WAVE PATTERN DIAGRAMS

by H. DUFFO
Captain (Ret.), French Navy

and M. Van HULSE,
Ingénieur Civil de l'Université de Liège

An earlier article of mine (DUFFO), carrying this same title was published in the *I.H. Review*, Vol. XLIII, No. 1, January 1966.

I dwelt there on a major shortcoming in the conventional methods used for working out wave pattern diagrams. In both these methods — the wave crest method, and the orthogonal method — the sea floor is represented by a tiered surface which is obviously very different from the actual surface.

The above article described a method where the relief is represented by a surface which we can make as similar to the actual surface as we like, but which does not include any of those abrupt discontinuities that cause us to question the reliability of traditional methods.

In that article the problem was solved satisfactorily from a theoretical point of view, but although the various steps were explained, the necessary working documents were not given.

Reading this earlier article we see that at least *two* special protractors are indispensable — one for increases and the other for decreases in depth. In point of fact it would be advisable to have *at least four* of these special protractors, but at that time none had been devised. I gave *one* diagram which when read in conjunction with an overlay could serve as a special protractor, but this was merely for a decrease in depth. Its object was simply to enable me to establish an example of a wave pattern diagram for an area where the upward of the bottom towards the coast was quite uninterrupted. This is by no means an exceptional occurrence, but we ought not to limit ourselves to studying places were the slope is continuously upwards.

A table or a graph for computing successive angles of incidence is also essential, but my earlier article supplied neither.

I was aware of these shortcomings — but at the time, for lack of means, I shrank from undertaking the enormous amount of work involved. Now, however, it has been possible to carry out this work, thanks to the computer at the University of Kinshasa.

There were also further objections to my method which I had not myself foreseen but about which I was informed later, and these criticisms certainly have their pertinency.

There has been strong criticism of the recommended preliminary work, i.e. of the choice of isobath interval, and especially of the drawing of contour lines with $\Delta C/C$ constant, although the latter procedure is recommended by Ingénieur Général LAVAL in his treatise on Marine Engineering.

Admittedly, it would be too much to hope that Hydrographic Offices, already more than fully occupied and short of funds, would consider starting an enterprise that has such a small likelihood of profitability as that of compiling the special charts essential for the correct drawing of consecutive contour lines with $\Delta C/C$ constant. The operator will therefore have to draw these contours himself, and he will be certain to make errors that will lead to still further distortion of the picture of an already arbitrarily simplified topography. Additionally, the isobaths on a chart — and these are as a rule the most reliable and representative element of the bottom configuration — will often be incorrectly utilized and thus poor results will be obtained since the contour lines which were adopted may be relatively too far away.

We have therefore envisaged a solution using *no more than* the contour lines shown on the chart (provided these are close enough together), or at all events to make use of *all* such lines. We shall thus be able to avoid, or reduce to the strict minimum, the necessity of tracing the intermediate isobaths upon which the conventional methods have to depend.

However, an exposition of this process would go far beyond the scope of a single article. We will thus confine ourselves to indicating a simple method which, if used judiciously, will nevertheless retain a very adequate measure of accuracy.

*
* *

We shall start by returning to a conventional characteristic of orthogonals for the particular case where the isobaths are rectilinear and parallel [LONGUET-HIGGINS (1956)].

Let us take two of these orthogonals, C_0 and C_α — indicated thus after their velocity (*) — and on the section of the orthogonal between them,

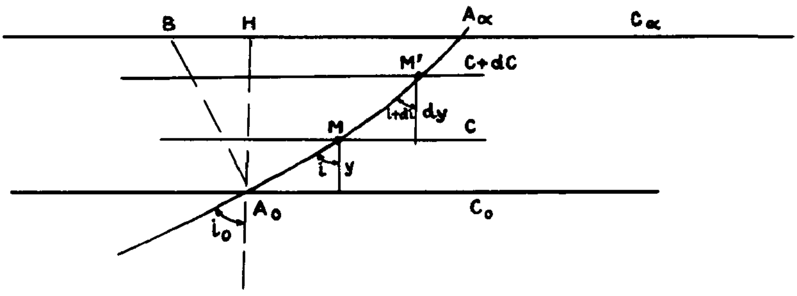


FIG. 1

(*) Velocity : rate of wave propagation.

$A_0 A_\alpha$, let M and M′ be almost adjacent points (see figure 1). Furthermore let the distance from M to A_0C_0 be designated y, C being the velocity of the isobath passing through M, and i the angle of incidence.

Likewise i_0 is this angle of incidence at A_0, and $y + dy$, $C + dC$, $i + di$ at M′. $MM' = ds$, as ds is an infinitesimal.

"In a narrow layer where the index for the angle of refraction is constant, this angle equals the angle of incidence made with the following layer" [LACOMBE (1964)]. Since sin i/C is constant all along the orthogonal we may write :

$$\frac{\sin i_0}{C_0} = \frac{\sin i}{C} = \frac{\sin (i + di)}{C + dC} = \frac{\sin (i + di) - \sin i}{dC}$$

and then

$$\cos i \,.\, \frac{di}{dC} = \frac{\sin i_0}{C_0} = \text{constant.}$$

Let us now compute the radius of curvature at M, *i.e.*

$$R = \frac{ds}{di}.$$

However, as

$$ds = \frac{dy}{\cos i} \;;$$

we may eliminate i :

$$R = \frac{dy}{dC} \,.\, \frac{C_0}{\sin i_0}$$

As R depends solely on the gradient of C in relation to y, if this gradient is constant between the two isobaths C_0 and C_α it will be equal to

$$\frac{C_0 - C_\alpha}{A_0H}$$

and the radius of curvature will not vary. The orthogonal is thus a circle of radius :

$$\frac{A_0H}{\dfrac{C_0 - C_\alpha}{C_0}} \cdot \frac{1}{\sin i_0}$$

The table shown in the Appendix yields values of C as a function of depth, and will facilitate computation.

In reality the contour lines are never exactly parallel and rectilinear, although they will be *nearly* straight and will have only a *very small* angle between them. The gradient will not be constant, but will only vary *very slightly* between the two contour lines if these have been chosen sufficiently close.

We shall obtain orthogonals that closely resemble a relief with nearly rectilinear and parallel isobaths, where the velocity between them varies little, by replacing this relief by a fictitious surface which approximates it very closely, one where the contour lines are exactly rectilinear and parallel and where between two sufficiently close lines, we shall adopt for the velocity a constant gradient that will equal the mean gradient :

$$\frac{C_0 - C_\alpha}{A_0 H}$$

However, this will only solve the problem for a case that is extremely rare. Basing ourselves on this case we will now try to find another solution that will be valid for the more general case which is where any contour line may be concerned.

*
**

Let us take an isobath of velocity C_0 (fig. 2). A wave front hits it at A_0 and is refracted back to the isobath C_α which it intersects at A_α. The tangents at A_0 and A_α are A_0T_0 and $A_\alpha T_\alpha$.

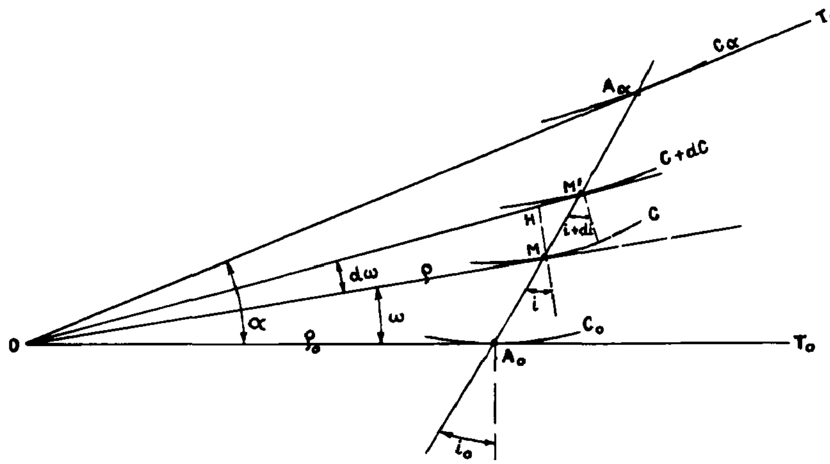


FIG. 2

On A_0A_α let us now take two neighbouring points M and M′ which are the intersections with the isobaths C and C + dC whose tangents at M and M′ make angles ω and $\omega + d\omega$ with the tangent A_0T_0, the tangents meeting at O. Although the figure shows it thus, we need not at present assume that the tangents A_0T_0 and $A_\alpha T_\alpha$ pass through point O; The angles of incidence are i and $i + di$.

We will assume that between the isobaths C_0 and C_α we may draw an infinity of isobaths having C and $C + dC$ sufficiently close to one another for us to state that the velocity between two consecutive isobaths is constant and equal to $C + dC/2$. At M the wave front will pass from a medium of speed C to a medium where up to M′ its velocity will be $C + dC/2$, the velocity interval adopted being extremely small.

We may write :

$$\frac{\sin i}{C} = \frac{\sin r}{C + \frac{dC}{2}} = \frac{\sin r - \sin i}{\frac{dC}{2}} = 2 \sin \frac{r - i}{2} \cos \frac{r + i}{2} \cdot \frac{1}{\frac{dC}{2}},$$

or

$$\frac{dC}{2C} \cdot \frac{\sin i}{\cos \frac{i + r}{2}} = 2 \sin \frac{r - i}{2}$$

But M and M′ are extremely close to one another, and $\frac{i + r}{2}$ is practically equal to i. Likewise, the principal value of $2 \sin \frac{r - i}{2}$ is $r - i$. Disregarding the infinitesimals of higher order we have as a result :

$$\frac{dC}{C} \tan i = 2 (r - i).$$

Now let us take into account the fact that the curvature of M — M′ is extremely small in reality. (Figure 3 is an enlargement of the centre portion of figure 2).

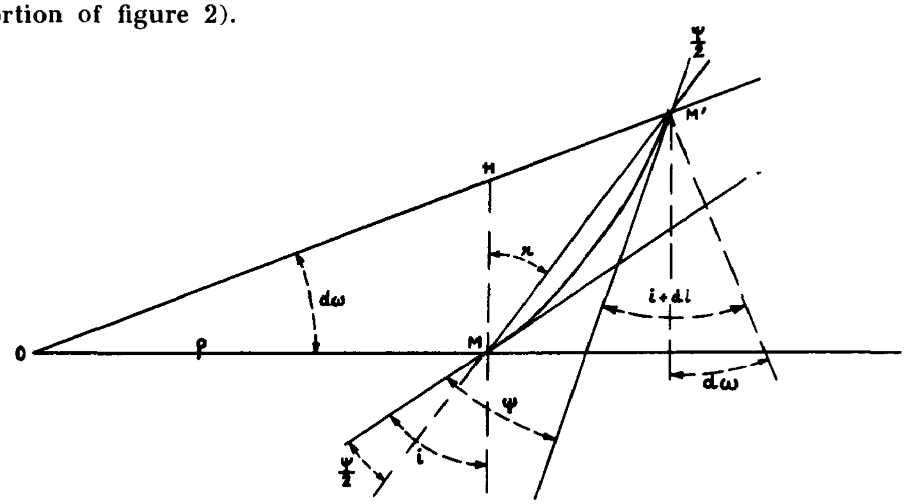


FIG. 3

The segment of line M—M′ is the chord of an infinitesimal curvature which can be considered as an arc of a circle, and it is seen that $2(r - i)$ is the angle by which the tangent to the orthogonal has rotated between M and M′. This angle is denoted Ψ on figure 3.

$$\Psi = \tan i \frac{dC}{C}$$

The radius of curvature at M will be the limit of the ratio between the arc MM′ = ds and the angle Ψ when M′ tends towards M, $d\omega$ tending to zero :

$$R = \frac{MM'}{\Psi} = \frac{ds}{dC} \cdot \frac{C}{\tan i}$$

$d\omega$ being infinitesimal, angle H of triangle MHM′ can be considered as a right angle since

$$MM' = ds = \frac{MH}{\cos i} = \frac{dy}{\cos i}$$

the infinitesimal MH being designated dy.

We then obtain the formula :

$$R = \frac{dy}{dC} \cdot \frac{C}{\sin i}$$

which is similar to the one computed above for the case of rectilinear parallel isobaths. However, $\overrightarrow{dy}$ no longer has a fixed direction. Moreover, even if between the isobaths C_0 and C_α dy/dC were strictly constant, the radius of curvature would vary since C and sin i vary without their ratio remaining constant.

Nevertheless we can make use of this formula. We can always take C_α close enough to C_0 so that between these two isobaths the velocity will vary very little, and will always take the same direction. It will then be permissible to put $dC/C = - K d\omega$, K being a non-dimensional function of $d'\omega$, it will always have the same sign and will vary very little.

We can therefore make a first approximation which will consist of assuming that K is a constant. This amounts once again to taking — instead of the actual relief during the course of the path of a wave front — a fictitious relief that resembles it very closely. We have already made a similar approximation for the case of isobaths that are both *nearly* rectilinear and *nearly* parallel. The formula for the radius of curvature becomes :

$$R = \frac{dy}{d\omega} \cdot \frac{1}{K \sin i}$$

a formula where we shall only be considering the *absolute values*. Furthermore, in order to simplify notation, we propose henceforward *systematically* only to consider absolute values.

K has a known value. Thus, if tangent $A_\alpha T_\alpha$ makes and angle α with tangent $A_0 T_0$ by integration we shall have :

$$K = \log_e \frac{C_0}{C_\alpha} / \alpha$$

In actual fact α is not known since A_α is unknown, its determination being one of the objects of this exercise. However, we have some idea of where α is to be found. We shall take A_α a little either to the left or right of the point where the intersecting curve at A_0 meets the isobath C_α, according to whether the velocity decreases or increases (i.e. whether the depth is decreasing or increasing). In the area we are considering, the position of the tangent will hardly vary, except when isobath C_α has a strong curvature. More often only a negligible variation in the value of angle α will be entailed when A_α is very slightly moved.

However, if the difference were to turn out too large, we could consider the first plotting as an approximation, and after plotting the orthogonal approximately we could rectify the value of both α and K.

Figure 2 shows that $dy/d\omega$ has geometrical significance. This significance is MO, since O is the point of contact of the tangent at M and the envelope of tangents to the contour lines at each point on the wave front. If we put $\rho = MO$, the formula giving R becomes :

$$R = \frac{\rho}{K \sin i}$$

Thus, quite naturally, we can think of using A_0 as the basis for determining the orthogonal's radius of curvature. Over a short path we can assimilate the orthogonal with its osculating circle from A_0 up to a point not far away, relatively speaking. Then from this point step by step we will draw a series of arcs up to the isobath C_α, arcs which will touch each other tangentially. In the classic methods the orthogonal is plotted as a broken line.

On the chart the intermediate contour lines between such isobaths as C_0 and C_α are not known, and in general we are only able to plot them very approximately and very arbitrarily. This uncertainty justifies the use of a particularly convenient second approximation which will enable us to adopt a final form for our fictitious relief. We will assume that on a wavefront path not only is K constant but also that the tangents converge, as point O is a fixed point. These two conditions determine an infinity of sections of intermediate isobaths. It will not be necessary to draw new contour lines unless the chart's contour lines were not close enough together to present a valid picture of the relief.

The choice of our fictitious surface has one important advantage : for *each point* on the orthogonal, ρ will be known as soon as *the position of this point* is known, since O is known. If K has been correctly determined — and this is possible as soon as α has been determined with sufficient approximation — R will be known both accurately and continuously all along the orthogonal.

If we take an angle φ where $\tan \varphi = K$ we shall obtain a pleasing construction, but we shall see that it will only rarely have a practical application.

R is then obtained by the formula :

$$R = \frac{dy}{d\omega} \cdot \frac{1}{K \sin i} = \frac{\rho}{\tan \varphi \sin i}$$

Starting from A_0; in the above formula i is given information and ρ is also known, since if we adopt a position for the tangent $C_\alpha T_\alpha$, point O is determined. With this angle α, we shall later in this article see how φ can be computed by means of a nomogram.

On figure 4 it can immediately be seen that A_0B is equal to $\rho_0/\sin i_0$, as B is the meeting point of the incident ray at A_0 with the perpendicular at O to line A_0O.

On the perpendicular to the incident ray at A_0, let Q be a point such that BQ makes an angle $\pi/2 - \varphi$ with BA_0. A_0Q is equal to A_0B. cotan φ or to

$$\frac{\rho_0}{\sin i \ \tan \varphi} = R$$

The centre of the curvature of the orthogonal at A_0 is then Q when the depth decreases, or its symmetrical counterpart in relation to A_0 when the depth is increasing.

An arc of an osculating circle is drawn as far as M. A_0M must subtend an angle Ψ taken small enough for the curve to merge with the osculating circle. To make sure that this is so we will compute the radius of curvature at M by the same formula but where ρ_0 will be

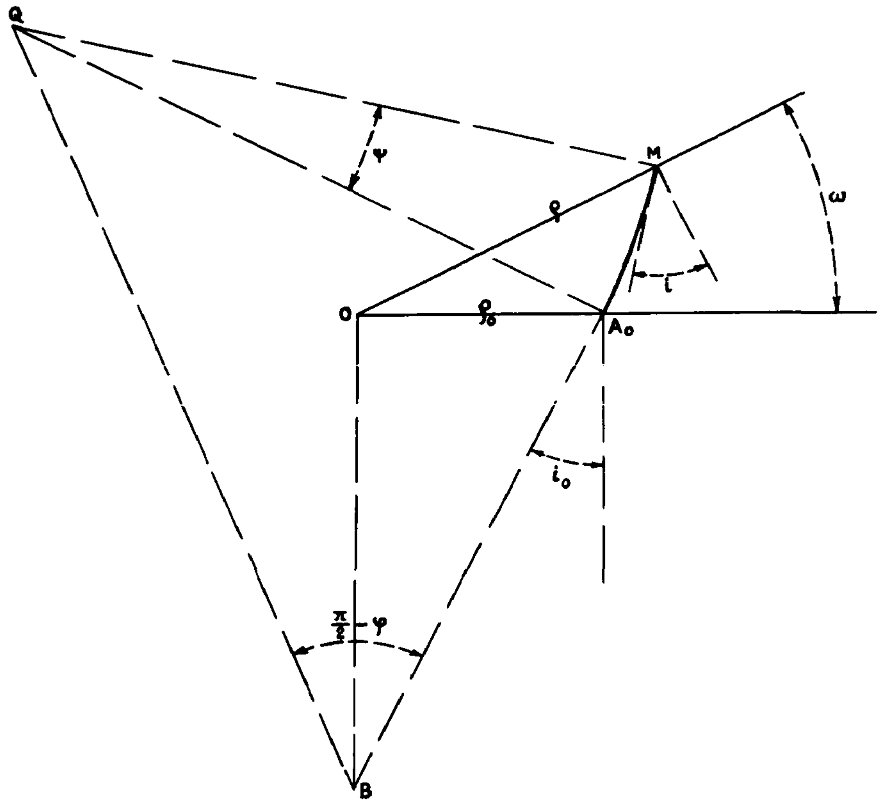


FIG. 4

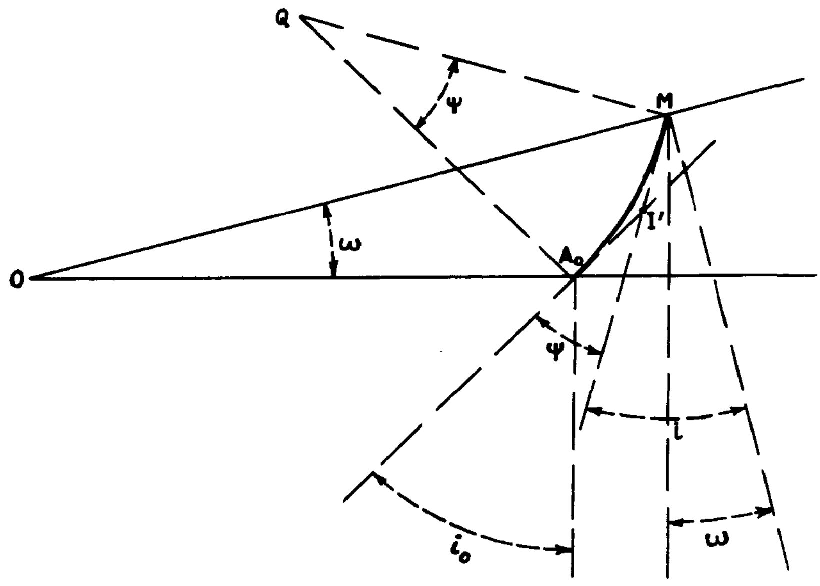


FIG. 5

replaced by ρ = OM and i_0 by $i = i_0 + \omega - \Psi$. Angle Ψ will be small enough when along arc A_0M, and to within the graphical accuracy, these two circles merge, the circles being tangent at A_0 at the beginning of the orthogonal, and their radii being, respectively, the radius of the curvature at A_0 and that computed for M. When this merging is attained we can continue step by step. If the difference is too great we can take a new point M closer to A_0.

However, the first *practical* difficulty that arises is that point O is not always within the graphical limits, and consequently neither ρ_0 nor the series of ρ's will be explicited.

There can also be a second difficulty. Even if O is within the graphical limits Q can be outside them if φ is very small. We therefore had to decide on another procedure where these difficulties will not arise.

We will write :

$$R = \frac{\rho \,.\, \alpha}{\sin i \log_e \frac{C_0}{C_\alpha}} \quad \text{since} \quad \tan \varphi = K = \frac{\log_e \frac{C_0}{C_\alpha}}{\alpha}$$

Taking the case, for example, of an arc of circle with A_0 as starting point, we shall have :

$$R_0 = \frac{A_0 H}{\sin i_0 \,.\, \log_e \frac{C_0}{C_\alpha}} \cdot \frac{\alpha}{\tan \alpha} \quad \text{since } \rho_0 = \frac{A_0 H}{\tan \alpha} \text{ (figure 6)}$$

Let us now take point M on a circle of radius R_0 that we shall assume already plotted. A_0M is subtended by angle with its apex at Q, making an angle $\Psi/2$ with the incident ray. To determine M it suffices to know the distance OM which we shall call d :

$$d = 2R_0 \sin \frac{\Psi}{2} = \frac{\alpha_0}{\tan \alpha} \cdot \frac{A_0 H}{\sin i_0} \cdot \frac{2 \sin \frac{\Psi}{2}}{\log_e \frac{C_0}{C_\alpha}}$$

Thus, if we attribute an arbitrary value to Ψ we can plot M anywhere on the osculating circle. The tangent at M to this circle will make an angle $\Psi/2$ with A_0M. For determining the arc of circle we can alternatively take point I where the tangent is parallel to A_0M. This will be the point of intersection of the bisectors of the angles A_0M and MA_0 made by these two tangents and of the mediatrix of A_0M.

However, it may be more advantageous to take another road, for example to choose d that we can take equal to A_0H/s and to compute the corresponding angle Ψ, s being an arbitrary number.

From :

$$d = \frac{A_0 H}{s} = \frac{\alpha}{\tan \alpha} \cdot \frac{A_0 H}{\sin i_0} \cdot \frac{2 \sin \frac{\Psi}{2}}{\log_e \frac{C_0}{C_\alpha}}$$

we deduce :

$$\sin \frac{\Psi}{2} = \frac{\tan \alpha}{\alpha} \cdot \frac{\sin i_0}{2} \cdot \frac{\log_e \dfrac{C_0}{C_\alpha}}{s}$$

We see that the $1/s$ factor can at once be computed. This gives sin $\Psi/2$ for $s = 1$. We shall obtain a sin $\Psi/2$ corresponding to an s that is not 1 by making use of the integer values 2, 3, 4, 5, etc. — or their reciprocals — and by dividing or multiplying sin $\Psi/2$ by 2, 3, 4, 5, etc.

We can then obtain points on the osculating circle — for instance M — whose distance to A_0 will equal A_0H divided or multiplied by an integer. Let us now draw the arc for one of these points. We shall consider this arc correctly chosen when, if from M we plot it towards A_0, it meets this osculating circle A_0 — to within the graphical accuracy.

If the difference is unacceptable, we conclude that A_0M is too large, and we will therefore select a new point M on the arc, for example at point I. We will then proceed to carry out the same checking process again.

This arrangement is valid for the case where $\alpha = 0$ (i.e. where the contour lines are rectilinear and parallel). The formula can, however, be simplified because $\tan \alpha/\alpha = 1$, and since it is very nearly equivalent to the formula found by direct means. In fact, when C_0 and C_α are close $\log_e C_0/C_\alpha$ differs very little from

$$\frac{C_0 - C_\alpha}{C_0}$$

Therefore the two fictitious reliefs, represented by the two formulas, merge almost exactly.

However, in order to be able to make a computation in the "reverse sense" — with a view to carrying out the check recommended above — it will be essential to know the direction of OM even when point O is outside the graphical limits.

On figure 6 we see that :

$$\tan \omega = \frac{MH'}{\rho_0 + A_0H'} = \frac{MH'}{\dfrac{A_0H}{\tan \alpha} + A_0H'}$$

or :

$$\tan \omega = \tan \alpha \cdot \frac{MH'}{A_0H} \cdot \frac{1}{1 + \dfrac{A_0H'}{A_0H} \cdot \tan \alpha}$$

or by putting

$$\tan \beta = \frac{A_0H'}{A_0H}$$

$$\tan \omega = \tan \alpha \cdot \frac{MM'}{A_0H} \cdot \frac{1}{1 + \tan \alpha \ \tan \beta}$$

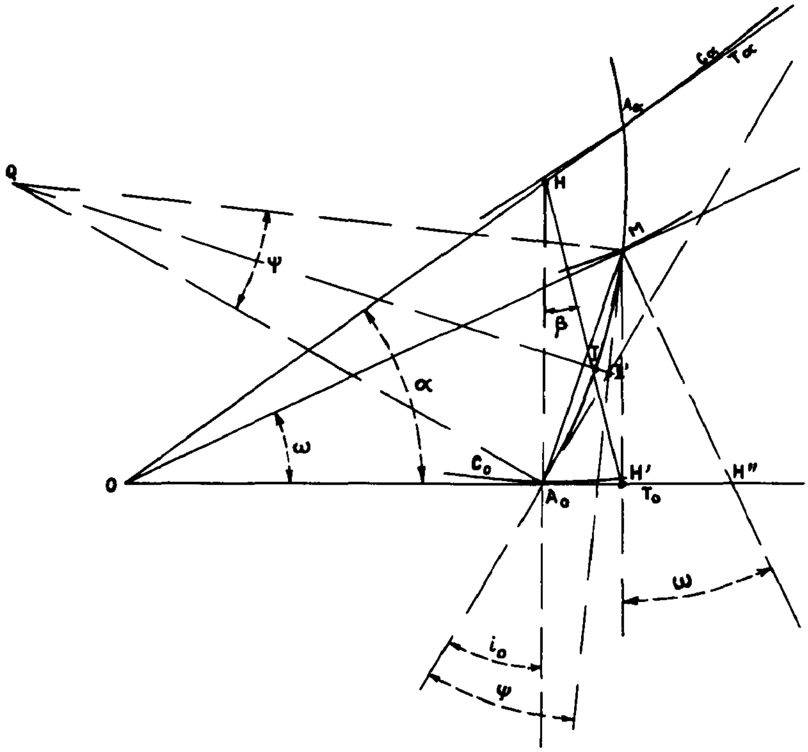


FIG. 6

A_0H' is positive on this figure 6, but it could equally well be negative, and the denominator of the last fraction would then become : $1 - \tan\alpha \tan\beta$. Moreover if α and β are small we may assume that we have :

$$\omega = \alpha \, . \frac{MH'}{A_0H}$$

For the computation of the new value of $\sin \Psi/2$ we shall use the same formula but we shall substitute $i = i_0 - \Psi + \omega$ for i_0 and MH'' for A_0H.

*
**

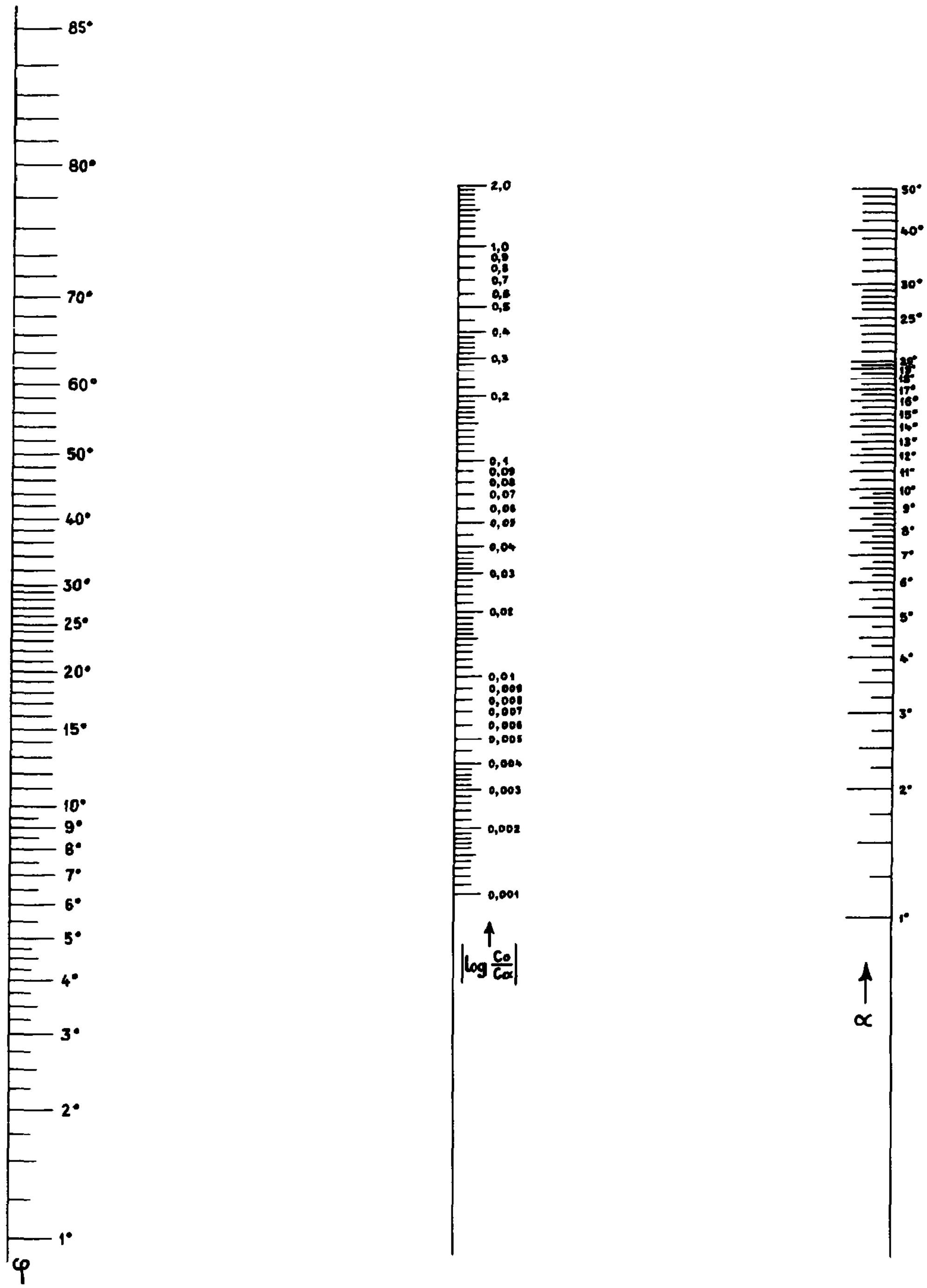
We must now supply the practical means necessary for the use of our two solutions, i.e.

— For the first solution, the means of obtaining φ

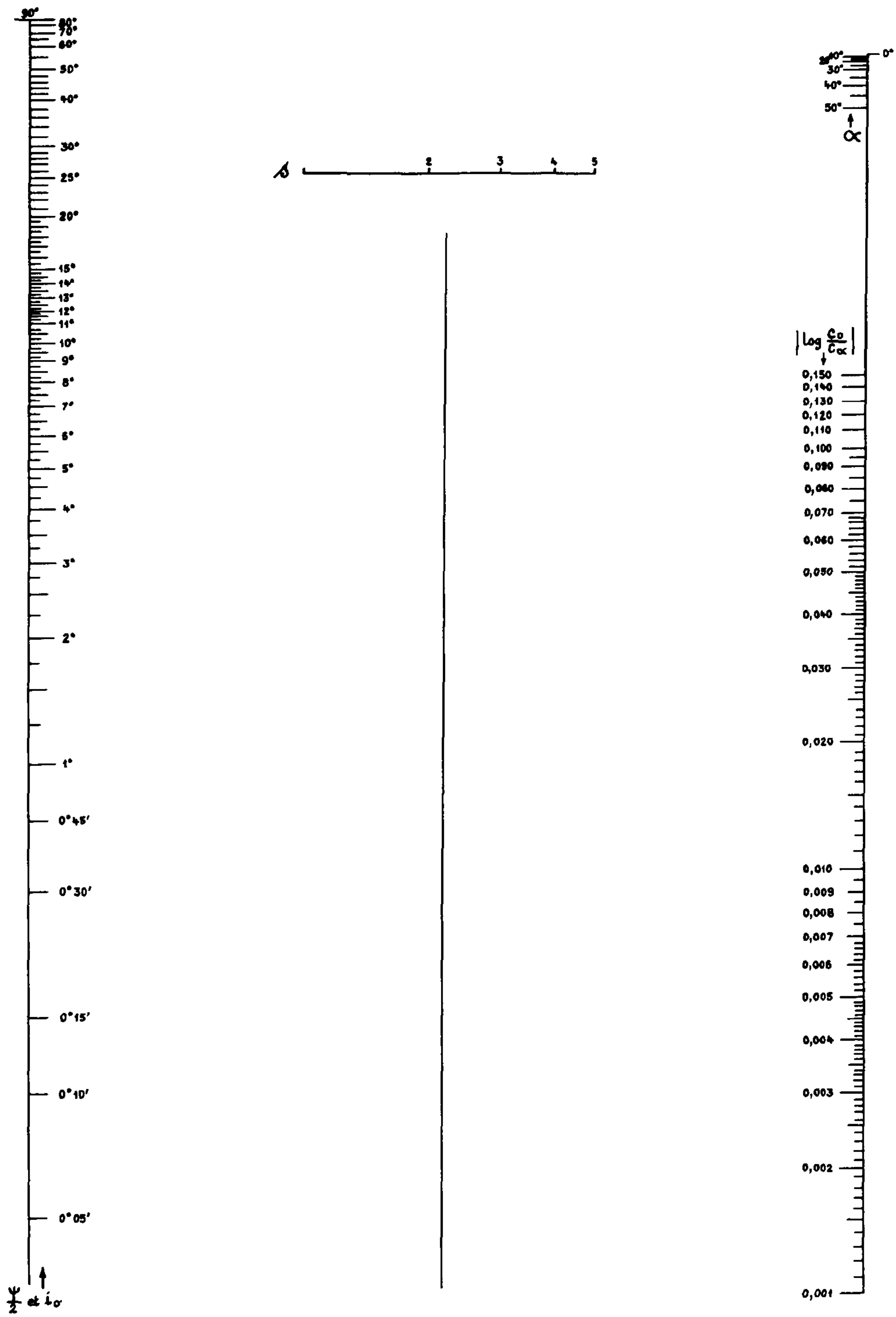
$$\text{from } \tan\varphi = \frac{\log_e \dfrac{C_0}{C_\alpha}}{\alpha}$$

— For the second solution, the means of obtaining Ψ

$$\text{from } \sin\frac{\Psi}{2} = \frac{\tan\alpha}{\alpha} \, . \, \frac{\sin i_0}{2} \, . \, \frac{\log_e \dfrac{C_0}{C_\alpha}}{s}$$



NOMOGRAM 1


90°
80°
70°
60°
50°
40°
30°
25°
20°
15°
14°
13°
12°
11°
10°
9°
8°
7°
6°
5°
4°
3°
2°
1°
0°45′
0°30′
0°15′
0°10′
0°05′
Ψ/2 et i_σ
s
2
3
4
5
0°
30°
40°
50°
α
|Log Co/Cα|
0,150
0,140
0,130
0,120
0,110
0,100
0,090
0,080
0,070
0,060
0,050
0,040
0,030
0,020
0,010
0,009
0,008
0,007
0,006
0,005
0,004
0,003
0,002
0,001

NOMOGRAM 2

The velocity values are supplied by a double entry table showing period and depth, this table being based on the now standard WIEGEL tables (1954). We confined ourselves to even periods between 4 and 20 seconds. As for depths, we limited ourselves to the contour lines shown on metric charts. Thus direct use can be made of our table for all routine purposes, and interpolations are possible. Only the logarithm of velocity will be used (except perhaps for $\alpha = 0$). We shall therefore give the common log which is the one we shall use for our nomograms.

For, in fact, to be able to use these two methods we shall require nomograms (the nomograms 1 and 2) whose size will remain within acceptable limits if we take account of the following :

— It will usually be reasonable to assume that the angle between tangents A_0T_0 and $A_\alpha T_\alpha$ will not be more than 50° (for an α less than 50°). In practice, however, this angle will almost always be much smaller. If, exceptionally, it is more than 50° it will suffice to choose an intermediate isobath.

— Likewise, the log C_0/C_α values are generally of very little interest outside the range 0.15 – 0.001. The table of velocities shows that for the neighbouring depths we have selected 0.15 is in fact never exceeded. The lower limit, moreover, will very rarely be reached, and if this should be the case then refraction will be nearly negligible.

In both methods, to obtain C_0/C_α, we start from the velocity table. The two isobath depths are known, and from these we deduce log C_0 and log C_α and find their difference.

In the first nomogram there are three scales, one each for the values φ, α, and log C_0/C_α. The line joining the α and the log C_0/C_α graduations meets the φ scale at a point whose graduation thus provides the value for φ.

As well as a "central hinge", the second nomogram includes two graduated scales, each of which has two graduations since there are four variables (α, Ψ, i_0, and log C_0/C_α. For the moment s will be taken as equal to 1).

On the left-hand scale which is graduated in sine logs, the i_0 and $\Psi/2$ graduations coincide. On the right hand scale the α and log C_0/C_α graduations will be sufficiently separated to avoid any possibility of confusion.

The line joining the log C_0/C_α graduation to the i_0 graduation meets the "central hinge" at a point which for convenience we may call P. The line joining the α graduation and this point P meets the left scale at a point whose graduation provides the $\Psi/2$ value, taking $s = 1$.

A third scale, a horizontal one, gives logs of 2, 3, 4 and 5. In order to obtain, for example, the value of $\Psi/2$ for $s = 2$, starting from the value found for $s = 1$, we plot the logarithm of 2 downwards. It would be plotted upwards if we wanted to find $\Psi/2$ for $s = 1/2$.

An expressly simplified example makes it possible to use both methods simultaneously, but here we shall only give prominence to the first method.

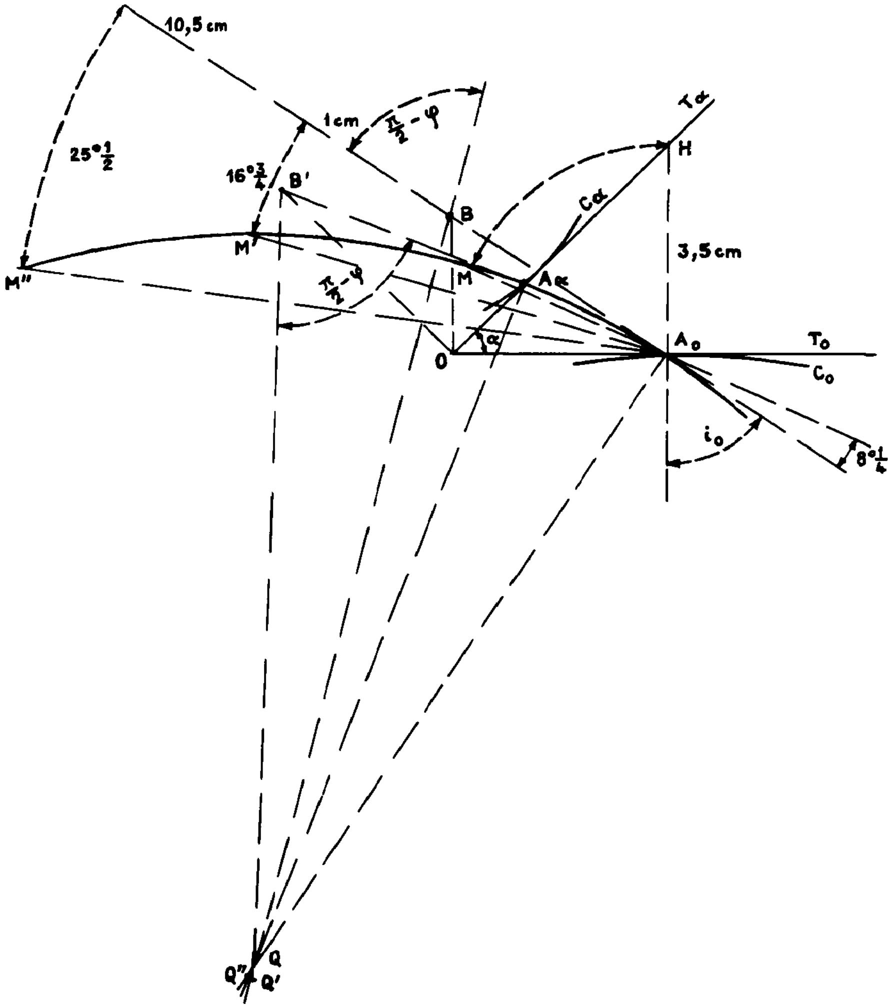


Fig. 7

On figure 7, let C_0 be the first isobath encountering a 10-second period wave front at A_0 with an angle of incidence of 56°.3 (The value of this angle will in fact only be required when we come to the second construction).

We will now draw the orthogonal between isobaths C_0 (for depths of 20 m) and C_α (for depths of 10 m).

The table of velocities gives C_0 and C_α and their logs.

$$\log C_0 = 1.08340$$
$$\log C_\alpha = 0.96534$$

$$\log C_0/C_\alpha = 0.11806$$

FIRST CONSTRUCTION

We now prolong the tangent to the wave front at A_0 up to C_α, and since the depths are decreasing let us take a point A_α slightly to the left. The tangent to isobath C_α cuts A_0T_0 at point O, with an angle $\alpha = 45°$. (This choice of angle is merely provisional). Nomogram No. 1 supplies $\varphi = 19°.1$. The normal to the tangent OA_0 at point O meets at B the extension of the tangent to the wave front at A_0. From B we draw a line making an angle $\pi/2 - \varphi$ with BA_0. This line will meet at Q the normal to the wave-front at A_0.

Q will be the centre of the curvature in relation to A_0 — provided always that α is correct.

We now draw an arc which we can see will meet C_α at a point where the tangent is practically coincident with OT_α.

Our osculating circle is thus correctly drawn. It remains to check whether between the two isobaths this circle can be identified with the orthogonal throughout its whole length.

Let us now compute the radius of curvature at A_α. The tangent to the orthogonal will be the normal to QA_α at this point. This normal is met at B′ by the normal at O to OA_α. O has not changed, and $\log C_\alpha/C_0$ is identical in absolute value to $\log C_0/C_\alpha$: thus φ has not changed.

From B′ we will draw a line making an angle of $\pi/2 - \varphi$ with $B'A_0$. This line meets $A_\alpha Q$ at Q′. $Q'A_\alpha$ is the radius of curvature at A_α.

Let us take Q″ as a point on A_0Q such that $Q''A_0 = Q'A_\alpha$. With this point Q″ as centre we will draw a circle passing through A_0. We shall see that on our diagram this circle coincides with the A_0 osculating circle over the whole length of the arc A_0A_α. Our check has thus been satisfactory. If this had not been the case we would only have used the portion where there was in fact this graphical coincidence, and starting from the point where it ended we would have continued the plotting. However, if circles centred on Q and Q″ meet isobath C_α at points some distance apart then, instead of using the tangent at A_α it will be possible to use the tangent at the median point. If α showed a tendency to differ greatly from the chosen value we could even start all over again with a better point O and a new angle α.

SECOND CONSTRUCTION

Point O will very often be outside the graphical limits. However, it remains a simple matter to determine α. We need only measure the difference between the azimuths at A_0 and A_α. When we have this difference we have all the necessary elements for using Nomogram No 2.

i_0 and $\log C_0/C_\alpha$ are given information. Point P will accordingly not change during any of the approximations we may need to make.

In our example $\alpha = 45°$. We find that $\Psi/2 = 8°\frac{1}{4}$ (for $s = 1$).

Let $A_0H = A_0O$, $s = 1$, and on the line making an angle of $8°\frac{1}{4}$ with the incident ray we now take a distance A_0M equal in length to A_0H. M will clearly fall on the arc of circle centred on Q, and extended beyond A_α. On this figure 7 the same check has been made for M′ and M″ corresponding respectively to $s = 1/2$ and $s = 1/3$. Note that the tangent to the orthogonal at M makes an angle of Ψ, $(16°\frac{1}{2})$ with the incident ray. Let us also determine point I. With three points and their tangents we will plot the curve A_0M as far as the C_α isobath.

Finally, however, we would strongly recommend that second procedure be utilized, for it is always preferable to employ only a *single* method since this decreases the risk of error, and with constant use it will become almost automatic. Moreover the first construction will often prove impracticable, more particularly for the most general case which is where α is small.

It should also be mentioned that the second method will permit a simple analysis of the error arising from the uncertainty regarding α. *In certain cases it can at once be seen that this error is negligible.*

*
* *

In conclusion we will explain why we thought it appropriate to spend a considerable amount of time working on wave pattern diagrams, a task that was often both difficult and irksome. It has been said "What is the use of establishing wave pattern diagrams when all that is needed is an aircraft to photograph the sea". It is of course true that aerial photographs are very useful, but they are exceedingly costly, particularly in terms of plane time. Moreover it will not suffice, as in land photography, merely to have fine weather and good visibility conditions; waves are also a prime necessity — and the waves of the moment are not always of the kind we need. Moreover, it is only rarely that waves are nearly monochromatic and with the desired direction and length. Often several superimposed waves will make interpretation of a photograph an impossibility.

In our opinion, however, the interest of wave front patterns is not merely that they help us to forecast what finally, given time and patience, we could probably ascertain on the spot.

What seems of much more interest is not that we should plot wave front patterns for the existing state of things at a particular site, but rather that we should give ourselves the possibility of extending the plotted patterns of actual wave fronts to the kind of pattern we would like them to have in order, for instance, to *improve the shelter afforded by a port* or to *lessen the strain on a mole.* It is well known that the destructive force of the sea is reduced considerably when the orthogonals spread out fanwise.

Modern powerful suction dredgers furthermore make it possible for material to be picked up from one side of a mole and dumped on the other, and thus to remodel the aspect of the bottom.

The huge sums already invested in ports prohibits the resiting of those that are badly placed to locations where they ought really to have been built in the first place — and even then they would not necessarily be perfect.

A better idea for port improvements might in many cases be the systematic use of modern dredgers. This would in the end be a less costly method than the lengthening and strengthening of a mole, or the addition of more and costly breakwaters which are both liable to damage and a hindrance to ship handling in this era of supertankers.

Henceforth we ought perhaps to envisage protecting a port not by imprisoning it inside moles — or at least not solely by this means — but rather by a carefully planned carving out of portions of the underwater landscape in its vicinity.

It will often be possible to achieve a basin of calm water more effectively and more attractively by means of an alteration in depth to divert the waves rather than by breaking their force with a mole.