A SIMPLE METHOD FOR THE PREDICTION OF THE TIME AND HEIGHT OF HIGH AND LOW WATER

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ABSTRACT

A program is described which searches for and locates the extrema of any type of tide.

INTRODUCTION

The character of the tide at a given harbour may be determined with the help of the form number F (DIETRICH, 1963). The quantity involves the ratio of the local values of the major harmonic constituents M_2, S_2, K_1 and O_1 :

$$F \equiv \frac{K_1 + O_1}{M_2 + S_2} \tag{1}$$

The tide is said to be

a) semidiurnal if $0 \leqslant F \leqslant .25$
b) mixed if $.25 < F \leqslant 3.00$
c) diurnal if $F > 3.00$

The time interval between successive high and low waters in a semidiurnal and a diurnal tide is approximately 6 and 12 hours respectively. The spacing between the extrema in a mixed tide is more complicated. Figure 1 shows the water level at Victoria, British Columbia for which $F = 2.1$, between July 18 and 24, 1968. In this instance shorter period fluctuations override the major diurnal oscillations with a continuous shift in their position and amplitude.

A program which could predict the time and height of the extrema of a semidiurnal or diurnal tide most successfully is likely to flounder hopelessly in the prediction of a mixed tide such as the one shown in figure 1. On the other hand it does not seem practical to develop two or three distinct programs which would essentially do the same job. After some experimentation we came up with the following general scheme of prediction.

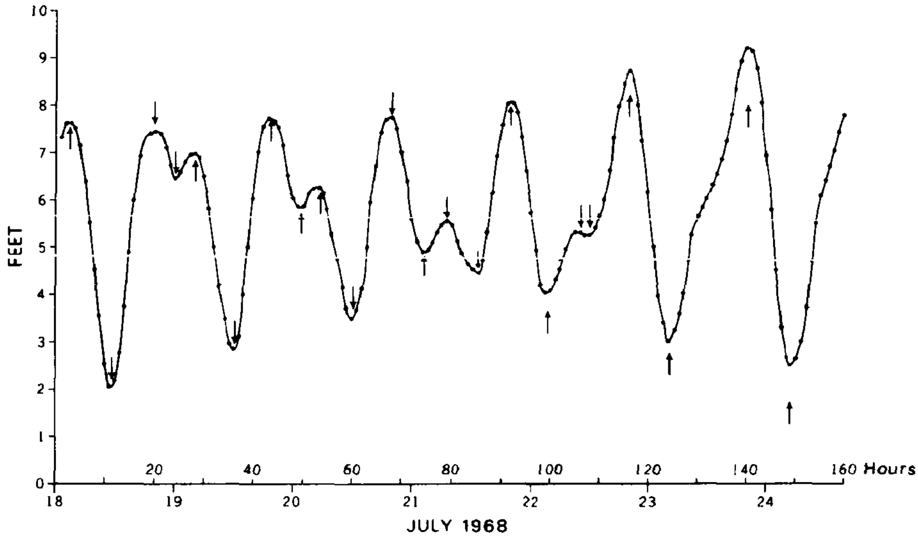

Fig. 1. — *One week of observations on the water level at Victoria, British Columbia covering the interval 18-24 July, 1968. The tide is of a mixed character with F = 2.1. In the sequence of highs and lows observed, the tide starts with diurnal characteristics, then becomes overriden by shorter period oscillations of smaller amplitudes and eventually reverts to a diurnal character.*
The arrows indicate the time and height of the extrema predicted using the method described in the paper.

THE SEARCH AND LOCATION OF EXTREMA

At any station the tide may be represented by a sum of harmonics of the form:

$$z(t) = \sum_{j=1}^{n} f_j A_j \cos (V_j + u_j - g_j + \sigma_j t) \qquad (2)$$

where:

A_j, g_j = the local value of the amplitude and Greenwich phase lag of the constituents;
f_j, u_j = the nodal modulation in amplitude and phase;
V_j = the astronomical argument at the time origin;
σ_j = the frequency;
t = the time.

A necessary condition for the occurrence of an extremum at time t_k in the function $z(t)$ is that its first derivative vanishes at this point:

$$D(t_k) \equiv z'(t_k) = - \sum_{j=1}^{m} f_j A_j \sigma_j \sin (V_j + u_j - g_j + \sigma_j t_k) = 0 \qquad (3)$$

The search for an extremum of $z(t)$ therefore consists in the search for the time t_k for which:

$$D(t_k) = 0 \tag{3a}$$

There exist naturally an infinite number of such points for (2) and in practice the problem consists in locating a given sequence of them in a minimum number of steps, taking a minimum amount of computing time and in such a way that all the relevant extrema are located without overlooking any of them. A program which succeeds in doing this for a mixed tide will *a fortiori* locate all the extrema of a diurnal or semidiurnal tide.

A telltale sign of the occurrence of a zero in $D(t)$ over a time interval (t_r, t_{r+1}) is that $D(t)$ will change its sign at the two end points:

$$\text{sgn } D(t_r) \neq \text{sgn } D(t_{r+1}) \tag{4}$$

We must ensure naturally that the time interval:

$$\Delta t = t_{r+1} - t_r \tag{5}$$

is small enough so that no zero is overlooked. (1) can help us determine the minimum time step Δt for a given type of tide. In the scheme of prediction, the constituents of the station are given and they are used to evaluate (1); a sensible time step will be chosen as:

a) Δt = 3 hours for semi-diurnal tide,
b) Δt = 0.5 hours for a mixed tide, (6)
c) Δt = 6 hours for a diurnal tide.

We select in this way a time step which cannot overlook any zero in the derivative, which is distant from its neighbouring zero by more than .5 hour. In a mixed tide the zeros may be closer than .5 hour at times, but for practical purposes, it makes sense to note just one of them. At the same time we satisfy up to a point the prerequisite that the time of computation be minimized since we jump in time by as large a time step as is allowable.

Once we have determined the appropriate Δt, our next task is to locate the zero. For this purpose we have to compute the derivative at t_r and t_{r+1}. The quickest way to do this is by table look up since there is ample storage space in memory when predictions are to be prepared. A table of 2 000 cosines values is stored for arguments between 0 and 360° with an extension to 450° so that sines and cosines may be looked up indifferently from the table with a minimum amount of manipulations.

We now have to find t_k where $D(t_k) = 0$ in a minimum number of steps. We tried to locate this zero by linear interpolation between $D(t_r)$ and $D(t_{r+1})$ or by using a Taylor series expansion; both approaches worked for diurnal and semidiurnal tides but they proved most hazardous with mixed tides. We had to resign ourselves eventually to Bolzano's method of bisection which does not in any way locate the zeros of $D(t)$ in a minimum number of steps but which at least locates all of them without any ambiguity whatsoever.

The technique goes as follows. One moves forward in time from the origin or the last extremum in steps Δt given by (6) till a change in the sign of the derivative is noted. Then one moves backward by a step $\frac{1}{2}\Delta t$;

depending on the sign of the derivative at the latter point, one moves either forward or backward by $\frac{1}{4}\Delta t$. This procedure is followed till the zero of D(t) is narrowed down to a time interval of width around 0.1 hour ($\sim$ 6 minutes or 1/64 Δt for diurnal, $\frac{1}{8}\Delta t$ for mixed and 1/32 Δt for semi-diurnal). The position of the zero of D(t), t_k, is inferred from linear interpolation between the last two values of D(t) and $z(t)$ is computed at this point from (2). Figure 2 illustrates an example of the manipulations involved in the search for a zero of D(t).

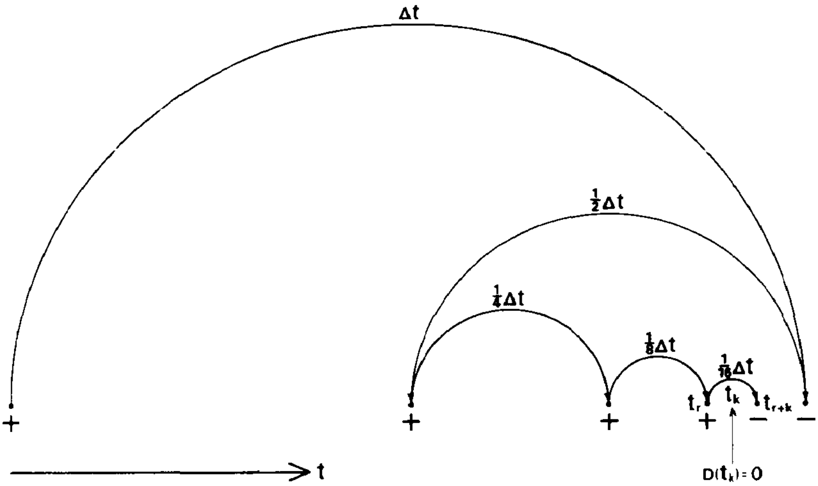

FIG. 2. — An example of the sequence of steps involved in locating a zero t_k of the derivative D(t). The sign of D(t) at the various points tested is denoted by + or —. After a step Δt, the sign has changed; by a retrogression of $1/2\Delta t$, the sign has reverted to plus, forcing a forward step of $1/4\Delta t$ where the sign is still unchanged. Two further forward steps of $1/8\Delta t$ and $1/16\Delta t$ locate the minimum width interval (t_r, t_{r+1}) over which the position of t_k is determined by linear interpolation from the values of D(t) at t_r *and* t_{r+1}.

We indicate by arrows in figure 1 the time of the extrema predicted for Victoria using the technique just described; the shaft of the arrow locates the time abscissa while the tip ends at the predicted height. We note that all the extrema of this very complicated tide are properly detected including the slight oscillation on the 22nd. The levels predicted at the end of the interval are lower than observed but there is some evidence that the mean level at Victoria was abnormally raised at that time.

SHALLOW WATER TIDES

Shallow water tides are strongly distorted by non-linear bottom effects. It was found necessary in the past to analyse directly the extrema in order to obtain some success in the predictions (LUBBOCK, 1836); lately HORN (1960) has developed a systematic method to handle such series of extrema. This method unfortunately requires nineteen years of continuous observations; we have found it more convenient in practice, whenever the tide is diurnal or semidiurnal in character, to search for a larger number of harmonic constituents in the hourly observations over fewer years (GODIN, 1973) and then afterwards to correct the predicted times and heights with the help of the 18 folded constituents identified by Doodson in his method for the analysis of shallow water tides (DOODSON, 1957). In this fashion it is possible to retain the scheme of prediction just described to find the appropriate position and value of the extrema; afterwards they are amended with the help of the Doodson corrections which are given by:

$$\Delta\,[E\,(t_k)] = \sum_{m=1}^{18} B_m \cos(V_m - G_m + \sigma_m\, k) \tag{7}$$

where:

$\Delta[E(t_k)]$ = the correction of the height or time of occurrence of the extremum labelled by the integer k;

B_m, G_m = the amplitude and Greenwich phase lag for a given correction constituent (different sets are used to correct the time and height of high and low water);

V_m = the astronomical argument at the time origin (the value of the solar perigee p' is dropped from such an argument);

σ_m = the frequency per half lunar day or per lunar day depending on whether the tide is semidiurnal or diurnal.

Quebec is a shallow water harbour for which it has always been most difficult to provide adequate predictions. We list in table 1 the values predicted and observed for its extrema during the first 14 days of June, 1968 applying the method just described. One hundred and seventy-four harmonics were used first to supply coarse values which were then refined with the help of the DOODSON corrections (7). We have chosen the month of June in this example because the perturbations in level caused by the weather and the fluctuations in the discharge are at a minimum then, so that we can judge more adequately the success of the prediction of the tide by itself. The table indicates that the times of occurrence of the extrema are predicted most accurately; there are occasional discrepancies between the predicted and observed heights. It is difficult to determine from such a short series if the discrepancies are systematic or not; we could establish with the help of longer series that they are indeed stochastic in character and that they are caused essentially by transient pressure gradients which occur over the St. Lawrence River and the Gulf.

TABLE 1

Predicted and Observed Time and Height of High and Low Water at Quebec, Province of Quebec during the first Two Weeks of June, 1968 Using 174 Harmonic Constituents and 4 Sets of 18 Doodson Correction Constituents

F = 0.2

Day	Time		Diff	Height		Diff	Time		Diff	Height		Diff
	Pred	Obs	min	Pred	Obs	feet	Pred	Obs	min	Pred	Obs	feet*
1	0403	0400	3	1.5	2.2	−0.7	0909	0915	−6	15.8	16.2	−0.4
	1705	1710	−5	1.1	1.5	−0.4	2202	2200	2	12.9	12.9	0
2	0449	0445	4	1.8	2.3	−0.5	1001	0955	6	15.3	16.0	−0.7
	1753	1750	3	1.3	1.5	−0.2	2252	2256	−4	12.7	13.0	−0.3
3	0542	0540	2	2.0	2.8	−0.8	1058	1050	8	14.9	15.1	−0.2
	1844	1830	14	1.4	1.7	−0.3	2347	2350	−3	12.9	13.8	−0.9
4	0644	0643	1	2.2	3.2	−1.0	1159	1156	3	14.5	15.2	−0.7
	1939	1935	4	1.5	2.0	−0.5						
5	0052	0055	−3	13.4	13.5	−0.1	0758	0755	3	2.1	2.2	−0.1
	1308	1300	8	14.2	13.7	0.5	2037	2040	−3	1.4	1.1	0.3
6	0154	0154	0	14.4	14.4	0	0923	0915	8	1.7	1.8	−0.1
	1417	1414	3	14.1	14.1	0	2136	2140	−4	1.4	0.8	0.6
7	0251	0245	6	15.5	15.1	0.4	1031	1020	11	1.1	1.1	0
	1519	1515	4	14.2	14.3	−0.1	2233	2228	5	1.2	0.8	0.4
8	0342	0330	12	16.6	16.7	−0.1	1131	1125	6	0.8	0.7	0.1
	1616	1610	6	14.6	14.5	0.1	2328	2325	3	1.2	0.6	0.6
9	0431	0428	3	17.7	17.9	−0.2	1229	1225	4	0.6	1.6	−1.0
	1709	1705	4	14.9	15.3	−0.4						
10	0023	0020	3	1.2	0.8	0.4	0519	0515	4	18.5	18.3	0.2
	1327	1315	8	0.5	0.5	0	1802	1755	7	15.1	15.6	−0.5
11	0115	0114	1	1.1	0.6	0.5	0609	0610	−1	19.0	19.1	−0.1
	1422	1413	9	0.5	0.5	0	1852	1850	2	15.0	15.6	−0.6
12	0206	0158	8	1.0	1.0	0	0659	0655	4	19.1	19.7	−0.6
	1514	1513	1	0.5	0.5	0	1941	1943	−2	14.9	15.5	−0.6
13	0255	0250	5	1.1	1.2	−0.1	0752	0750	2	18.9	19.8	−0.9
	1604	1600	4	0.6	0.8	−0.2	2034	2030	4	14.6	14.9	−0.3
14	0347	0345	2	1.2	1.3	−0.1	0845	0845	0	18.2	19.2	−1.0

* The heights are in "feet" a unit of measurement still used in Canada.

CONCLUSION

The method of prediction just described, although not the most sophisticated nor the most rapid, does locate unerringly practically all the extrema of any type of tide. We have checked this fact by reproducing the extrema observed in a segment of the tide at Victoria, British Columbia. The method can also predict the extrema at shallow water harbours where the tide has a definite diurnal or semidiurnal character.

The program embodying this method, written in Fortran IV, is called PRED and is available to anyone who requests it. The program takes

78 seconds on the CDC 6400 to prepare one year of prediction for Victoria using 62 constituents and a time step of 0.5 hours; for Quebec, it takes exactly 2 minutes to prepare coarse predictions using 174 constituents and a time step of 3 hours, 48 seconds to evaluate the DOODSON corrections with the help of 4 sets of 18 folded DOODSON correction constituents and an infinitesimal amount of time to apply these corrections to the coarse values.

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