RHO-RHO LORAN-C COMBINED WITH SATELLITE NAVIGATION FOR OFFSHORE SURVEYS *

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ABSTRACT

The Loran-C system of navigation recently became usable in the rho-rho (range-range) mode with the installation of atomic clocks at all the Loran-C stations. This paper outlines briefly the principles of rho-rho Loran-C operation and the associated problems of geodesy and radio wave propagation. It describes the procedure for using rho-rho Loran-C alone and in combination with Satellite Navigation, as developed over the past two years at the Bedford Institute of Oceanography. Results of accuracy tests are given along with estimates of the magnitudes of the various sources of errors in Loran-C range measurements for both the stand-alone and satellite aided modes of operation. The results generally show a 2σ absolute ranging accuracy of 270 m using rho-rho Loran-C alone with an improvement to about 180 m when combined with Satellite Navigation. The Bedford Institute of Oceanography rho-rho Loran-C system has been used routinely with Satellite Navigation at ranges of 2 500 km (1 250 n.m.). Based on this experience the rho-rho Loran-C coverage would include most of the North Atlantic Ocean and large areas of the Pacific.

INTRODUCTION

Loran-C is a long range, 100 kHz, radio navigation system operated by the United States Coast Guard. It had its beginning during the waning years of World War II. After the war, testing of a similar system continued on the east coast of the United States, and in 1957 this exper-

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imental set up became the first operational Loran-C chain. Eight chains are now in operation throughout the world, providing coverage for most of the North Atlantic Ocean and large areas of the Pacific. Figure 1 shows the rho-rho (range-range) Loran-C coverage of the North Atlantic Ocean.

The United States Department of Transportation has proposed that Loran-C be the primary navigation aid for the coastal/confluence zone as described in the U.S. National Plan for Navigation (U.S. Department of Transportation, 1970). If this plan is approved by the Office of Management and Budget it will mean that the existing Loran-C chains will be updated and that possibly 10 new stations will be added to provide Loran-C coverage for the Great Lakes, in the Gulf of Mexico and along the U.S. west coast (P. Klass, 1972). A new Loran-C station is expected to be constructed shortly in Presque Isle, Maine and will probably be operational for test purposes by 1975. According to the United States Coast Guard this will be the prototype of the new generation of Loran-C stations.

Although Loran-C is designed and operated as a hyperbolic navigation system the recent installation of atomic clocks at all stations has made it usable as a ranging system as well. Rho-rho operation gives strong position line geometry over a larger area and only requires two shore stations for a fix compared with the three stations needed for a hyperbolic fix. Furthermore, in much of the coverage area where three stations can be received there is redundant information, making the range-range fix stronger still. Rho-rho operation has the weakness of accumulating error due to the relative drift between the atomic clocks. A further weakness of not only Loran-C but all radio positioning systems is the uncertainty of the phase lag correc-

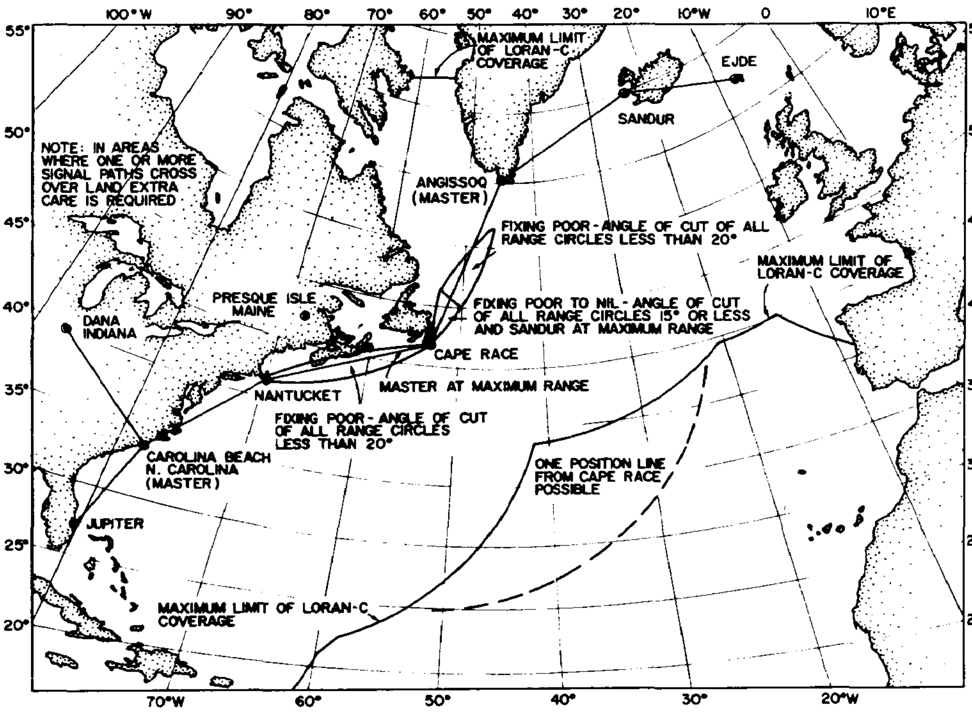

Fig. 1. — Rho-Rho Loran-C coverage of the North Atlantic Ocean.

tion, particularly if one of the signal paths is over land. Fortunately these problems can be solved by Satellite Navigation position information from which frequent checks on both the value of the overland phase lag corrections and the relative clock drift can be obtained. The most likely cause of error in Satellite Navigation positions of a moving ship is inaccurate course and speed information (STANSELL, 1969). This weakness of the Satellite Navigation system can be solved to a large degree by Loran-C. Hence the two systems complement one another.

In May 1971, an Austron 5000 rho-rho Loran-C receiver, rented from Offshore Navigation Incorporated, was evaluated onboard the Canadian Scientific Ship *Baffin*, operating out of the Bedford Institute of Oceanography. The evaluation was carried out on the Grand Banks, off the coast of Newfoundland, (see figure 1) and a Decca 12f survey chain was used as reference. The results of this trial are given in detail by EATON and GRANT (1972). In April 1972 an Austron 5000 system was purchased by the Bedford Institute of Oceanography, and has been used since then almost continuously in combination with Satellite Navigation for hydrographic, geophysical and oceanographic surveys onboard the Canadian Scientific Ship *Hudson*. Current effort is being directed toward the further development of techniques for using Loran-C and Satellite Navigation together.

PRINCIPLES OF OPERATION

The Loran-C system is described in a number of references: e.g., International Hydrographic Bureau (1965); POWELL and WOODS (1967). The Austron 5000 rho-rho Loran-C system is described by MARCHAL (1971). As shown in figure 2, it consists of a receiver and a Hewlett-Packard cesium beam frequency standard. A Digital Equipment Limited PDP-8/E computer marries the frequency standard to the receiver. The computer, in addition to converting the Loran-C ranges to latitude and longitude, also controls the tracking circuits of the receiver; the receiver cannot be operated independently of the computer. A teletype is the operator's primary means of communicating with the computer and hence with the receiver.

Each station in a Loran-C chain, starting with the master, transmits a series of eight pulses as shown in figure 3. The transmission sequence is repeated at a regular interval called the Group Repetition Period (GRP), which is different for each chain. The secondary station transmissions, until recently, were controlled by having them transmit a set time interval (coding delay) after receiving the master signal. The total delay (called the emission delay) from the time the master transmitted until the secondary station transmitted was therefore made up of the time for the master signal to travel to the secondary station (baseline travel time) plus the coding delay. (Emission Delay = Baseline Travel Time + Coding Delay) Now that atomic clocks have been installed at each of the Loran-C stations throughout the world, the secondary transmissions are controlled in-

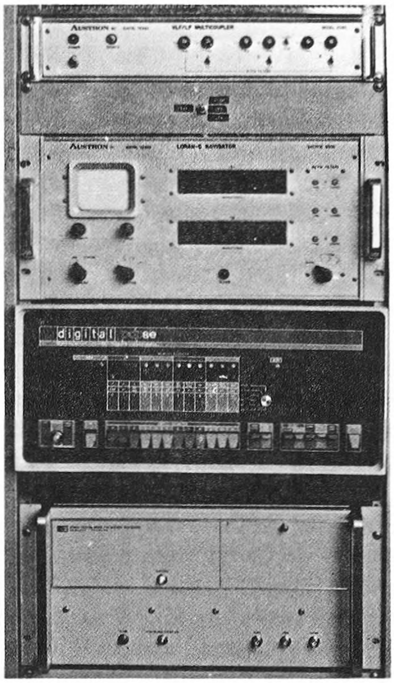

FIG. 2. — Austron Model 5000 Rho-Rho Loran-C Navigation System consisting of an Austron Loran-C VLF/LF Multicoupler and receiver (top), a Digital Equipment Limited PDP-8/e mini computer (middle) and a Hewlett-Packard Model 5061A Cesium Beam Frequency Standard (bottom).

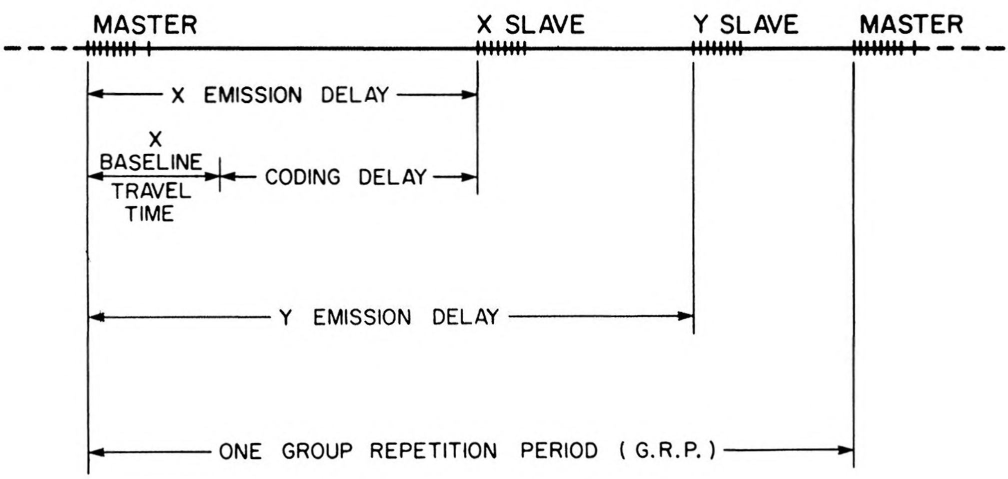

FIG. 3. — Timing of Loran-C Signals.

dependently by information obtained from these clocks and are timed to transmit one emission delay after the master transmits (a constant delay is assumed for the baseline travel time).

Atomic frequency standards are capable of reproducing the frequency of another standard or any given frequency to within a few parts in 10^{12}. The difference between the frequency of one atomic standard and another is commonly called the frequency offset and is due to variations in the conditions affecting atomic resonance (e.g., ambient magnetic field, temperature) as well as differences in the electrical components of the standards If an atomic frequency standard is used as a timing device (i.e. atomic clock) it will drift in time at a rate determined by the frequency offset. For example, a frequency offset of 1 part in 10^{11} is equivalent to a gain (or loss) of 1 μsec. in 10^{11} μsec. or 0.86 μsec/day. (Throughout this paper the terms "clock drift" and "drift rate" refer to the drift in time between two atomic clocks resulting from their frequency offset (frequency difference). Atomic frequency standards do not normally exhibit frequency drift.) Because of the frequency offset between the master atomic clock and the secondary station clocks, occasional (1 to 3 times daily) step adjustments (local phase adjustments, LPA's) of 0.05 or 0.10 μsec. are made to the secondary station transmission times to keep them synchronized with the master transmissions. If the LPA's were not applied the Loran-C hyperbolic patterns would shift as the secondary station got more and more out of synchronization with the master transmissions. A second result of the LPA's is that all transmissions in each Loran-C chain drift in time at the drift rate of the master station atomic clock. Since Loran-C is used for time dissemination (PAKOS, 1969) as well as for navigation, all Loran-C chains are monitored and the daily differences between the master transmission and the U.S. Naval Observatory master clock are recorded and are available from the U.S. Naval Observatory. The differences for the U.S. East Coast and North Atlantic Loran-C chains for the first eight months of 1972 are shown in figure 4.

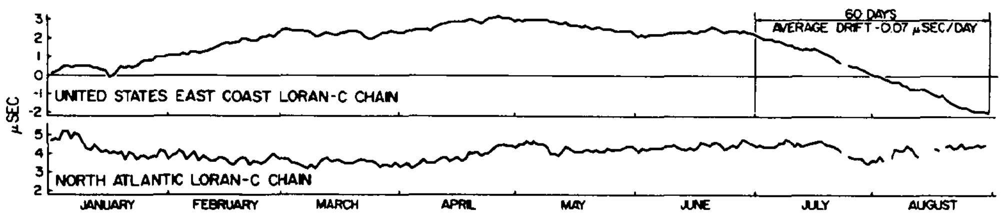

FIG. 4. — Timing Error of U.S. East Coast and North Atlantic Loran-C Chains against U.S. Naval Observatory (Daily Phase Values, Series 4, U.S. Naval Observatory).

RHO-RHO OPERATION

In the rho-rho mode the mobile receiver has its own atomic clock which is used to predict the transmission times of the master and secondary stations. Before rho-rho operation can begin the receiver atomic clock must be synchronized with the start of a GRP. To do this the radio wave travel time from at least one of the Loran-C stations to the receiver must be known. The radio wave travel time is found by:

1) Determining the receiver position;
2) Computing the geodetic distance from the Loran-C station to the receiver;

3) Converting this distance to travel time at the velocity of light in a vacuum (299792.5 km/sec), and
4) Applying a phase lag correction to allow for the variation of radio wave velocities over the earth's surface from the vacuum velocity. The phase lag correction is a complex function of distance, frequency and the electromagnetic properties of the atmosphere and the earth's surface. It is usually obtained either from tables or graphs. The tables used by the Bedford Institute of Oceanography for finding over water phase lag were computed by P. BRUNAVS of the Canadian Hydrographic Service, Ottawa, using the algorithm described in BRUNAVS and WELLS (1971). A procedure for calculating phase lag over inhomogeneous terrain is MILLINGTON's method described by MILLINGTON (1949), BIGELOW (1965), and POTTS and WIEDER (1972).

If synchronizing by a master signal, the beginning of a GRP is found by subtracting the master radio wave travel time from the time of arrival of the master signal. If synchronizing by a secondary signal, the time of master transmission, signifying the beginning of a GRP, is found by subtracting the sum of the radio wave travel time and the secondary station emission delay from the time of arrival of the secondary signal.

The difference between the predicted and observed times of master transmission is entered into the computer along with the GRP and emission delays and thereafter the receiver atomic clock is used to predict the time of transmission of each of the stations in the Loran chain. The clock continues to predict transmission times as the receiver moves about the coverage area and the time intervals measured by the receiver between transmission and reception of each signal, multiplied by the propagation velocity and corrected for phase lag, give the ranges to the transmitting stations.

These ranges are the shortest distances between the Loran-C transmitters and the ship measured over the actual surface of the earth. To the extent that a reference ellipsoid approximates the surface of the earth these distances are then geodesics on that ellipsoid. The position of the receiver in ellipsoid co-ordinates (latitude and longitude) can therefore be computed using a suitable algorithm. The rho-rho Loran-C system at the Bedford Institute of Oceanography has been programmed to compute the latitude and longitude iteratively, using two or three ranges. The algorithm is described in the appendix.

ATOMIC CLOCK DRIFT (FREQUENCY OFFSET)

A frequency offset between two atomic frequency standards of one part in 2×10^{11} will result in a linear change (drift) in the observed Loran-C ranges of about 0.5 μsec/day (150 m/day). The drift rate is usually determined by keeping the receiver stationary and logging the Loran-C ranges for two or three days. A graph of the Loran-C ranges against time gives the drift rate. Thereafter a clock drift correction can be applied to all readings. The accuracy of the clock drift correction improves with the length of the rating period. If the signal is noisy a longer rating period may be necessary and when rating on a secondary signal instead of a master the rating period should be about one third longer to smooth out the effect of the secondary station LPA's. Logging the Loran-C ranges at shorter intervals during the rating period will also improve the accuracy of the clock drift correction.

During the 1971 trial, it was found that a minimum rating period of 1½ to 2 days was necessary using a logging interval of 10 minutes. Figure 5 shows the ranges of the Carolina Beach, Cape Race and Nantucket stations of the U.S. East Coast Loran-C chain plotted against time as observed at Halifax during a three day period in August, 1972.

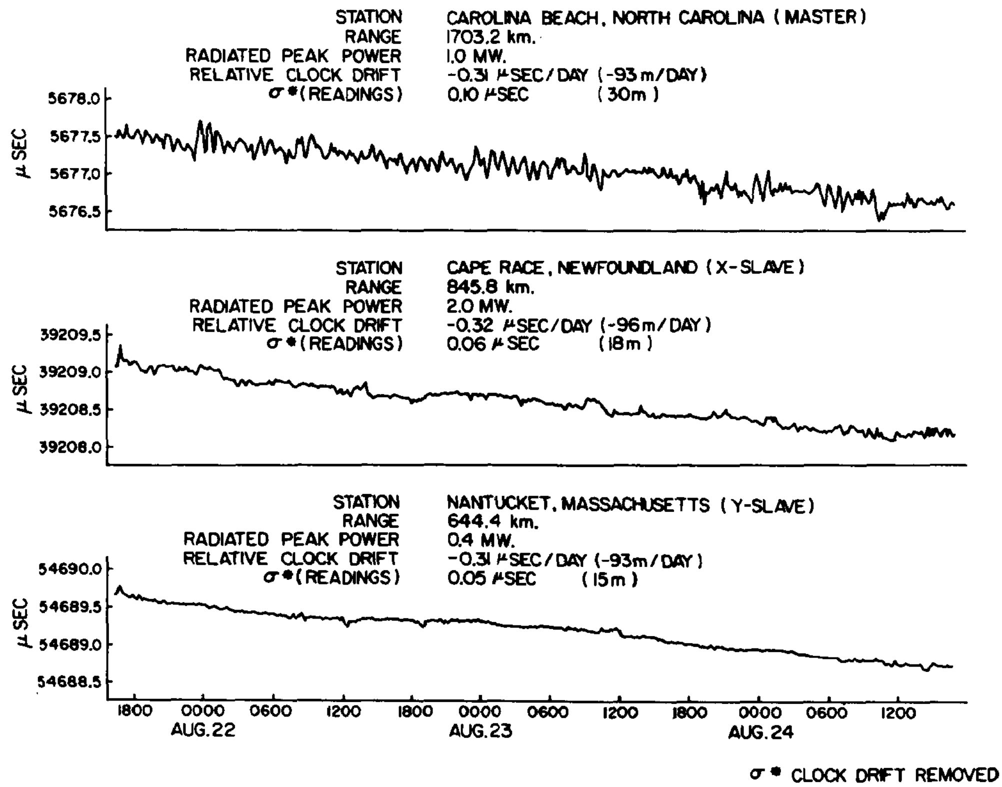

FIG. 5. — Carolina Beach, Cape Race and Nantucket Loran-C Ranges observed at Halifax, 21-24 August 1972.

The accuracy of the clock drift correction depends on the two standards maintaining the same relative frequency difference between them. Not only do the receiver clocks change their frequencies with changing conditions but, as figure 4 shows, changes occur in the frequencies of Loran-C chains. If a rho-rho Loran-C user has rated his clock on the U.S. East Coast chain just before 30 June, 1972 he would have had a clock drift error of 0.07 μsec/day (21 m/day).

Generally the clock drift determined during a rating period predicts future relative drift to better than 0.05 μsec/day (1 part in 1.7×10^{12}). On two different occasions while using the Bedford Institute of Oceanography rho-rho Loran-C system the difference between the average clock drift observed during the cruise and the drift obtained from the pre-cruise clock rating were found to be 0.003 μsec/day (0.9 m/day) and 0.02 μsec/day (0.9 m/day) and 0.02 μsec/day (6 m/day).

GEODETIC CONSIDERATIONS

The Loran-C data sheets published by the U.S. Coast Guard from information supplied by the U.S. Naval Oceanographic Office give the geographic positions of the Loran-C stations based on the Mercury 1960 Datum. This is a geocentric datum using the Fischer 1960 spheroid; $a = 6378166.0$ m and $f = 1/298.3$. The 1927 North American Datum on which most North American charts are based uses a Clarke 1866 spheroid ($a = 6378206.4$ m and $b = 6356583.8$ m) and is not geocentric. The translation components used by the United States Coast Guard between the North American Datum and the Mercury Datum are:

$$X_0 = 3 \text{ m}, \; Y_0 = 111 \text{ m and } Z_0 = 225 \text{ m where}$$
$$[(X_0, Y_0, Z_0) = (X, Y, Z) \text{ mercury } -(X, Y, Z)_{NAD}].$$

The Satellite Navigation positions used in conjunction with the Loran-C positions for determining overland phase lag and clock are presently computed with respect to a third datum using a geocentric spheroid with $a = 6378144.0$ m and $f = 1/298.23$. The translation components between NAD and the satellite datum, for the Atlantic Provinces of Canada, are: $X_0 = -45$ m, $Y_0 = 164$ m $Z_0 = 190$ m. (Krakiwsky, Wells and Kirkham, 1972). In calculating a position from Loran-C ranges, the 1960 Fischer spheroid parameters should be used (in which case the position obtained will be with respect to the Mercury 1960 Datum). Alternatively the Loran-C station positions should first be transformed using formulas similar to those found in Heiskanen and Moritz (1967) so that the fix computation can be performed on a local datum such as NAD, 1927. To demonstrate the size of the error that could be introduced by not taking these precautions: the Cape Race, Newfoundland, Loran-C transmitter on the Mercury Datum differs by 80 m from the same position expressed with respect to the NAD, 1927 datum and by 30 m from the value expressed with respect to the current satellite datum.

RHO-RHO LORAN-C ERROR ESTIMATES

In this section the sources of errors in Loran-C range measurements are described and one sigma estimates of their magnitudes are given.

Some of the errors are obtained from PAKOS (1969) who dealt with the error budgets for users of the Loran-C system for time and frequency dissemination. Other error sources applicable to users of rho-rho Loran-C have been added as a result of Bedford Institute of Oceanography experience in using a rho-rho Loran-C system over the past two years. These errors are described below and examples are given to illustrate how they are used to predict ranging errors.

User Prediction Error:

$\sigma_{pe} = 0.1$ μsec (30 m) (Water Path)
$\sigma_{pe} = 0.4$ μsec (120 m) (Land Path)

— results from the uncertainty in propagation velocity of the signal from the transmitter to the receiver. The water path estimate applies to users who have all over-water path between the transmitter and receiver. The land path estimate is larger because of the uncertainty in land conductivities and hence the phase lag correction. The larger over-land estimate also allows for the fact that the user may be experiencing anomalies due to cliffs, mountains, etc., locally or along the path. POTTS and WIEDER (1972) give details on this source of error.

Groundwave Propagation Anomaly (overland):

$\sigma_{pa} = 0.2$ μsec (60 m)

— results from the variability in propagation velocity over land due to weather effects. One of the main causes of the variability in propagation velocity is the difference in conductivity between wet and dry soil. This source of error becomes negligible for most users at sea.

Secondary Station Synchronization Error:

$\sigma_{ss} = 0.05$ μsec (15 m)

— refers to the synchronization accuracy between the master and the secondary station transmissions.

User Measurement Error:

$\sigma_{me} = 0.1$ μsec (30 m) (normally)
$\sigma_{me} = 0.2$ μsec (60 m) (night, long range)

— results from the uncertainty in the time measurement due to the equipment (error estimates assume good quality equipment). This error is also used here to include errors due to radio noise which varies with weather conditions, local interference, receiver sensitivity and the radiated power of and distance to the transmitter.

Receiver Synchronization Error:

$$\sigma_{rs} = (\sigma_{pe}^2 + \sigma_{pa}^2 + \sigma_{ss}^2 + \sigma_{me}^2)^{1/2}$$

— is the difference between the actual time of the start of a GRP and the time predicted by the receiver atomic clock as determined by the synchronization procedure described earlier in the section on rho-rho operation. The error estimates in this formula should correspond to the conditions under which the synchronization takes place.

Clock Drift Error:

$$\sigma_{cd} = 0.05\,\alpha\ \mu\text{sec}\ (15\,\alpha\ \text{m})$$

where α is the number of days since synchronization.

— results from the inability to predict exactly the future relative drift of the receiver atomic clock with respect to the chain master clock.

Ranging Error :

$$\sigma_r = (\sigma_{rs}^2 + \sigma_{cd}^2 + \sigma_{pe}^2 + \sigma_{pa}^2 + \sigma_{ss}^2 + \sigma_{me}^2)^{1/2}$$

— is the total error in a rho-rho Loran-C range measurement. The values for σ_{pe}, σ_{pa}, σ_{ss}, and σ_{me} in the Ranging Error calculation are determined by the conditions under which the range measurement is made and will generally be different from the values used in the synchronization error calculation.

These errors are summarized in table I.

TABLE I

Rho-Rho Loran-C Error Estimates

Symbol	Name	Estimated 1σ Errors	
		Good Conditions μsec (m)	Bad Conditions μsec (m)
σ_{pe}	User Prediction Error	0.1 (30)	0.4 (120)
σ_{pa}	Groundwave Propagation Anomaly	0	0.2 (60)
σ_{ss}	Secondary Station Synchronization Error	0	0.05 (15)
σ_{me}	User Measurement Error	0.1 (30)	0.2 (60)
σ_{rs}	Receiver Synchronization Error	0.14 (42)	0.49 (148)
	$= (\sigma_{pe}^2 + \sigma_{pa}^2 + \sigma_{ss}^2 + \sigma_{me}^2)^{\frac{1}{2}}$		
σ_{cd}	Clock Drift Error	α (0.05)	
	(α = number of days since synchronization)		
	for α = 5 days	0.25 (75)	
σ_r	Ranging Error		
	$= (\sigma_{rs}^2 + \sigma_{cd}^2 + \sigma_{pe}^2 + \sigma_{pa}^2 + \sigma_{ss}^2 + \sigma_{me}^2)^{\frac{1}{2}}$	0.32 (96)	0.74 (221)

An appreciation of how these errors affect the ranging accuracy of Loran-C can be obtained by considering an example of an operation.

The first step is the synchronization of the receiver clock with the chain master clock. This is normally done in harbour, and so there will usually be land path between the receiver and transmitter, which may introduce both terrain effects and propagation anomalies due to changes in weather conditions. Assuming that a secondary station is within several hundred kilometres, the estimate of the receiver synchronization error would be:

$$\sigma_{rs} = (\sigma_{pe}^2 + \sigma_{pa}^2 + \sigma_{ss}^2 + \sigma_{me}^2)^{\frac{1}{2}}$$
$$= (0.4^2 + 0.2^2 + 0.05^2 + 0.1^2)^{\frac{1}{2}}$$
$$= 0.46 \ \mu\text{sec} \ (138 \text{ m})$$

A slight improvement in this figure is theoretically possible if signals from other stations in the same chain can also be received. By correcting each of the observed readings by the mean of the differences between the observed and predicted values the error in synchronization can be reduced to approximately $\sigma_{rs}/\sqrt{n}$ where n is the number of stations. For $n = 2$ the synchronization error becomes:

$$\sigma_{rs} = 0.33 \ \mu\text{sec} \ (98 \text{ m}).$$

Having synchronized and rated the clock, the ship sails. At sea, where there is normally negligible land path between the transmitter and the receiver, the error due to Groundwave Propagation Anomalies (σ_{pa}) will be zero and the value of the User Prediction Error will tend to 0.1 μsec. For a user who had synchronized on two secondary stations, as described above, and who has been at sea for 5 days, the absolute ranging accuracy of Loran-C from a secondary station within about 1 700 km is estimated to be:

$$\sigma_r = (\sigma_{rs}^2 + \sigma_{cd}^2 + \sigma_{pe}^2 + \sigma_{pa}^2 + \sigma_{ss}^2 + \sigma_{me}^2)^{\frac{1}{2}}$$
$$= (0.33^2 + 0.25^2 + 0.1^2 + 0^2 + 0.05^2 + 0.1^2)^{\frac{1}{2}}$$
$$= 0.43 \ \mu\text{sec} \ (130 \text{ m})$$

If there is land between the transmitter and the receiver there may be errors due to the Groundwave Propagation Anomalies (σ_{pa}) and the User Prediction Error (σ_{pe}) will be increased to about 0.4 μsec. Under these circumstances the ranging accuracy would be:

$$\sigma_r = (0.33^2 + 0.25^2 + 0.4^2 + 0.2^2 + 0.05^2 + 0.1^2)^{\frac{1}{2}}$$
$$= 0.62 \ \mu\text{sec} \ (186 \text{ m})$$

During the rho-rho Loran-C trial east of Newfoundland in 1971 (Eaton and Grant, 1972) over 2 000 comparisons were made between the observed Loran-C ranges and the same ranges calculated from positions obtained from a Decca 12f survey chain while the ship was in good Decca coverage. The long term stability of Loran-C was investigated by calculating the standard deviation of the differences between the observed and calculated Loran ranges about the mean difference, after the mean clock drift had been removed. The result, for the Master signal at about 1 700 km over water, after subtracting the estimated variation due to Decca, was a long term stability of $\pm$ 80 m. This figure corresponds to a Ranging Error calculated using only the User Prediction Error (σ_{pe}) and the User Measurement Error (σ_{me}). A value of 80 m for σ_r would be obtained if σ_{pe} and σ_{me} were both 0.10 μsec (57 m). These are reasonable estimates under the circumstances and therefore demonstrate that, for this particular case, the error estimates agree with the observed errors.

RHO-RHO LORAN-C COMBINED WITH SATELLITE NAVIGATION

Rho-rho Loran-C and Satellite Navigation are well suited for use together. Positions obtained from the Satellite Navigation system provide frequent checks on the Loran-C overland phase lag, clock drift and synchronization corrections but to do so it requires the accurate velocity information provided by Loran-C. In this section the methods used by the Bedford Institute of Oceanography to combine these two systems are described and estimates of the improvement in the rho-rho Loran-C ranging accuracy are compared with observed results.

Since Loran-C is used during the satellite pass to provide velocity information for the Satellite Navigation fix calculation it is necessary to ask how accurately Loran-C can determine the course and speed of the vessel during a satellite pass. The only Loran-C errors affecting velocity calculations are the short term errors due to noise which have been grouped under the User Measurement Error (σ_{me}). To illustrate the size of the velocity errors, if $\sigma_{me} = 0.1$ μsec (30 m) and the angle of cut of the two range circles is 45 deg., the relative positioning accuracy is about 95 m. The errors in course and speed calculated between two positions ten minutes apart, for a ship making ten knots, would be about ± 2½ deg. and ± 0.4 knot. Using STANSELL's (1969) rule of thumb that the positioning error of a Satellite Navigation fix is 0.25 nautical mile for every knot of velocity error, the Satellite Navigation positions should be accurate to about $\sigma_{sn} = 180$ m (0.6 μsec).

Satellite Navigation can be used to synchronize the receiver with the chain master transmissions and determine the clock drift correction in much the same way as these operations are carried out with the ship alongside. To see how Satellite Navigation can be used to establish synchronization, assume for the moment that the phase lag and clock drift errors are zero. The only sources of error in a Loran-C range measurement to a master station will be the Receiver Synchronization Error (σ_{rs}) and User Measurement Error (σ_{me}). Although one Satellite Navigation fix will only be accurate enough to establish synchronization to within about 0.6 μsec (180 m), if the average of a number of fixes is used the error in the receiver synchronization correction after n passes will improve to about :

$$[(\sigma_{sn}^2 + \sigma_{me}^2)/n]^{\frac{1}{2}}$$

where σ_{sn} = Satellite Navigation Error $\simeq$ 180 m,
σ_{me} = User Measurement Error $\simeq$ 30 m.

After 4 days, assuming a satellite fix accurate to ± 180 m is obtained every 3 hours, the Receiver Synchronization Error would be about 0.1 μsec (30 m).

The clock drift correction is determined by plotting the differences between each observed Loran-C range and the ranges calculated from the

Satellite Navigation positions against time; the slope is the drift rate. After 4 days the clock drift correction should be accurate to about 0.1 μsec/day (30 m/day) and will improve with time, the rate of improvement being determined by the frequency and quality of the satellite fixes. The Loran-C range used to determine the clock drift correction should be the range with the minimum amount of land path because of the larger errors that are introduced by land path.

When the receiver has been synchronized and the clock drift correction has been applied to all ranges the remaining differences between the observed ranges and those calculated from the Satellite Navigation positions must be due to inaccurate corrections for overland phase lag. The most probable cause of errors in overland phase lag corrections is incorrect estimates of land conductivity. Several comparisons between Loran-C and Satellite Navigation should give a better estimate of the land conductivity and new corrections for overland phase lag can be calculated.

This procedure was used during a hydrographic and geophysical survey off the coast of Labrador onboard the Canadian Scientific Ship *Hudson* in September and October, 1972. There was considerable land path for the Cape Race, Newfoundland signal while the Angissoq, Greenland signal path was all over water and was therefore used in comparison with Satellite Navigation to measure the receiver synchronization and clock drift corrections. After the Cape Race Loran-C range had been corrected for clock drift and receiver synchronization, all additional differences between the observed Loran-C and Satellite Navigation positions were assumed to be due to errors in the overland phase lag corrections. During the first few days it became apparent that the corrections for overland phase lag computed prior to the cruise, assuming a ground conductivity of 0.002 mhos/m, were too high. They were continually adjusted as the cruise progressed. The overland phase lag corrections varied from about 2.4 μsec (720 m; 84% land along the path) near the coast where the signal from Cape Race crossed Newfoundland and Labrador, to 0.5 μsec (150 m; 9% land path) 500 km from the coast.

As an indication of the accuracies that were being obtained, during post analysis 70 satellite fixes were selected at random from the vicinity of 55° N, 56° W and compared with the corresponding Loran fixes. The results show a standard deviation of 157 m in latitude and 225 m in longitude which, if the Satellite Navigation positions were exact, corresponds to a ranging error in all ranges of about 150 m (0.5 μsec).

Two independent checks on the ranging accuracy of Loran-C were made when the ship checked at a buoy, in shallow water, that had been fixed by Hi-Fix to better than ± 40 m. Prior to both checks the Loran-C synchronization and clock drift corrections had been determined entirely by Satellite Navigation comparisons. The errors in the Angissoq ranges on the two occasions were 0.59 μsec (177 m) and 0.07 μsec (21 m). The mean overland phase lag correction to the Cape Race range from the two buoy checks was 2.40 μsec (720 m). This value was within a few tenths of the values determined by Satellite Navigation.

FURTHER OPERATIONAL CONSIDERATIONS

The Bedford Institute of Oceanography rho-rho Loran-C system currently uses three ranges, where possible, in its fix calculation. Each of three ranges is weighted approximately according to its signal-to-noise ratio as determined visually from the oscilloscope. The residuals (differences between the observed ranges and those ranges corresponding to the position chosen by the computer program to minimize the inconsistencies) are used along with the Satellite Navigation comparisons to check on the error growth due to clock drift and phase lag.

Ranges of 2600 km (1400 n.m.) and greater have been obtained on several occasions with the Austron 5000 system and it has been used routinely with Satellite Navigation at ranges of 2200 to 2400 km (1200 to 1300 n.m.). However, there is danger of the tracking point jumping by one cycle (10 μsec, 3 km) at ranges greater than about 2000 km. and without an independent system such as Satellite Navigation to check on "cycle skips" the system must be used with great care. One further problem when using Loran-C at long range is the difficulty with re-acquiring a signal once it has been lost. This problem becomes more serious when the receiver position is not known to better than 3 km, in which case the correct cycle cannot be acquired. Figure 1, showing the rho-rho Loran-C coverage of the North Atlantic, was constructed using a maximum range of 2400 km. (1300 n.m.).

Loran-C skywave was found to be totally unsuitable for surveying operations. On several occasions the skywave delay was quite stable for a few hours but generally the delay was both unpredictable and unstable.

CONCLUSIONS

Two years of experience has shown clearly that rho-rho Loran-C is an effective, accurate survey system at ranges up to 2400 km and that its capabilities are greatly enhanced by combining it with Satellite Navigation. The estimated absolute ranging accuracies of rho-rho Loran-C, after 5 days at sea are : (all estimates at 95% confidence interval).

WITHOUT SATELLITE NAVIGATION

± 0.9 μsec (270 m) - no land path.
± 1.25 μsec (375 m) - land path

WITH SATELLITE NAVIGATION

± 0.6 μsec (180 m) - no land path
± 0.9 μsec (270 m) - land path

It is clear at this stage in the development of the Bedford Institute of Oceanography rho-rho Loran-C system that greater accuracy is possible from the further integration of Loran-C and Satellite Navigation. The improved accuracy would result not only from using more sophisticated mathematical techniques but also because the possibility of human error would be reduced. However, there may be a limit beyond which integration will cease to be an advantage. This limit results from increased cost of computer programming for an integrated system and the added maintenance problems. An allowance would also have to be made to operate the two systems independently should the need arise.

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APPENDIX

An Iterative Algorithm for Computing Geographic Co-ordinates from a Multi-Range Radio Positioning System

In figure 6, the estimated position is represented by p; the true position, as indicated by the ranging system, is represented by the point R. L_0, L_1, and L_2 represent the observed geodetic distances from point R to the three shore stations while S_0, S_1 and S_2 are the computed geodetic distances from the estimated position to each of the three stations.

From spherical trigonometry the distance (D) between two points on the surface of a sphere is found by :

$$D = a \cdot U \qquad (1)$$

where a = radius of the sphere, and U = angle subtended at the centre of the sphere by the arc between the two points.

U can be found from the formula :

$$\cos U = \sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2 (\lambda_2 - \lambda_1) \qquad (2)$$

ϕ_1, λ_1 and ϕ_2, λ_2 are the latitudes and longitudes of the two end points.

If one end of the line is held fixed and a is kept constant the distance

(D) is a function only of the latitude and longitude of the other end point. Equation (1) can therefore be rewritten as :

$$D = F(\phi_2, \lambda_2)$$

or :

$$F(\phi_2, \lambda_2) - D = 0 \tag{3}$$

where :

$$F(\phi_2, \lambda_2) = a . \text{arc cos} [\sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2 \cos (\lambda_2 - \lambda_1)] \tag{4}$$

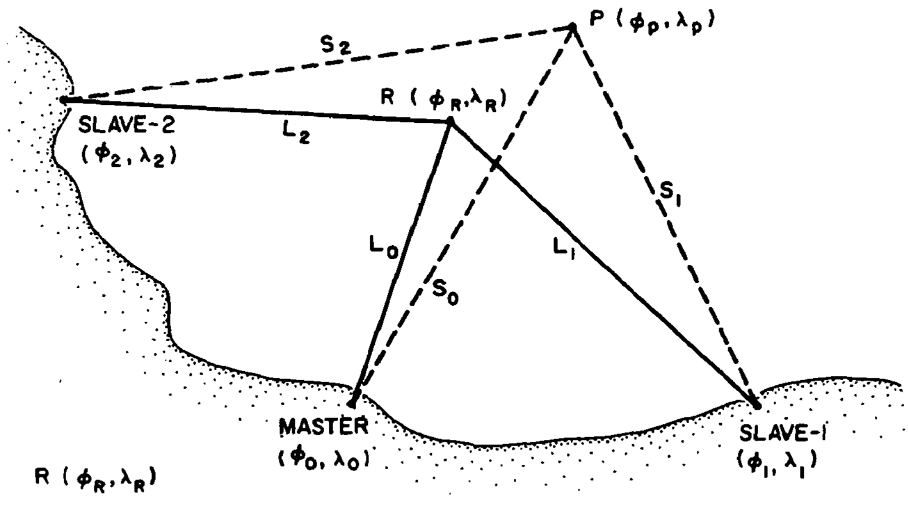

FIG. 6. — Three Range Fix.

From a reasonable initial approximate position (p) corrections ($\Delta\Phi$, $\Delta\lambda$) to Φ_p and λ_p are desired that yield a new estimated position closer to (R). The corrections are found by first expanding equation (3) in a Taylor series about the approximate position (p) and observed distance (L). Retaining only linear terms, equation (3) becomes :

$$F(\phi, \lambda) - D \simeq F(\phi_p, \lambda_p) + \frac{\partial F}{\partial \phi}(\phi_p, \lambda_p) \Delta\phi + \frac{\partial F}{\partial \lambda}(\phi_p, \lambda_p) \Delta\lambda - L = 0 \tag{5}$$

where $d\Phi$ and $d\lambda$ have been replaced by $\Delta\Phi$ and $\Delta\lambda$ since it was assumed that the first approximate position was reasonable and therefore these differences are small. Evaluation of equation (5) for each line from the three shore stations to (p) gives the three observation equations

$$\frac{\partial F_0}{\partial \phi} \Delta\phi + \frac{\partial F_0}{\partial \lambda} \Delta\lambda = L_0 - S_0 \tag{6a}$$

$$\frac{\partial F_1}{\partial \phi} \Delta\phi + \frac{\partial F_1}{\partial \lambda} \Delta\lambda = L_1 - S_1 \tag{6b}$$

$$\frac{\partial F_2}{\partial \phi} \Delta\phi \quad \frac{\partial F_2}{\partial \lambda} \Delta\lambda = L_2 - S_2 \tag{6c}$$

where : $$\frac{\partial F_i}{\partial \phi} = \frac{\partial}{\partial \phi}[a\,u] = a\left[\frac{-\cos\phi_p \sin\phi_i + \sin\phi_p \cos\phi_i \cos(\lambda_p - \lambda_i)}{\sin u}\right] \quad (7a)$$

and :

$$\frac{\partial F_i}{\partial \lambda} = \frac{\partial}{\partial \lambda}[a\,u] = a\left[\frac{\cos\phi_p \cos\phi_i \sin(\lambda_p - \lambda_i)}{\sin u}\right] \quad (7b)$$

The $S_i = F_i(\Phi_p, \lambda_p)$ are found from equation (4) where the lines are calculated between each of the three shore stations and the point (p). The L_i are the observed distances.

In matrix notation, equations (6) become :

$$\begin{bmatrix} \frac{\partial F_0}{\partial \phi} & \frac{\partial F_0}{\partial \lambda} \\ \frac{\partial F_1}{\partial \phi} & \frac{\partial F_1}{\partial \lambda} \\ \frac{\partial F_2}{\partial \phi} & \frac{\partial F_2}{\partial \lambda} \end{bmatrix} \begin{bmatrix} \Delta\phi \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} L_0 - S_0 \\ L_1 - S_1 \\ L_2 - S_2 \end{bmatrix} \quad (8)$$

If there are only two shore stations the solution is :

$$\begin{bmatrix} \Delta\phi \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} \frac{\partial F_0}{\partial \phi} & \frac{\partial F_0}{\partial \lambda} \\ \frac{\partial F_1}{\partial \phi} & \frac{\partial F_1}{\partial \lambda} \end{bmatrix}^{-1} \begin{bmatrix} L_0 - S_0 \\ L_1 - S_1 \end{bmatrix} \quad (9)$$

and the new estimate of the receiver position is found from the equations :

$$\begin{aligned} \phi &= \phi_p + \Delta\phi \\ \lambda &= \lambda_p + \Delta\lambda \end{aligned} \quad (10)$$

The new estimate is then treated as an initial approximation and the above procedure is repeated to obtain yet another new estimate of the receiver position. This process is continued until $\Delta\Phi$ and $\Delta\lambda$ are insignificant, at which point the estimated position is the receiver position.

All two range positioning systems have two points where the readings are the same; the points are on opposite sides of the baseline between the two shore stations. This iterative algorithm will not resolve the ambiguity. However, the problem does not arise if the initial approximate position is on the same side of the baseline as the receiver position.

How does one find values for $\Delta\Phi$ and $\Delta\lambda$ when there are three or more ranges ? The most commonly used method is based on the least squares criterion; that the sum of the squares of the inconsistencies be minimized. Rewriting equation (8) in the form :

$$AX = L \quad (11)$$

where :

$$A = \begin{bmatrix} \frac{\partial F_0}{\partial \phi} & \frac{\partial F_0}{\partial \lambda} \\ \frac{\partial F_1}{\partial \phi} & \frac{\partial F_1}{\partial \lambda} \\ \frac{\partial F_2}{\partial \phi} & \frac{\partial F_2}{\partial \lambda} \end{bmatrix} \qquad X = \begin{bmatrix} \Delta\phi \\ \Delta\lambda \end{bmatrix}$$

and :

$$L = \begin{bmatrix} L_0 - S_0 \\ L_1 - S_1 \\ L_2 - S_2 \end{bmatrix}$$

it can be shown (e.g. WELLS and KRAKIWSKY, 1971) that the weighted least squares estimate ($\hat{X}$) of (x) is equal to :

$$\hat{X} = \begin{bmatrix} \Delta\hat{\phi} \\ \Delta\hat{\lambda} \end{bmatrix} = (A^T PA)^{-1} A^T PL \qquad (12)$$

where P is the 3×3 (or $n \times n$ if there are n ranges) weight matrix. The simplest form of the weight matrix is the identity matrix, i.e.,

$$P = I = \begin{bmatrix} 100 \\ 010 \\ 001 \end{bmatrix}$$

where all three (or more) ranges are used equally. Or, if one of the ranges is not usable, the corresponding diagonal element in the weight matrix can be set to zero and that range will be ignored, e.g., if secondary station 1 was not usable the weight matrix could be set to :

$$P = \begin{bmatrix} 100 \\ 000 \\ 001 \end{bmatrix}$$

Intermediate values of the diagonal elements may also be used.

The iterative process for three or more ranges is identical to the two range case except that equation (12) is used instead of equation (9).

The speroidal shape of the earth is taken into account in the calculation of the geodesic distances (S) from the approximate position to the fixed shore stations. Instead of equation (4) the following equation, called the Andoyer-Lambert long line formula, is used :

$$S_i = a\,(U + du) \qquad (13)$$

where U is obtained from equation (2) and

$$du = \frac{f}{4}\left[\left(\frac{u + 3 \sin u}{1 - \cos u}\right)(\sin \phi_p - \sin \phi_i)^2 + \left(\frac{u - 3 \sin u}{1 + \cos u}\right)(\sin \phi_p + \sin \phi_i)^2\right] \quad (14)$$

f is the flattening of the ellipsoid and is equal to $(a\text{-}b)/a$ where a and b are the semi-major and semi-minor axes of the ellipsoid. This formula is derived by THOMAS (1965).

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