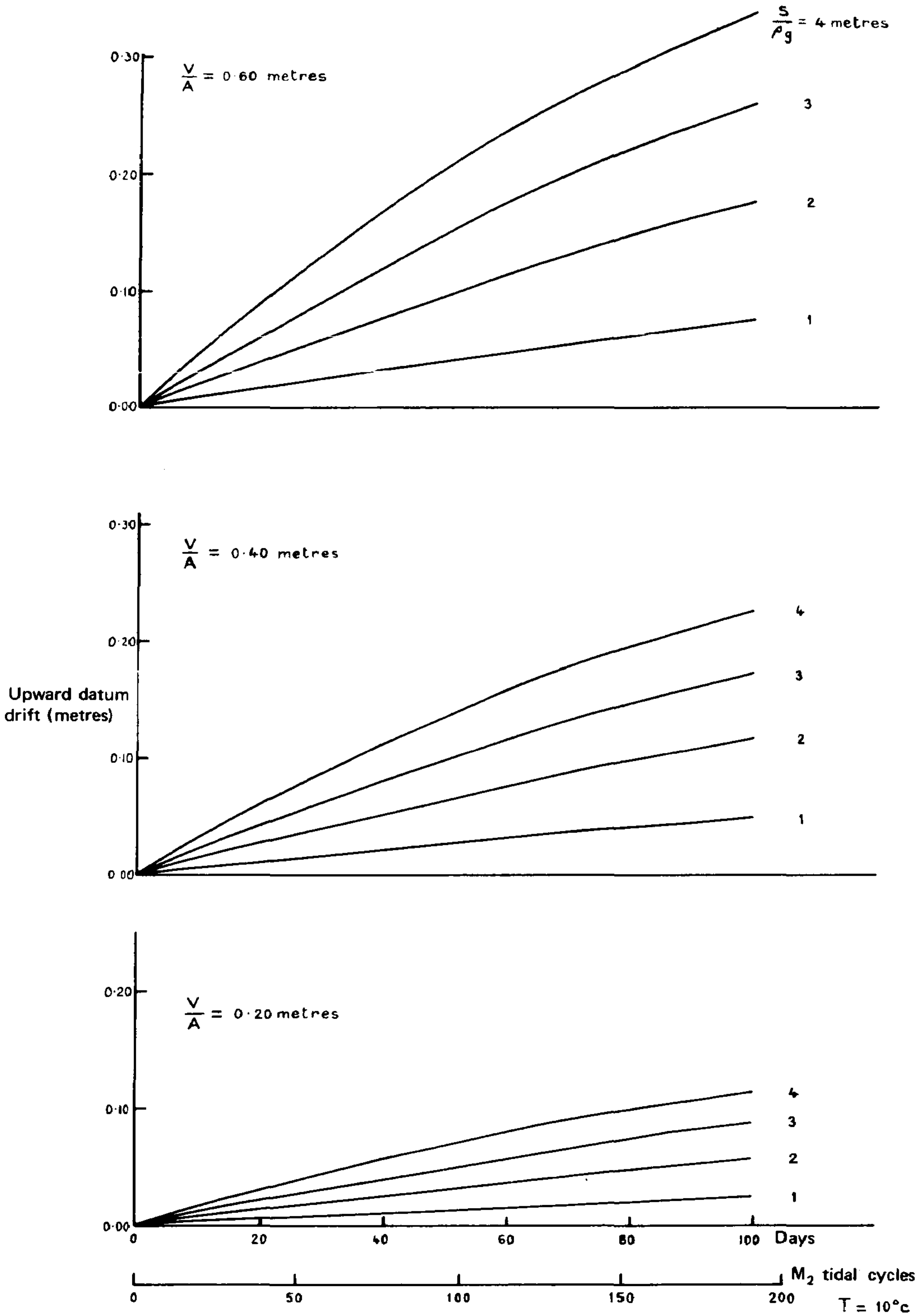

FIG. 7. — Curves for estimating non-bubbler gauge datum drift for harmonic tidal waves of amplitude $S/\rho g$ on a mean water level of $(S/\rho g + 2)$ metres. Absorption factor $\beta = 1.0$ for air at 10 °C.

Over N tidal cycles the (V/A) term is progressively reduced to :

$$\left(\frac{V}{A}\right)\psi^N$$

so that :

$$\text{datum shift over N cycles} = \left(\frac{V}{A}\right)(1 - \psi^N) \tag{24}$$

The Hilbre Island data (figure 6) was used to compute an appropriate value for the absorption factor β. This value was 1.02, which shows that water mixing during the exchange process was sufficiently vigorous for all of the available undersaturation to be used for gas removal. The continuous line through the data has been drawn for a coefficient of unity. This agreement means that a coefficient of unity may be assumed for design purposes : on this basis a family of curves may be plotted (figure 7) showing datum drift for different values of (V/A) and for different tidal ranges. For minimum datum drift the (V/A) factor must be kept low. Even for (V/A) as low as 0.05 metre the loss of gas by molecular diffusion is too small to influence these calculated drift rates.

Because the solubility of gases in liquids generally decreases with rising temperature, the curves of figure 7 should be adjusted for temperatures differing from 10 °C. From HORNE, 1969, the lumped coefficients for nitrogen and oxygen are :

	5°C	10°C	15°C	20°C
air solubility in metres^3 per metre^3 (x 10^{-3})	24.2	21.7	19.2	18.6

for sea water of chlorinity 20 ‰, at atmospheric pressure.

For temperatures below 10 °C drift rates should be increased by approximately 10 % for each 5 °C fall of temperature, while above 10 °C drift rates should be reduced by this amount. For more exact calculations the original solubility tables should be consulted.

The curves of figure 7 are intended for use in the design of non-bubbler gauges. Once a gauge is installed it is probably better in practice to reinflate the system periodically and to observe the extent of the datum drift, as changes due to other causes, such as small air leaks, are then detected.

3. Automatic correction of non bubbler gauge records

Although the correction term which must be added to all non-bubbler gauge data (figure 5) is easily included during computer processing of data, in some cases there is a requirement for the recorder to display correct water levels at all stages of the tidal cycle. An example of this is where the information is required to control shipping movements. The pressure measuring device normally records on a chart which is calibrated in metres; if this scale were made slightly non-linear, then the non-bubbler correction term (and also any regular water density changes during the tidal cycle) could be allowed for. The chart would then read the correct level at all times. However, as the production of specially graduated charts is expensive, a better solution is to design an underwater buffer volume with a cross-sectional area which decreases as water enters further into the system. This gives a (V/A) factor which increases with increasing water head pressure and thus, by choosing suitable values for A, a linear correction term in P_m is obtained. The correction term itself is then balanced out by operating the gauge at a greater sensitivity than would be required for normal bubbler operation.

An example of this is shown by the straight line correction in figure 5; this has been computed for a gauge operating on a fresh water sensitivity scale when actually measuring elevations of sea water of density 1028 kilogrammes per cubic metre. For a bubbler gauge readings would be high by this amount. However for a non-bubbler gauge the error is the difference between this line and the appropriate non-bubbler correction curve as the corrections are of opposite sign. If the (V/A) factor is 0.50 metre, then the corrections balance to within 0.03 metre for water elevations up to 10 metres, and this would be sufficiently accurate for many installations. For the gauge to read accurately to within 0.01 metre over the whole scale a bell-shaped outlet volume is required. The inset in figure 5 shows the cross section of a suitable buffer volume where a long length (volume 11.4×10^{-3}) of connecting tube is required.

THE DESIGN OF A PNEUMATIC TIDE GAUGE SYSTEM

Confronted with all of the above theoretical considerations the prospective user of a pneumatic tide gauge system may well feel that the simplest solution would be to measure water levels on a flight of tide poles. This would be an unfortunate reaction as pneumatic systems are fundamentally simple and effective provided they are properly designed : the purpose of this final section is to show how the theory should be applied to the design of an actual installation. Wherever there is a requirement to measure tidal changes of water level, certain characteristics of the situation are fixed while others must be selected by the designer to give the required accuracy of measurement with the minimum of system complications. Figure 8 summarises the design parameters and their relationships starting from the basic requirements and ending with calculations of minimum connecting tube radius and bubbling rates.

Overall system design accuracy at this Institute is generally 0.01 metre but for non-scientific measurements lower tolerances may well be acceptable. Ideally a system should be designed to give correct water head pressures directly at the measuring device, but this is only possible for short lengths of connecting tube and low rates of increasing water level. The two parameters over which the designer has no control are this maximum rate of increase of water level, and the amplitude of the tidal variations to be measured.

Once these three basic parameters have been established, the next stage is to find a suitable site for gauge installation, and to select a suitable gauge.

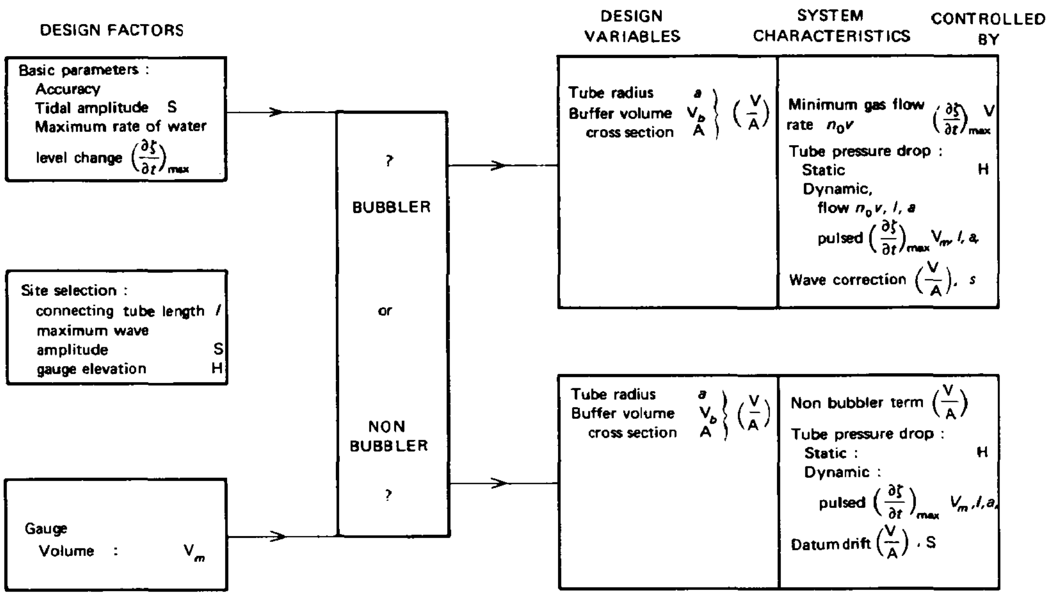

FIG. 8. Summary chart of relations between pneumatic tide gauge installation characteristics.

Details of the selection of suitable sites are given elsewhere (PUGH, 1971); for these considerations it is sufficient to emphasise the primary requirement : to have as short a length of connecting tube as possible, as this length is involved in almost all subsequent calculations. The maximum wave amplitude expected may be reduced by picking a sheltered site, but care should be taken to ensure that the site chosen is well connected hydraulically to the location for which the tidal measurements are to be representative. The elevation of the recorder above the outlet point should be kept as small as possible although this is only of secondary importance.

A recorder which measures the pressure differential between the system and the atmosphere is preferable to a system which measures absolute pressure, as absolute pressure measurements must be corrected for changes in barometric pressure. With a differential system only water head pressures are measured; variations in atmospheric pressure are automatically balanced out. The type of measuring system chosen, apart from having sufficient range and sensitivity to match the basic requirements, should have as small a volume as possible and this volume should not change appreciably with the pressure being measured.

Where only a short length of connecting tubing is required the practice at this Institute is to use a bubbler configuration as this gives directly the correct water head pressures at the recorder. A constant flow of dry gas along the tube also helps to avoid condensation forming within it. Where it is not possible to install a tube which falls continuously from the recorder to the outlet, any condensation would collect in the depressions and could give errors due to the formation of local water heads and their attendant changes in pressure. Over longer distances the non-bubbler may be used to advantage because of the smaller pressure gradients and, although the response is generally slightly non-linear, the correction term is simple to apply. The non-bubbler also has the advantage of not needing a wave correction, but it does require periodic checking of the datum stability. The next stage is therefore to calculate the minimum tube radius compatible with the required accuracy for both bubbler and non-bubbler systems, to see whether both, only the non-bubbler, or neither is practicable.

For bubbler systems the rate of gas supply will be set so that (7) is satisfied. Suppose that ζ_R is sufficiently high for the bubbling rate to be independent of the system pressure, P_m. Substituting into (11) and rewriting the pressure ratio in terms of water elevations :

$$\Delta P = \frac{8\eta l}{\pi a^4}\left[\frac{V}{10}\left(\frac{\partial \zeta_i}{\partial t}\right)_{max} - \frac{1}{(\zeta_i + 10)}\left(\frac{\partial \zeta_i}{\partial t}\right)\left(V_m + \frac{\pi a^2 l}{2}\right)\right]$$

ΔP will be greatest if $(\partial\zeta_i/\partial t)$ has its largest negative value when $\zeta_i = 0$. Although this is improbable for tidal measurements it gives an extreme case for design purposes. Suppose, also, that the maximum rates of rise and fall are equal and that the recorder volume is very small compared with the tube volume. Then the maximum pressure difference across the tube is :

$$\Delta P = \frac{8\eta l}{10\pi a^4}\left(\frac{\partial \zeta_i}{\partial t}\right)_{max}\left(V_b + \frac{3}{2}\pi a^2 l\right)$$

The buffer volume V_b must be sufficiently large to absorb volume changes due to waves so that no water enters the connecting tubing. For a wave as previously defined (16) it is easily shown that the minimum volume is :

$$V_b = \pi a^2 l \left(\frac{2s}{P_0}\right) \qquad (25)$$

The maximum pressure difference is therefore :

$$\Delta P = \left(\frac{8\eta}{10}\right)\left(\frac{\partial \zeta_i}{\partial t}\right)_{max}\left(\frac{l}{a}\right)^2\left(\frac{2s}{P_0} + \frac{3}{2}\right) \qquad (26)$$

This expression is most revealing : it shows that the maximum pressure drop for a bubbler gauge is proportional to the square of the length of connecting tube, and inversely proportional to the square of the tube radius.

For the non-bubbler gauge we may rewrite expression (21) at zero water head, in terms of length :

$$\Delta P = \left(\frac{8\eta}{10}\right)\left(\frac{\partial \zeta_i}{\partial t}\right)_{max}\left(\frac{l}{a}\right)^2\left(\frac{1}{2}\right) \qquad (27)$$

where, as before, the minimum and maximum rates of change of water level are assumed equal. In general the maximum pressure drop along the connecting tube is :

$$\Delta P = \left(\frac{8\eta}{10}\right)\left(\frac{\partial \zeta_i}{\partial t}\right)_{max}\left(\frac{l}{a}\right)^2 \phi \qquad (28)$$

where $\phi = 0.5$ for a non-bubbler gauge and normally lies between 1.5 and 2.5 for a bubbler gauge. Thus there is at least a threefold advantage here

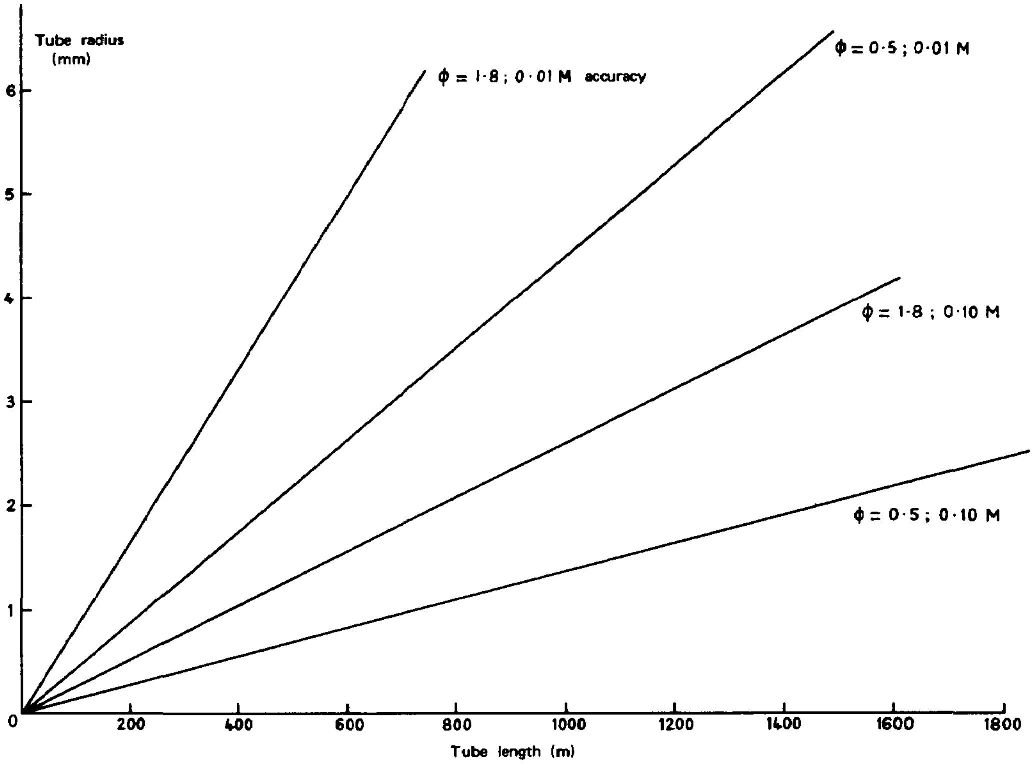

FIG. 9. — Lines showing the minimum tube radius necessary for various accuracies and tube lengths, without corrections, over the whole tidal range, when the maximum rate of water level change is 1 metre per hour.
$\Phi = 0.5$ is the non-bubbler mode. $\Phi = 1.8$ is a typical bubbler mode (with up to 3 m amplitude waves on 10 metres of water head). For other rates of water level change the minimum radius must be multiplied by (rate of change in m/hr)$^{1/2}$.

in using a non-bubbler gauge. The design procedure is to use expressions (26) and (27) to calculate the minimum tube radius compatible with the required accuracy (expressed as ΔP) and then to choose whether the system shall be a bubbler or non-bubbler mode. Table 1 summarises the maximum pressure gradients for various wave design parameters, tube lengths and radii. Figure 9 may be used to calculate the minimum tube radius necessary for accuracies of 0.10 and 0.01 metre.

If a bubbler mode is selected, the outlet volume is calculated from (25), and the minimum bubbling rate from (7). The measurements should then be corrected using (10), (11) and (17). Expression (25) is also used to calculate the outlet volume for a non-bubbler mode; in this case corrections should be made using (10) and (19), and for datum drift (24) where necessary. The final design parameter, the cross-sectional area of the buffer volume, A, should be as large as practicable for both the bubbler and non-bubbler gauges as in the former this helps to minimise wave corrections, and in the latter to reduce the non-bubbler correction term and the rates of datum drift.

An ingenious method whereby some of the advantages of both the bubbler and non-bubbler gauges may be used simultaneously was suggested as early as 1905 by M. ROLLET de L'ISLE (see ROUMÉGOUX, 1964). The

TABLE 1

Maximum pressure drop across the connecting tube for different pneumatic gauge systems.

		l = 200 metres a = 2 mm	l = 200 metres a = 10 mm	l = 1000 metres a = 2 mm	l = 1000 metres a = 10 mm
BUBBLER					
(1) No wave buffer					
$V_b = 0$	$\Delta P = 1.5\left(\frac{8\eta}{10}\right)\left(\frac{\partial \zeta_i}{\partial t}\right)_{max}\left(\frac{l}{a}\right)^2$	0.006	0.0003	0.150	0.006
(2) Buffer for 3 metre amplitude waves on 10 metres of water					
$V_b = \frac{3}{10}\pi a^2 l$	$\Delta P = 1.8\left(\frac{8\eta}{10}\right)\left(\frac{\partial \zeta_i}{\partial t}\right)_{max}\left(\frac{l}{a}\right)^2$	0.007	0.0003	0.180	0.007
(3) Emergency non-bubbler for 10 metre tidal range					
$V_b = \pi a^2 l$	$\Delta P = 2.5\left(\frac{8\eta}{10}\right)\left(\frac{\partial \zeta_i}{\partial t}\right)_{max}\left(\frac{l}{a}\right)^2$	0.010	0.0004	0.250	0.010
NON-BUBBLER					
Independent of V_b	$\Delta P = 0.5\left(\frac{8\eta}{10}\right)\left(\frac{\partial \zeta_i}{\partial t}\right)_{max}\left(\frac{l}{a}\right)^2$	0.002	0.0001	0.050	0.002

Note. $(\partial\zeta_i/\partial t)_{max} = 1$ metre per hour; $\eta = 17.5 \times 10^{-6}$ Newtons per m^2/sec (air at 10 °C). Units are metres of sea water head.

system uses one tube for supplying gas to the underwater outlet and a second tube for pressure transmission to the measuring device, so that there is no datum drift and tube pressure gradients are minimised. In this case the minimum bubbling rate (7) should be calculated using the volume of both tubes but the pressure drop is calculated as for a non-bubbler (20) through the single tube. Static pressure head corrections should be applied as usual. Wave corrections must also be made (17) but the appropriate system volume need not include the volume of the tubes if these are connected to the outlet volume through suitable filters. If high rates of gas flow are needed then the gas cylinder may be replaced by an air compressor motor.

The gas in pneumatic systems is normally dry air; however, it would be better to use a gas of lower viscosity provided it were not too soluble in sea water. Theoretically some of the lower paraffin hydrocarbons (for example, methane — $\eta = 10 \times 10^{-6}$ N/M^2 sec at 10 °C) would be a suitable replacement.

This work was part of the research programme of the Institute of Coastal Oceanography and Tides and is published with the permission of the Director, Dr. J. R. ROSSITER.

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APPENDIX 1

Summary list of principal symbols used

A : horizontal cross-sectional area of buffer volume;
a : radius of connecting tube;
g : gravitational acceleration;
H : gauge elevation above outlet level;
l : length of connecting tube;
M : mass of gas in system;
ΔM : incremental change in mass of gas in system;

N : total number of tides over which datum drift is calculated;
n : number of bubbles entering system through counter in unit time;
n_0 : as above, with the system at atmospheric pressure;
P_A : atmospheric pressure;
ΔP : pressure drop along connecting tube;
P_m : instantaneous pressure at measuring device;
ΔP_m : incremental change in P_m in unit time;
P_i : instantaneous water head pressure at outlet level;
P_0 : water head pressure at outlet below the trough of a wave;
P_I : non-bubbler gauge inflation pressure;
P_R : low pressure from which gas enters the system;
R : universal gas constant;
S : pressure amplitude of a tidal wave;
s : pressure amplitude of short period wave; $s/P_0 \ll 1$;
t : time variable;
T : absolute system temperature;
ΔT : incremental change of system temperature;
V : total system volume to outlet level; $V = V_m + V_t + V_b$;
ΔV : volume of gas removed from a non-bubbler gauge during a tidal cycle, at pressure P_I.
V_b : volume of outlet buffer container;
V_m : volume of measuring device. Ideally V_m is small and independent of P_m;
V_t : volume of connecting tube;
V_i : instantaneous value of system volume;
v : volume of bubbles entering system through counter from compressed gas supply;
w : water vapour pressure;
α : s/P_0;
β : gas absorption factor for non-bubbler gauge datum drift;
γ : ratio between specific heats of the gas at constant pressure and at constant volume;
ζ_A : water level equivalent of $P_A \cdot (P_A = \rho g \zeta_A)$. To sufficient accuracy. $\zeta_A = 10$ metres;
ζ_i : instantaneous water level above gauge outlet;
$\Delta\zeta_i$: incremental change in water level above gauge outlet;
ζ_R : water level equivalent of $P_R \cdot (P_R = \rho g \zeta_R)$;
ζ_0 : water level at system pressure $P_0 \cdot (P_0 = P_A + \rho g \zeta_0)$;
η : gas viscosity at system temperature. $\eta = 17.5 \times 10^{-6}$ N/M² seconds, for air at 10 °C;
ρ : sea water density;
ρ_{gA} : system gas density at atmospheric pressure and system temperature;
ρ_{aA} : air density at atmospheric pressure and system temperature;
ϕ : tube pressure drop factor for system design;
ψ : non-bubbler datum drift factor;
ω : angular speed of harmonic water level changes.

APPENDIX 2

Wave corrections on the non-bubbler gauge

Only second order wave corrections are applicable for non-bubbler gauges. Because this is not immediately obvious the detailed argument is given here.

Suppose that in the presence of waves the true water head pressure above the outlet is given, as previously, by :

$$P_i = P_0 + s\,(1 + \sin wt)$$

so that the true mean water head pressure is $(P_0 + s)$.

On a chart recorder, because of the waves, the trace will have a finite width; during record reduction the centre of this trace is taken as the measured pressure and then corrected for the non-bubbler effect. At the trough of the wave, from (19), and neglecting, as previously, any attenuation due to the connecting tube, the measured pressure is :

$$P_0 - \rho g\left(\frac{V}{A}\right)\left(1 - \frac{P_I}{P_0}\right)$$

while at the wave peak the measured pressure is :

$$P_0 + 2s - \rho g\left(\frac{V}{A}\right)\left(1 - \frac{P_I}{P_0 + 2s}\right)$$

At the centre of the trace the measured pressure is therefore :

$$\bar{P}_m = P_0 + s - \rho g\left(\frac{V}{A}\right)\left(1 - \frac{P_I}{P_0}\,\frac{(P_0 + s)}{(P_0 + 2s)}\right) \qquad (A_1)$$

This mean pressure is converted to mean water head pressure by adding the non-bubbler corrections :

$$\rho g\left(\frac{V}{A}\right)\left(1 - \frac{P_I}{\bar{P}_m}\right)$$

Evaluating this correction term using the same iterative procedure as for the bubbler gauge wave correction, i.e. setting $\bar{P}_m = P_0 + s$, the calculated mean water head pressure from (A_1) and (A_2) is approximately :

$$P_0 + s + \rho g\,\frac{V}{A}\,\frac{P_I}{P_0}\,\frac{s^2}{(P_0 + s)\,(P_0 + 2s)} \qquad (A_2)$$

Comparing this with the true mean water head pressure, the error is the final term.

To take an extreme example : for $s = 3$ metres on 5 metres of water depth with $P_I = 10$ metres, the computed water head pressure will be too high by $(0.016)\,\rho g\,(V/A)$; for $(V/A = 0.50$ metre this is 0.018 metre.