

GEOMETRIC AND MATHEMATICAL PROPERTIES OF A MODIFIED HYPERBOLIC PATTERN

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INTRODUCTION

When using hyperbolic Hi-Fix or Sea-Fix chains, normally two or three Slave stations are disposed around a central, or common, Master Station.

It is not always realised, however, that with only a small re-arrangement of the wiring in the ship's receiver any Slave can be used as the common station without the need to modify that particular Slave station.

Figure 1 shows that new hyperbolic patterns can be created merely

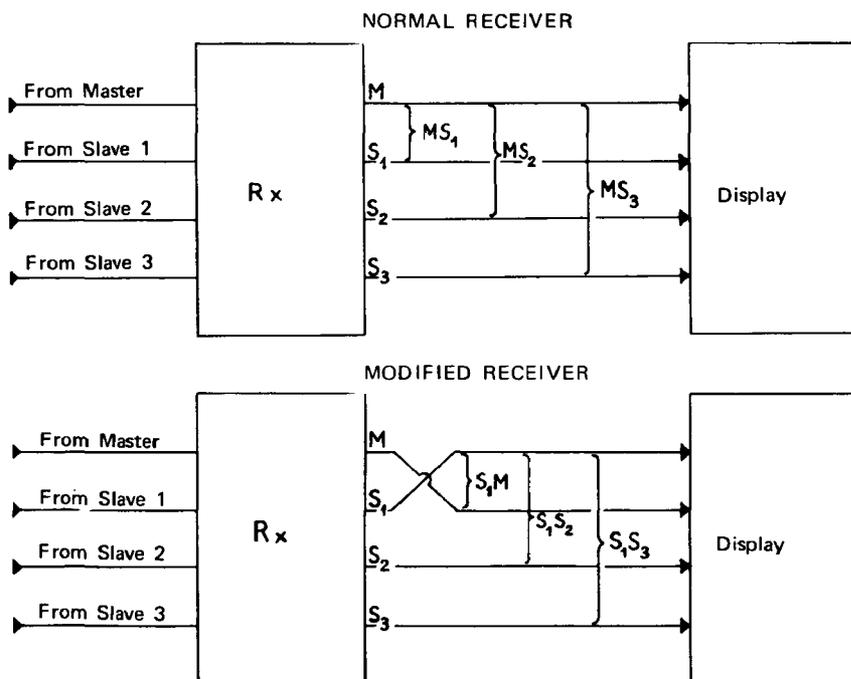


FIG. 1.

by interchanging the wiring of Master and Slave 1 receiver channels to the display unit. Thus in the case of this receiver Slave 1 becomes the common station, acting as "Master" (figure 2).

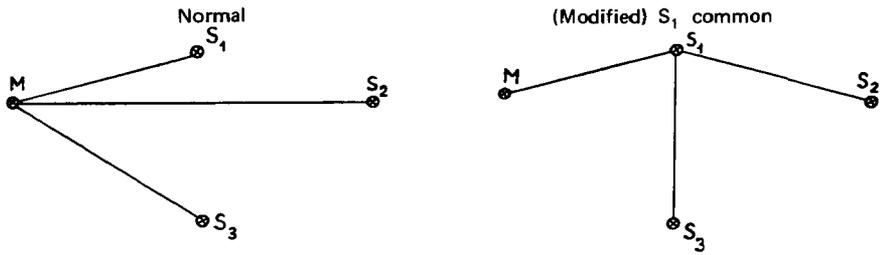


FIG. 2.

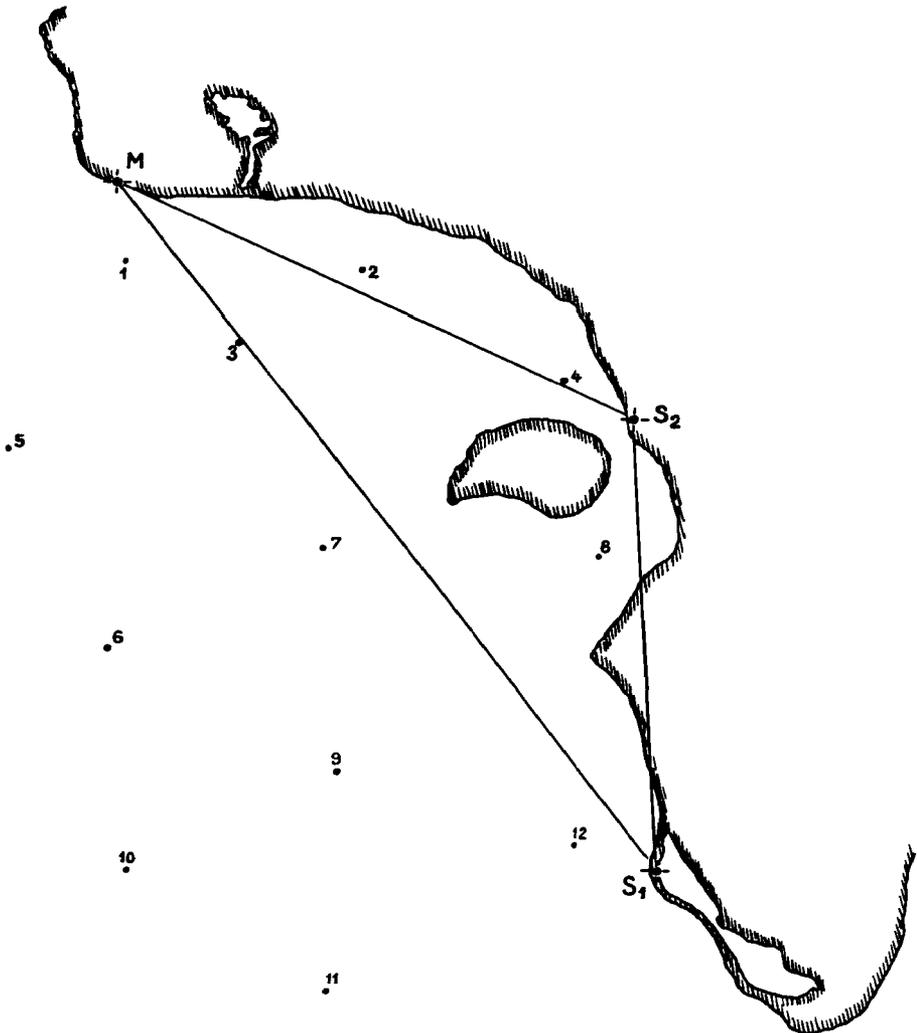


FIG. 3. — Sea-Fix trials : Bonaire, Netherlands Antilles, May 1971.

This modification has been built into the receivers of a Sea-Fix chain operated by the Hydrographic Service of the Royal Netherlands Navy. At present these receivers are only wired for the modified pattern in which Slave 2 can act as Master. In the following derivations this will be the only possibility considered. The equations for patterns in which Slave 1 or Slave 3 act as Master can be obtained by interchanging the Slave numbers.

GEOMETRIC PROPERTIES

Needless to say, with the possibility of interchanging Master and Slaves a much more flexible system is obtained. With Master and Slaves

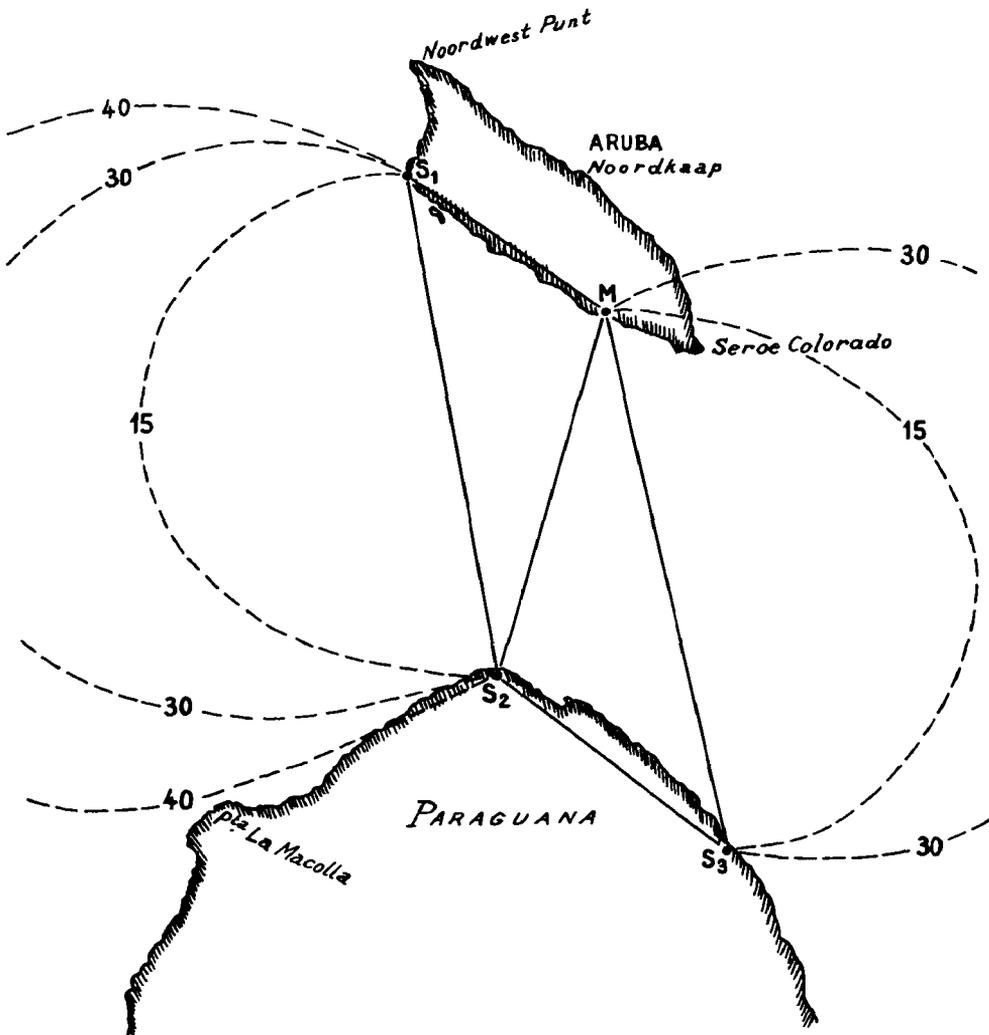


FIG. 4.

located as in figure 3, both baselines are over water, while if the Master station were located at the present Slave 2 site, the Master — Slave 1 baseline would lie partly over land.

As several tall radio masts were located on this baseline re-radiation effects would have been suffered. Moreover, due to the coastline configuration there would have been a likelihood of boundary effects.

With the Master sited as in figure 3, and the receiver switched to "Slave 2 as Master", both Slaves will receive undisturbed trigger signals from the Master. The pattern geometry, however, remains as if the Master occupied the centre position.

Figure 4 shows the sea area between Aruba and the Paraguana Peninsula, Venezuela. Here a three-Slave chain was installed.

As the ground conductivity in Paraguana is very poor (on account of sand dunes and rock), it was unlikely that an S_2S_3 baseline could be chosen. The Master station was therefore sited on Aruba and the Slaves 2 and 3 could thus be triggered over a full sea path. With the receiver switched to "normal" for the Western part and to " S_2 as Master" for the Eastern part, the whole area could be covered.

In cases like the one mentioned here — where the survey area is often shifted in relation to the stations — the need for re-stationing is obviated by this modification.

Another advantage is that when moving along a coast it will not be necessary to shift complete sets of stations, since only one station at a time need be moved.

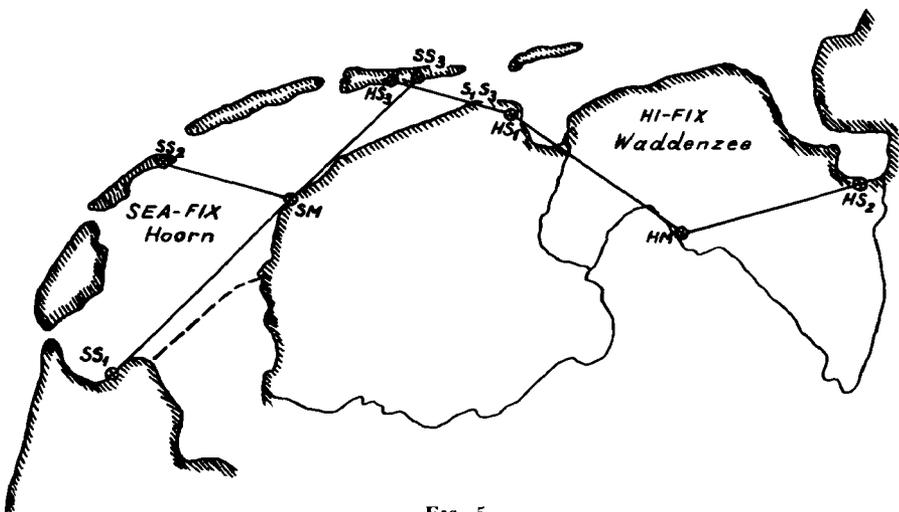


FIG. 5.

In the Netherlands itself the modification has been used for filling a gap in the coverage of the existing Waddenzee Hi-Fix chain and a newly installed Sea-Fix chain (figure 5).

For certain reasons it was important not to alter the positions of the existing Hi-Fix Slaves.

The timing of the Hi-Fix chain was first modified in order that it could be operated with three Slaves. It was then possible to install the third Slave, and thus with the modified receiver the gap between the Hi-Fix and Sea-Fix chains could be covered by the Hi-Fix pattern S_1S_3 .

MATHEMATICAL PROPERTIES

In figure 3, MS_1 and MS_2 give rise to normal hyperbolic patterns. With the receiver switched to " S_2 as Master " the modified patterns S_2M and S_2S_1 are obtained.

Since such parameters as the geographic coordinates of the transmitters, the transmitted frequency, and the propagation speed are the same for both normal and modified patterns it would seem obvious that there must be a mathematical relationship between these two kinds of pattern.

Notation :

MS_1 : distance Master to Slave 1.

ϕ : phase of transmitted signal.

$\phi_{S_1(P)}$: phase of Slave 1 signal, received at point P.

F : frequency of transmitted signal.

V : propagation speed.

$L_{MS_1(P)}$: lane number for pattern MS_1 at point P.

L^{obs} : observed lane number.

L^{comp} : computed lane number (from theodolite observations).

N : total number of lanes in a pattern.

n : total number of full lanes in a pattern ($n =$ integer part of N).

SC : synchronisation constant, a constant phase angle which is added to the Slave signal in the control unit of the Slave transmitter.

1. Equations for a lane number in the normal patterns

a) *Pattern MS_1 .*

At an arbitrary point P, the phase of the received Master signal is :

$$\phi_{M(P)} = \phi_{M(M)} + \frac{F}{V} MP \tag{1}$$

and the received Slave signal :

$$\varphi_{S_1(P)} = \varphi_{S_1(S_1)} + \frac{F}{V} S_1 P \quad (2)$$

Since the Slave 1 transmissions are phase-locked to those of the Master :

$$\varphi_{S_1(S_1)} = \varphi_{M(M)} + \frac{F}{V} MS_1 + SC \quad (3)$$

From equations (2) and (3) we obtain :

$$\varphi_{S_1(P)} = \varphi_{M(M)} + \frac{F}{V} MS_1 + SC + \frac{F}{V} S_1 P \quad (4)$$

From equations (1) and (4), the equation giving the lane number is :

$$L_{MS_1(P)} = \varphi_{M(P)} - \varphi_{S_1(P)} = \frac{F}{V} (MP - MS_1 - S_1 P) - SC \quad (5)$$

It is usual to adopt the lane number maximum at the Slave baseline extension and zero at the Master site. In order to do this we make :

$$SC = - 2 \frac{F}{V} MS_1 \quad (6)$$

From equations (5) and (6) we obtain :

$$L_{MS_1(P)} = \frac{F}{V} (MS_1 + MP - S_1 P) \quad (7)$$

The receiver can only measure fractions of a lane ($\varphi \leq 360$ degrees of phase). A counter is employed for registering the number of full lanes. Thus the synchronization constant given in equation (6) needs only to be expressed as a fraction of a lane.

At the Master baseline extension the full lane counter will then be switched to zero.

Note that the quantity

$$2 \frac{F}{V} MS_1$$

also represents the total number of lanes (N) in the MS_1 pattern.

b) *Pattern* MS_2 .

In the same way as for the MS_1 pattern described above the equation for a pattern MS_2 lane number becomes :

$$L_{MS_2(P)} = \frac{F}{V} (MS_2 + MP - S_2 P) \quad (8)$$

for which the synchronization constant is :

$$- 2 \frac{F}{V} MS_2 \quad (9)$$

2. Equations for a lane number in the modified patterns

a) Pattern S_2M .

With equations (4) and (1), but now for Slave 2, we find :

$$L_{S_2M(P)} = \varphi_{S_2(P)} - \varphi_{M(P)} = \varphi_{M(M)} + \frac{F}{V} MS_2 + SC + \frac{F}{V} S_2P - \varphi_{M(M)} - \frac{F}{V} MP$$

or :

$$L_{S_2M(P)} = \frac{F}{V} (MS_2 + S_2P - MP) + SC \quad (10)$$

From this equation it can be seen that if the chain is calibrated for the modified pattern S_2M , the synchronization constant must be zero, in order to have a zero reading at the Master— S_2 baseline extension. If, on the other hand, the chain is calibrated for the normal pattern MS_2 , equation (10) leads to :

$$L_{S_2M(P)} = -\frac{F}{V} (MS_2 + MP - S_2P) + 2\frac{F}{V} MS_2 + SC$$

or :

$$L_{S_2M(P)} = -L_{MS_2(P)} + 2\frac{F}{V} MS_2 + SC \quad (11)$$

Here

$$2\frac{F}{V} MS_2$$

represents the total number of lanes in the MS_2 pattern (or the S_2M pattern) and equals N .

Now, equation (9) showed that $-SC =$ fraction of N . Thus $N + SC = n$, the number of full lanes in pattern MS_2 .

If the chain is calibrated for the normal pattern MS_2 we may write the relation between patterns MS_2 and S_2M as :

$$L_{S_2M(P)} = n - L_{MS_2(P)} \quad (12)$$

It can be seen from this equation that an increase in the L_{MS_2} lane number results in an equivalent decrease in the L_{S_2M} lane number : i.e. *fixed corrections for the MS_2 pattern can be applied, with opposite signs, to the S_2M pattern.*

If for a point P a pattern correction ΔL_{MS_2} has been established, we may write :

$$L_{MS_2(P)}^{obs} + \Delta L_{MS_2(P)} = L_{MS_2(P)}^{comp} \quad (13)$$

and for the modified pattern :

$$L_{S_2M(P)}^{obs} - \Delta L_{MS_2(P)} = L_{S_2M(P)}^{comp} \quad (14)$$

This is of course if L_{MS_2} has been computed with equation (7) and L_{S_2M} with equation (10), the SC being

$$-2\frac{F}{V} MS_2.$$

b) *Pattern S₂S₁.*

The lane number for the Slave-Slave pattern S₂S₁ can be written as :

$$\begin{aligned} L_{S_2S_1(P)} &= \varphi_{S_2(P)} - \varphi_{S_1(P)} \\ &= \varphi_{M(M)} + \varphi_{S_2(P)} - \varphi_{M(M)} - \varphi_{S_1(P)} \\ &= \{ \varphi_{M(M)} - \varphi_{S_1(P)} \} - \{ \varphi_{M(M)} - \varphi_{S_2(P)} \} \end{aligned}$$

or

$$L_{S_2S_1(P)} = L_{MS_1(P)} - L_{MS_2(P)} + x \quad (15)$$

in which x is a positive, integer quantity representing a multiple of 360° of phase.

Thus, the lane number of the modified pattern S₂S₁ is equal to the difference between the lane numbers of the normal patterns MS₁ and MS₂ plus a constant.

If the chain is calibrated for the normal pattern, that is to say, if Slaves 1 and 2 are synchronized for patterns MS₁ and MS₂ respectively, then :

$$L_{MS_1(P)} = \frac{F}{V} (MS_1 + MP - S_1P)$$

and

$$L_{MS_2(P)} = \frac{F}{V} (MS_2 + MP - S_2P).$$

With these equations, equation (15) becomes :

$$L_{S_2S_1(P)} = \frac{F}{V} (MS_1 + MP - S_1P - MP + S_2P) + x$$

or

$$L_{S_2S_1(P)} = \frac{F}{V} (MS_1 - MS_2 + S_2P - S_1P) + x \quad (16)$$

On the S₁S₂ baseline extension L_{S₂S₁} is minimum. At any point on this extension L_{S₂S₁}(P) + Δφ = 0, Δφ being a fraction of 360 degrees. And we have :

$$S_2P - S_1P = -S_2S_1$$

Under these conditions equation (16) becomes

$$L_{S_2S_1(P)} + \Delta\varphi = \frac{F}{V} (MS_1 - MS_2 - S_2S_1) + x + \Delta\varphi = 0$$

or

$$-\frac{F}{V} (S_2S_1 + S_2M - S_1M) + x + \Delta\varphi = 0$$

$$x + \Delta\varphi = L_{S_2S_1(M)} \quad (17)$$

Thus, $x + \Delta\varphi$ represents the lane number for the Master station in the modified pattern S₂S₁, if it is computed with the general equation

$$L_{S_2S_1(P)} = \frac{F}{V} (S_2S_1 + S_2P - S_1P).$$

This lane number can be considered as a pattern constant; it is composed of an integer part, x , and a fraction, $\Delta\varphi$.

$$\begin{aligned} x &= \text{Int. } L_{S_2S_1(M)} \\ \Delta\varphi &= \text{fraction } L_{S_2S_1(M)} \end{aligned} \tag{18}$$

With formulas (17) and (18), the relation (15) between the modified pattern S_2S_1 and the normal patterns MS_1 and MS_2 becomes :

$$L_{S_2S_1(P)} = L_{MS_1(P)} - L_{MS_2(P)} + \text{Int. } L_{S_2S_1(M)} \tag{19}$$

This equation shows that an increase in the pattern S_2S_1 lane number is equal to the difference between the increases in the MS_1 and MS_2 lane numbers, i. e. *the fixed pattern correction for the modified pattern S_2S_1 is the difference between pattern corrections for the normal patterns MS_1 and MS_2 .*

With equations (16) and (17) we obtain :

$$\begin{aligned} L_{S_2S_1(P)} &= \frac{F}{V}(MS_1 - MS_2 + S_2P - S_1P) + L_{S_2S_1(M)} - \Delta\varphi \\ &= \frac{F}{V}(MS_1 - MS_2 + S_2P - S_1P + S_2S_1 + S_2M - S_1M) - \Delta\varphi \end{aligned}$$

or

$$L_{S_2S_1(P)} = \frac{F}{V}(S_2S_1 + S_2P - S_1P) - \Delta\varphi \tag{20}$$

In other words, if $L_{S_2S_1}$ is computed with the general equation

$$L_{S_2S_1(P)} = \frac{F}{V}(S_2S_1 + S_2P - S_1P)$$

and if Slaves 1 and 2 are synchronized for the normal patterns MS_1 and MS_2 , a correction $-\Delta\varphi = -\text{fraction of } L_{S_2S_1(M)}$ has to be applied to the computed lane number.

If $\Delta L_{MS_1(P)} = \text{pattern correction for pattern } MS_1 \text{ at point } P$ and $\Delta L_{MS_2(P)} = \text{pattern correction for pattern } MS_2 \text{ at point } P$, then with equation (20) we obtain :

$$L_{S_2S_1(P)}^{obs} + (\Delta L_{MS_1(P)} - \Delta L_{MS_2(P)}) = L_{S_2S_1(P)}^{comp} - \Delta\varphi \tag{21}$$

Resume

When in a two-slave hyperbolic chain, Slaves 1 and 2 are synchronized to obtain the normal patterns MS_1 and MS_2 and the receiver is switched to position " Slave 2 as Master ", the following set of equations is satisfied for the modified patterns :

a) *Mathematical model.*

$$(10) \quad L_{S_2M(P)} = \frac{F}{V} (S_2M + S_2P - MP) + SC_{(MS_2)}$$

$$(20) \quad L_{S_2S_1(P)} = \frac{F}{V} (S_2S_1 + S_2P_1 - S_1P) - \Delta\varphi$$

b) *Relation between modified and normal patterns.*

$$(12) \quad L_{S_2M(P)} = n - L_{S_2M(P)}$$

$$(19) \quad L_{S_2S_1(P)} = L_{MS_1(P)} - L_{MS_2(P)} + \text{Int. } L_{S_2S_1(M)}$$

c) *Observation equations.*

$$(14) \quad L_{S_2M(P)}^{\text{obs}} - \Delta L_{MS_2(P)} = L_{S_2M(P)}^{\text{comp}}$$

$$(21) \quad L_{S_2S_1(P)}^{\text{obs}} + (\Delta L_{MS_1(P)} - \Delta L_{MS_2(P)}) = L_{S_2S_1(P)}^{\text{comp}} - \Delta\varphi$$

In these equations

$$SC_{(MS_2)} = -2 \frac{F}{V} MS_2$$

$$\Delta\varphi = \frac{F}{V} (S_2S_1 + S_2M - S_1M)$$

SC and $\Delta\varphi \leq 360$ degrees

ΔL = pattern correction.

Note : Naturally, it is also possible to calibrate the modified pattern direct. Synchronization will then be carried out in such a way that the following equations are satisfied :

$$L_{S_2M(P)} = \frac{F}{V} (S_2M + S_2P - MP)$$

$$L_{S_2S_1(P)} = \frac{F}{V} (S_2S_1 + S_2P - S_1P)$$

The pattern corrections ΔL_{S_2M} and $\Delta L_{S_2S_1}$ can then be established with these equations.

3. Field trials

The first trials with the Sea-Fix chain using the modified receivers took place along the coast of Bonaire Island, Netherlands Antilles (figure 3).

The four patterns MS_1 , MS_2 , S_2M , and S_2S_1 were observed simultaneously with two receivers at 12 calibration positions, one receiver being switched to "normal" and the other to "S₂ as Master". At a given radio command the geographical coordinates were fixed by theodolite from trig-points ashore.

A phase-lag curve was established by crossing the baseline extensions at various distances.

The normal calibration computations were carried out and propagation speeds (over land and sea), monitor readings, and fixed corrections were established.

As it had been decided to leave the Slave gonio-settings at zero, the fixed corrections also include the synchronization constant.

a) *Normal pattern.*

As shown in tables 1 and 2 the observations of the normal patterns were corrected for monitor and phase lag errors. The computations were carried out using wave propagation speed over sea and the computed lane numbers were corrected for the difference between this speed and the propagation speed over land.

The pattern corrections ΔL_{MS_1} and ΔL_{MS_2} (computed minus observed) were obtained by computing the difference between the computed-and-corrected and the observed-and-corrected lane numbers.

TABLE 1
Pattern MS_1 .

	⑧	⑨	⑩	⑪	⑫	⑬	⑭
F I X	Obs.	Monitor + Phase Lag Corr.	Obs. and Corr.	Comp.	Propaga- tion Speed Corr.	Comp. and Corr.	$\Delta L_{MS_1} =$ C-O
N°			⑧ + ⑨			⑪ + ⑫	⑬ - ⑩
1.	15.64	+ 0.04	15.68	15.49	0	15.49	- 0.19
2.	77.33	+ 0.03	77.36	77.18	0	77.18	- 0.18
3.	59.42	0	59.42	59.29	0	59.29	- 0.13
4.	124.41	+ 0.01	124.42	124.30	- 0.04	124.26	- 0.16
5.	64.07	+ 0.01	64.08	63.93	0	63.93	- 0.15
6.	116.70	+ 0.01	116.71	116.55	0	116.55	- 0.16
7.	132.27	+ 0.01	132.28	132.15	0	132.15	- 0.13
8.	168.97	+ 0.01	168.98	168.79	+ 0.02	168.81	- 0.17
9.	178.10	0	178.10	177.89	0	177.89	- 0.21
10.	162.55	0	162.55	162.30	0	162.30	- 0.25
11.	205.09	0	205.09	204.87	0	204.87	- 0.23
12.	239.66	- 0.04	239.62	239.47	0	239.47	- 0.15

TABLE 2
Pattern MS₂.

	①	②	③	④	⑤	⑥	⑦
F I X	Obs.	Monitor + Phase Lag Corr.	Obs. and Corr.	Comp.	Propaga- tion Speed Corr.	Comp. and Corr.	$\Delta L_{MS_2} =$ C-O
N°			① + ②			④ + ⑤	⑥ - ③
1.	12.30	+ 0.04	12.34	12.14	+ 0.01	12.15	- 0.19
2.	86.43	+ 0.02	86.45	86.25	+ 0.03	86.28	- 0.17
3.	56.52	0	56.52	56.36	+ 0.02	56.38	- 0.14
4.	140.94	0	140.94	140.78	- 0.03	140.75	- 0.19
5.	36.78	+ 0.01	36.79	36.64	+ 0.03	36.67	- 0.12
6.	70.51	+ 0.01	70.52	70.42	- 0.06	70.36	- 0.16
7.	101.54	+ 0.02	101.56	101.47	- 0.06	101.41	- 0.15
8.	149.83	- 0.02	149.81	149.48	+ 0.07	149.55	- 0.26
9.	107.54	+ 0.02	107.56	107.33	- 0.03	107.30	- 0.26
10.	87.22	+ 0.02	87.24	87.01	- 0.03	86.98	- 0.26
11.	110.15	+ 0.02	110.17	109.89	- 0.01	109.88	- 0.29
12.	135.97	+ 0.02	135.99	135.71	- 0.01	135.70	- 0.29

b) *Modified pattern.*

With the calibration results from the normal pattern, the following constants were computed :

$$N_{MS_2} = 161.28 \quad n = 161 \quad SC = - 0.28$$

$$L_{S_2S_1(M)} = 20.132 \quad \text{Int.} \times L_{S_2S_1(M)} = 20 \quad \Delta\varphi = 0.13$$

Tables 3 and 4 list the observations for the modified patterns S₂M and S₂S₁.

Columns 17 and 25 show that by using equations (12) and (19) respectively, the differences between the lane numbers observed in the modified patterns and the corresponding lane numbers theoretically converted from the normal patterns are only small and can be attributed to instrumental errors.

In general these differences will be even smaller, since at the time of our calibration the radiated pattern was not very stable, due to installation trouble.

The observed lane numbers were corrected for errors discovered by the monitor and for phase-lag errors. The lane numbers were again computed using wave propagation speed over sea, and then corrected for the difference between this speed and the speed over land.

As the computations were carried out with the general equations, the lane numbers were also corrected using the synchronization constant and $\Delta\varphi$ [see equations (10) and (20) respectively].

TABLE 3
Pattern S_2M .

	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
F I X	Obs.	$n - L_{MS_2}$	Diff.	Comp. and Corr.	Comp.	Comp. and Corr.	$\Delta L_{(S_2M)} = C-O$	Diff. = $\Delta L_{MS_2} + \Delta L_{S_2M}$
N°	(16) - (1)	(16) - (1)	(16) - (15)	(15) - (2)	(19) - (5)	(19) - (5) - 0.28	(20) - (18)	(7) + (21)
1.	148.71	148.70	- 0.01	148.67	149.14	148.85	+ 0.18	- 0.01
2.	74.57	74.57	0	74.55	75.03	74.72	+ 0.17	0
3.	104.48	104.48	0	104.48	104.93	104.63	+ 0.15	+ 0.01
4.	20.07	20.06	- 0.01	20.07	20.50	20.25	+ 0.18	- 0.01
5.	124.19	124.22	+ 0.03	124.18	124.64	124.33	+ 0.15	+ 0.03
6.	90.50	90.49	- 0.01	90.49	90.86	90.64	+ 0.15	- 0.01
7.	59.46	59.46	0	59.44	59.81	59.59	+ 0.15	0
8.	11.17	11.17	0	11.19	11.80	11.45	+ 0.26	0
9.	53.45	53.46	+ 0.01	53.43	53.96	53.71	+ 0.28	+ 0.02
10.	73.77	73.78	+ 0.01	73.75	74.27	74.02	+ 0.27	+ 0.01
11.	50.83	50.85	+ 0.02	50.81	51.40	51.11	+ 0.30	+ 0.01
12.	25.03	25.03	0	25.01	25.57	25.28	+ 0.27	- 0.02

TABLE 4
Pattern S₂S₁.

	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
F		MS ₁ - MS ₂ +	Diff.	Obs.	Comp.	Comp.	L _(S₂S₁)	Diff. =
I		Int. =		and		and	C-O	$\Delta L_{S_2S_1}$
X		L _{S₂S₁(M)}		Corr.	Comp.	Corr.		ΔL_{MS_1} +
N ^o		(8) - (1) + 20.0	(24) - (23)	(23) + (9) - (2)		(27) + (12) - (5)	(28) - (26)	ΔL_{MS_2}
1.	23.38	23.34	- 0.04	23.38	23.48	12.34	- 0.04	- 0.04
2.	10.91	10.90	- 0.01	10.92	11.06	10.90	- 0.02	- 0.01
3.	22.91	22.90	- 0.01	22.91	23.07	22.92	+ 0.01	0
4.	3.47	3.47	0	3.48	3.66	3.52	+ 0.04	+ 0.01
5.	47.28	47.29	+ 0.01	47.28	47.42	47.26	- 0.02	+ 0.01
6.	66.17	66.19	+ 0.02	66.17	66.26	66.19	+ 0.02	+ 0.02
7.	50.76	50.73	- 0.03	50.75	50.81	50.74	- 0.01	- 0.03
8.	39.11	39.14	+ 0.03	39.14	39.44	39.26	+ 0.12	+ 0.03
9.	90.54	90.56	+ 0.02	90.52	90.70	90.60	+ 0.08	+ 0.03
10.	95.33	95.33	0	95.31	95.42	95.32	+ 0.01	0
11.	114.95	114.94	- 0.01	114.93	115.12	115.00	+ 0.07	+ 0.01
12.	123.70	123.69	- 0.01	123.64	123.89	123.77	+ 0.13	- 0.01

The observed pattern corrections ΔL_{S_2M} and $\Delta L_{S_2S_1}$ could then be established.

Comparing the pattern corrections derived from ΔL_{MS_1} and ΔL_{MS_2} with the observed corrections, it is seen that the small difference can once again be attributed to an instrumental error (see columns 22 and 30).

4. Two-range mode

Although this has not yet been tested, it should also be possible to switch a receiver to the modified mode while working in the ranging mode (Master on board ship).

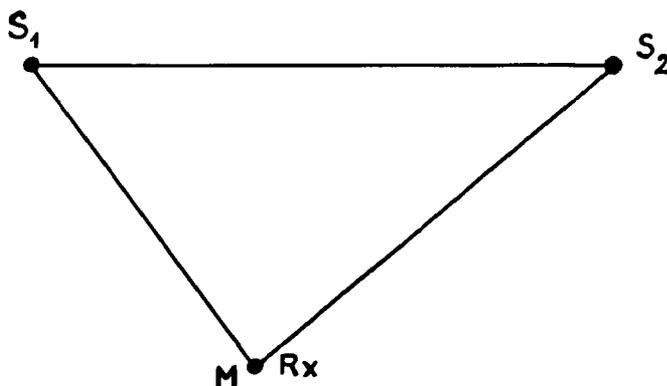


FIG. 6.

For Slave 1 we have :

$$\varphi_{M(S_1)} = \varphi_{M(M)} + \frac{F}{V}MS_1 \tag{22}$$

and

$$\varphi_{S_1(S_1)} = \varphi_{M(S_1)} + SC_1 \tag{23}$$

For the ship's receiver (R) we have :

$$\varphi_{S_1(R)} = \varphi_{S_1(S_1)} + \frac{F}{V}RS_1 \tag{24}$$

With equations (22), (23) and (24) we obtain :

$$\varphi_{S_1(R)} = \varphi_{M(M)} + \frac{F}{V}MS_1 + SC_1 + \frac{F}{V}RS_1$$

In the two-range mode $MS_1 = RS_1$:

$$\varphi_{S_1(R)} = \varphi_{M(M)} + 2\frac{F}{V}RS_1 + SC_1 \tag{25}$$

Analogously, for Slave 2 :

$$\varphi_{S_2(R)} = \varphi_{M(M)} + 2\frac{F}{V}RS_2 + SC_2 \tag{26}$$

Now, with the receiver switched to " S_1 as Master " :

$$L_{S_1 S_2(R)} = \varphi_{S_1(R)} - \varphi_{S_2(R)} = 2 \frac{F}{V} R S_1 - 2 \frac{F}{V} R S_2 + S C_1 - S C_2$$

or :

$$L_{S_1 S_2(R)} = 2 \frac{F}{V} (S_1 S_2 + R S_1 - R S_2) - 2 \frac{F}{V} S_1 S_2 + S C_1 - S C_2 \quad (27)$$

The term

$$- 2 \frac{F}{V} S_1 S_2 + S C_1 - S C_2$$

is a constant C.

Equation (27) then becomes :

$$L_{S_1 S_2(R)} = 2 \frac{F}{V} (S_1 S_2 + R S_1 - R S_2) + C \quad (28)$$

Note : Although the term

$$2 \frac{F}{V} S_1 S_2$$

has been introduced into (27) S_1 and S_2 are not interdependent, i. e. *from a propagation point of view pattern $S_1 S_2$ is independent of path $S_1 S_2$.*

Comparing equation (28) with (7), it is seen that a hyperbolic pattern between Slaves 1 and 2 is created, and that it has double the number of lanes. *In other words, a pattern is formed having a lanewidth equal to a quarter of the wave-length (figure 7).*

The other pattern for the modified receiver is $\varphi_{S_1} - \varphi_M$ which equals the normal range MS_1 .

This combination of a hyperbolic and a circular pattern of course improves neither the accuracy of position fixing nor the coverage.

The hyperbolic pattern may find its application as a steering pattern for survey work.

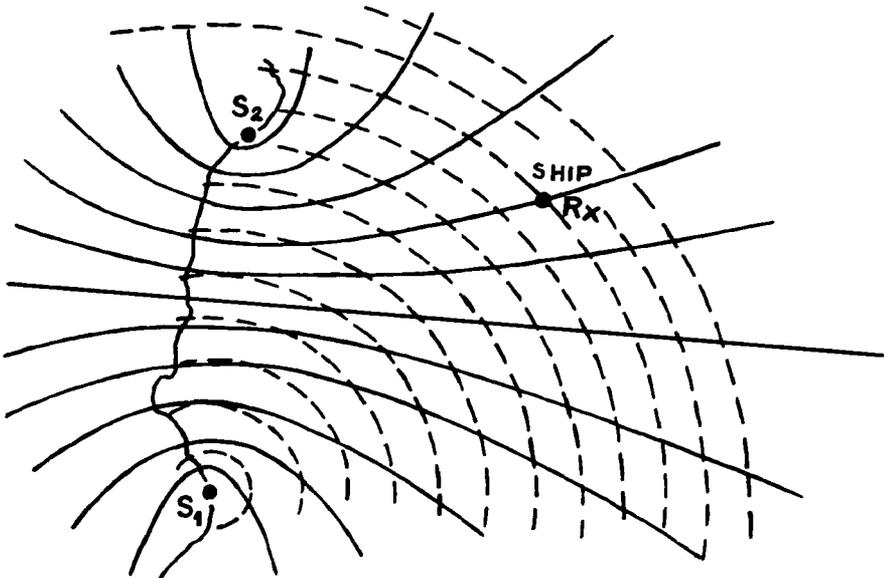


FIG. 7.