GEODETIC HYDROGRAPHY AS RELATED TO MARITIME BOUNDARY PROBLEMS

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GEOMETRY OF LINES AND CURVES

The dividing line in most existing treaties, agreements or decrees concerning either maritime boundaries (such as fishery limits, continental shelf delimitations, territorial water dividing lines, or concession limits) or straight baseline systems is stated in one of the following ways : as a great circle arc, a loxodromic line, a small circle arc, or even more commonly as a 'straight line' between given points in terms of geographical coordinates, or else by bearing and distance to a fixed point. These are the methods used, although in fact it is the spheroidal geodesic which represents the shortest distance between two points on the spheroidal ellipsoid (i.e. the oblate ellipsoid of revolution used as the reference surface in cartography.)

Apart from the equator and the meridians, which are closed, plane geodesics, all other geodesics are open curves of double curvature oscillating between two parallels of latitude symmetric to the equator. Furthermore they satisfy the condition that $\cos b . \sin a$ is constant along the curve (b being the reduced latitude, a the azimuth). In other words, they are relatively complicated curves and so far they have been of only modest use to the navigator, or to others concerned with the marine environment.

The straight line of the early "treaties" was normally interpreted by mariners as a loxodrome or compass line, for the simple reason that this curve happens to be a straight line on ordinary nautical charts (if they are on the Mercator projection). In more recent years, however, there has been a tendency to utilize great circle arcs in the wording of these treaties, or for their interpretation. The reason is, of course, that the great circle arc is considered a better representation of the curve of shortest distance. Under certain circumstances this is true, but it should be recalled that the great circle as such is not defined on the spheroid. Hence its use is meaningless, unless the corresponding sphere is identified by radius, centre, etc., and the appropriate coordinate conversions duly specified. In most treaties these are not mentioned, hence confusion and difficulties sometimes arise in the interpretation of positions in relation to the boundary line.

It is not easy to generalize about the differences between the various curves, due to the fact that both relative and absolute positions have decisive effects. The example below, gives an idea of the differences in terms of distance between two points A and B whose geographic positions are respectively (58°00′ N, 0°00′ E) and (62°00′ N, 10°00′ E) for the spheroidal loxodrome, the great circle arc and the spheroidal geodesic.

Distance along :		
(a) the spheroidal geodesic (International ellipsoid)	approx.	712 851 m
(b) the great circle arc, its radius having either :		
a Gaussian curvature at 60° N (mean latitude)	approx.	712 722 m
or meridional curvature at 45° N	approx.	710 221 m
(c) the spheroidal loxodrome (computed by numerical integration)	approx.	713 531 m

As can be seen, the various curves do not vary greatly as regards length. It is interesting to note, however, that the distance along the great circle arc (*) — as mariners understand the term — is less than that along the geodesic. Using the correct radius (Gaussian curvature at 60°) the great circle distance will work out at about 130 m less than the geodetic distance, which is in fact the more reasonable.

Figures 1 and 2 are illustrations based on the aforementioned posi-

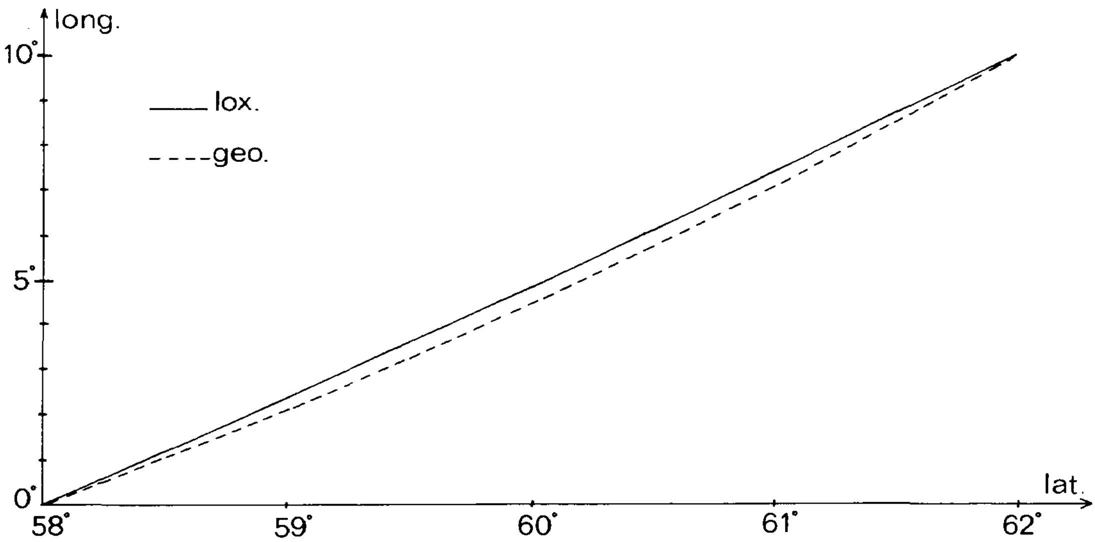

FIG. 1. — Longitude as a function of latitude :
(a) For a loxodrome;
(b) For a geodesic (coinciding almost exactly with a great circle arc with its radius at 45°)
between points A (58°00′ N, 0°00′ E) and B (62°00′ N, 10°00′ E).

(*) See (b) above : 1 nautical mile is equivalent to 1 minute of arc in latitude = 1852 m.

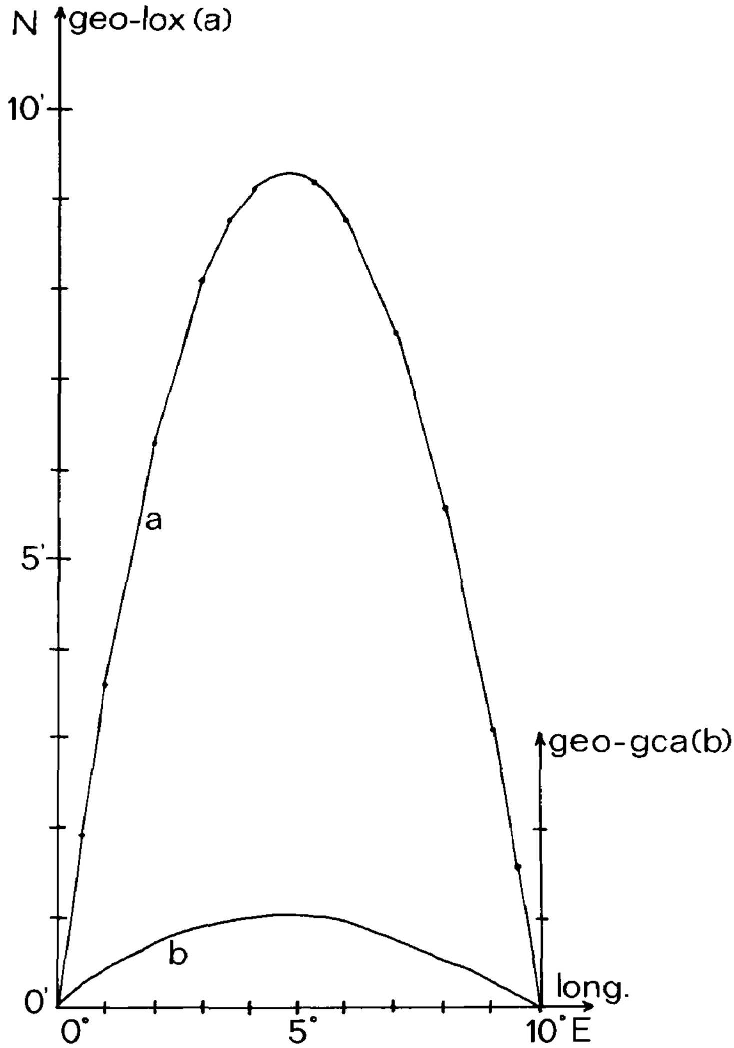

FIG. 2. — Differences in latitude as a function of longitude :
(a) between a geodesic and its corresponding loxodrome;
(b) between a geodesic and its corresponding great circle arc (with radius at 45°).

tions. They show the differences in the course of the geodesic, the corresponding loxodrome and the great circle arc (radius at 45° N).

Figure 1 depicts longitude as a function of latitude. Figure 2 shows the difference in latitude as a function of longitude. Curve (a) is referred to the left hand scale in minutes of arc, and curve (b) to the right hand scale which is in seconds of arc.

When examining the "cut off" arcs between the various lines their differences in area are seen to be more spectacular.

(a) The area between the spheroidal geodesic and the great circle arc (radius at 45° N) is approximately 10 km².

(b) The area between the spheroidal geodesic and the spheroidal loxodrome amounts to approximately 7100 km².

Case (b) gives particular emphasis to the desirablility of having boundary lines, or segments of these boundary lines, defined in a uniform and unequivocal way. Although no doubt it is wishful thinking to suppose that in future all "treaties" will define a boundary line as being a geodesic (on the International ellipsoid, or any other recognized reference ellipsoid), much could be gained if there was at least a tacit understanding to this effect amongst hydrographers themselves.

CONSIDERATIONS REGARDING ACCURACY

Besides the choice of a mathematical line or curve between a sequence of points there is also the question of accuracy of definition and of position when determining a boundary line or segments of such a line.

True accuracy in the first context can be defined as the degree of precision of the geographical coordinates for boundary turning points, i.e. in full minutes of arc, in tenths of a minute of arc, or in seconds of arc. The coordinates should be expressed to the same degree of precision as those for the control points in the surrounding coastal area for both the case of a boundary line obtained by geodetic calculations (without loss of accuracy) on the basis of coastal control points and for that of a line merely "negotiated" (that is to say, one not based on mathematic criteria).

If the boundary line is to be regarded as provisional, due to the state of the charting of the area, then it is advisable to allow for future changes by defining a boundary zone.

Let us now consider the real, or positional, accuracy. This can be defined as the accuracy of fixing at sea by navigational, hydrographic or geodetic means.

The general need for high positional accuracy at increasingly greater distances from the coast is underlined by the fact that exploration and exploitation of natural resources both on the sea bed and in the subsoil, as well as other marine activities are now feasible beyond the 200 m isobath. This again emphasises the necessity for unequivocal and accurate definition of all relevant boundary lines, be they close inshore or far out to sea. This should be done now, in order that disputes between nations and individuals may be avoided later on when exploitation interests may complicate the issue.

It is well known that the International Conference on the Law of the Sea, scheduled for 1974, visualizes an international regime for the oceanic sea bed and subsoil beyond the outer limits of national jurisdiction. Any agreement reached on this subject is bound to urge nations to delimit their outer shelf boundary, thus enabling them to invite bids for exploration and exploitation at the earliest possible date. Failure to reach international agreement on the other hand is unlikely to prevent a nation either from extending the limits of its shelf or exploring what is outside the shelf, as defined defined in the Geneva Convention of 1958.

This situation would be much more dangerous, since there would then be no general agreement upon outer boundaries adjacent to the high seas.

We may ask what degree of absolute positional accuracy is actually required. Undoubtedly the most ambitous demand comes from the oil drilling companies who wish to know exactly where to put down their drills in order not to risk trespassing on neighbouring property. From this point of view even a few metres could be significant, especially in the case where a single geologic structure extends across a dividing line. In general, the same absolute accuracy is needed when it is a question of locating such a geologic feature, or of relocating the casing of a well or some object left behind on the sea floor.

What then are the possibilities of meeting these requirements ? Up to the present time it has not been possible to determine a position at sea by existing hydrographic or geodetic means to an accuracy of within a few metres, and certainly not at some distance from the coast out of sight of land.

However, this fact does not prevent us from defining the boundary line with the same accuracy of definition as that of the fundamental material such as maps, photographs, etc. on which the computations are based. As advances are made in hydrographic and geodetic methods — and nowadays progress is very rapid — this accuracy of definition will become more and more substantial, and eventually we shall end up by being able actually to mark the boundary line at sea with such systems as acoustic transponders, cables, or active or passive markers anchored on the sea bed.

HYDROGRAPHIC OPERATIONS

When the hydrographer is called upon to define a boundary line according to certain criteria — which may be either purely mathematical or of a less accurate nature such as a "negotiated line" — he will often have to make constructions with his ruler and dividers and/or make a series of complicated computations.

Years ago the median and equidistance lines were often simply constructed on ordinary Mercator charts without taking into account the distortion factor. Thus the parties were indirectly accepting an inaccuracy of definition for the line, with a probable error of as much as 1-2 nautical miles depending on the scale of the chart used.

As commercial interests increased, in particular in the continental shelf areas, it became obvious that for the definition of a dividing line between nations or between concessions an inaccuracy of such a magnitude was not acceptable to the parties concerned. It is of course always possible to "add zeros" at the end of imprecise coordinates, but this does not hide the fact that the sea bed will not be properly divided.

Some of the frequently employed methods will now be discussed in some detail.

THE BILATERAL EQUIDISTANCE LINE

Depending on the circumstances, an equidistance line — or a median line — may normally be determined either by computation alone or by means of a combination of accurate construction and computation. In both cases the equidistance line should be based on equal geodetic distances to surrounding low water marks on the coastline or from straight baselines — i.e. the baselines should be regarded as geodesics. As the low water mark is not normally included in the geodetic grid, the coordinates for the coastal control points or the baseline turning points must be recorded with the highest possible degree of accuracy, and they should be taken from either a large scale chart (preferably one at 1/25 000 or larger) or from vertical aerial photographs. In the case of a long straight baseline, if the identification of relevant control points is difficult the coordinates for the low water marks should be registered to an accuracy that is compatible with the overall accuracy. Points along straight baselines should in fact not be selected, because the line running between the various turning points is already defined as a geodesic.

In most cases the low water line is extremely irregular, due either to off-lying islands or a pronounced sinuosity of the coast. This in turn means that a great number of control points will have an influence on the course of the true equidistance line between the two states. The line may thus be delineated by a set of points forming many curves, and so will not be easy to define.

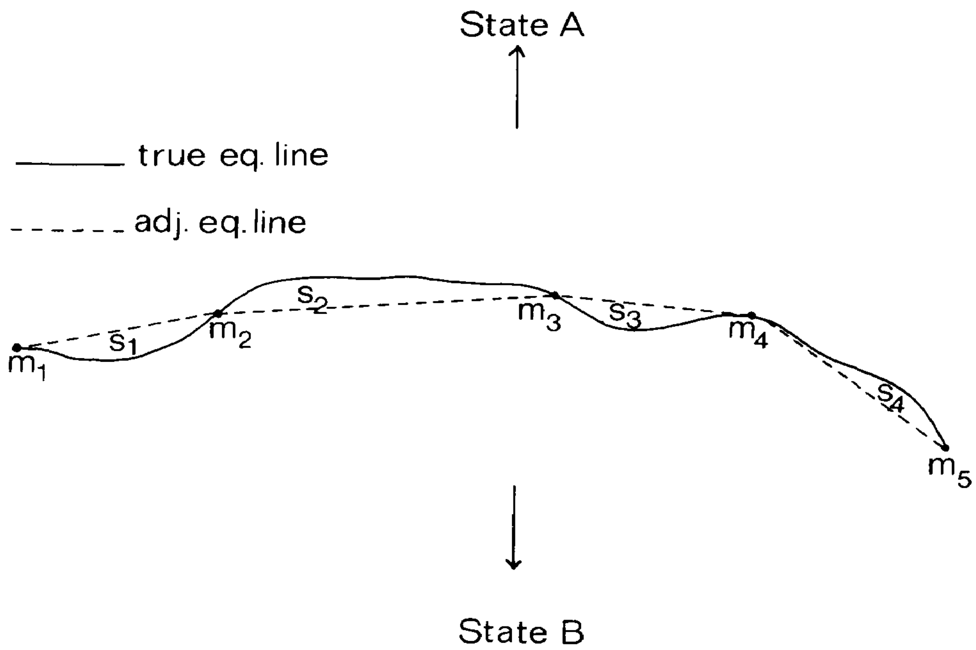

Fig. 3

In such cases the respective states may for administrative purposes agree to an adjusted or approximate equidistance line that follows only roughly the general contour of the respective coastlines — in other words to an "equidistance" line having only few, and relatively long, legs.

Figure 3 illustrates this situation. In this procedure a number of areas of different size and shape (s_1, s_2, s_3, s_4) between the true and the adjusted equidistance lines will be "cut off". From the point of view of equity of area it then seems advisable to choose the adjusted line according to an equal area "cut off" principle. This presupposes that each basic unit in the boundary waters has the same potential value. If this is not the case, then the contested area may be subdivided qualitatively, i.e. the small areas can be weighted and compared in order to delineate a final line of division.

Appendix A contains a short description of an accurate method of determining equidistance points using a combination of plotting methods and calculation.

Appendix B describes computer programs, some at present under development, for automated calculation of a sequence of equidistance points.

MEASUREMENT OF AREAS

For many purposes in connection with maritime boundaries it is of importance to be able to measure with accuracy areas of different shapes and sizes. Direct measurement of these areas on charts, either by subdivision into triangles or trapezia, or by planimetry, is laborious and often inaccurate even if large-scale "equal area" charts and maps are available. However, provided the limiting lines are well defined, it is always possible to use the computational method to define these areas with a high degree of accuracy by the method of subdivision into triangles. Appendix B describes a computer program for automated computation of areas on the spheroid that are bounded by a curve, or curves.

DETERMINATION OF A LINE A GIVEN DISTANCE FROM THE COAST — THE UNILATERAL EQUIDISTANCE LINE

Some maritime boundaries are defined as curves located at a given distance from the low water mark line and/or straight baselines. Territorial water lines, fishery limits, customs limits, pollution zone limits are examples of curves based on distance criteria. Mathematically such a curve is the envelope of small consecutive circles with a given distance as radius and centered on the low water mark line. From the point of view of plotting, this curve can be easily constructed using dividers and a curvetracer. However, only moderate accuracy is obtained by this method, depending on the scale and projection used. When a higher degree of accuracy is needed — or when the chart or map does not cover the whole area in question — it will be necessary to use an electronic ocmputer.

It may be recalled that the United Nations Seabed Commission in its preparatory work has indicated that it anticipates that any delimitation of an international and/or intermediate zone on the oceanic seabed and its subsoil will be based on straight distance criteria, probably in combination with a depth criterion.

APPENDIX A

MANUAL DETERMINATION OF TRUE EQUIDISTANCE POINTS

An earlier articles of mine (*) mentioned the development of an electronic computer program able to cope with all possible geographical possibilities. The development of this program is, however, very involved and continues to demand considerable effort. For most cases encountered in practical life a sufficiently accurate result may be obtained within a reasonable time by using a procedure involving a combination of plotting and manual calculation.

Figure 4 depicts at small scale two states (A and B) having several capes and prominent headlands. For reasons of simplicity it has been supposed that only one low water mark point (control point) on each cape or headland influences the course of the true equidistance line between the two states. These points are marked successively a_1, a_2, ... a_6 and b_1, b_2, ... b_5. Using the method of construction presented by R.H. KENNEDY (**) the approximate positions of the equidistance turning points m_1, m_2, ... m_9 (which are the points of intersection between neighbouring bisector lines) are found and their geographical coordinates are listed (approximate accuracy; to a full minute of arc). If a Mercator chart or other non-conformal chart is used allowance must be made for possible scale factors. Point m_1 in this example is equidistant from control points a_1, b_1, b_2 and so forth — to within the precision just indicated.

In the next step the precise UTM or geographical coordinates for the control points a_1, a_2, ... a_6 and b_1, b_2, ... b_5 are read from a large scale geodetic map (preferably at 1/25 000 or larger) to within the desired, or possible, order of accuracy. To find the exact coordinates for points m_1, m_2, ... m_9 the procedure is as follows (see figure 5) :

(*) Notes on hydrographic assistance to the solution of sea boundary problems. *Int. Hydr. Rev.*, Vol. XLVIII, (2), 1971, pp. 149-159.

(**) Brief remarks on median lines and lines of equidistance and the methods used in their construction. Paper presented at the Geneva Conference on the Law of the Sea, 2 April 1958.

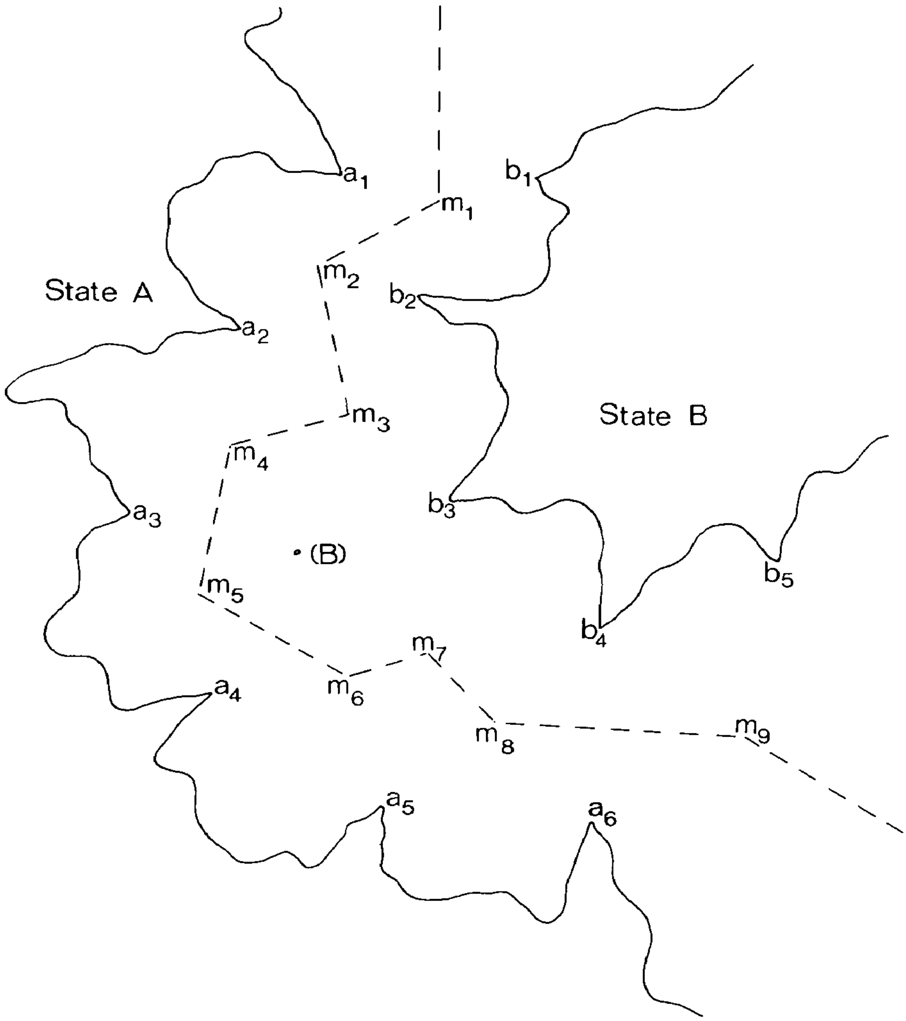

FIG. 4. — Generalized plot on a small scale
(For reasons of simplicity straight baselines have not been introduced).

On the International ellipsoid (or any other reference ellipsoid) the geodetic distances and their forward (and if required their back) azimuths between m_1 and the three respective control points a_1, b_1 and b_2 are computed either manually or on an electronic computer. One method would be to use the formulae suggested by P.D. THOMAS (*). In cases where there is some doubt as to the relevance of a chosen control point, the distance from this to m_1 and from m_1 to potential neighbouring control points should be calculated and compared as a basis on which to make a choice.

A Cartesian grid is used (see figure 6), and point m_1, together with the three forward azimuths of the geodesics are plotted with m_1 as centre.

(*) Spheroidal Geodesics, Reference Systems and Local Geometry, U.S. Naval Oceanographic Office, SP 138, 1970.

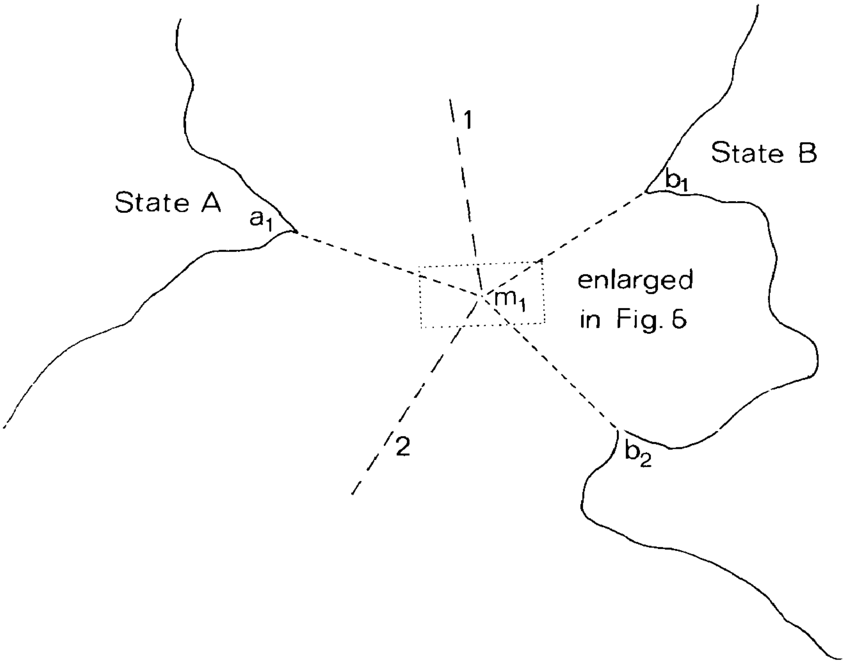

Fig. 5. — Detail of portion of figure 4.

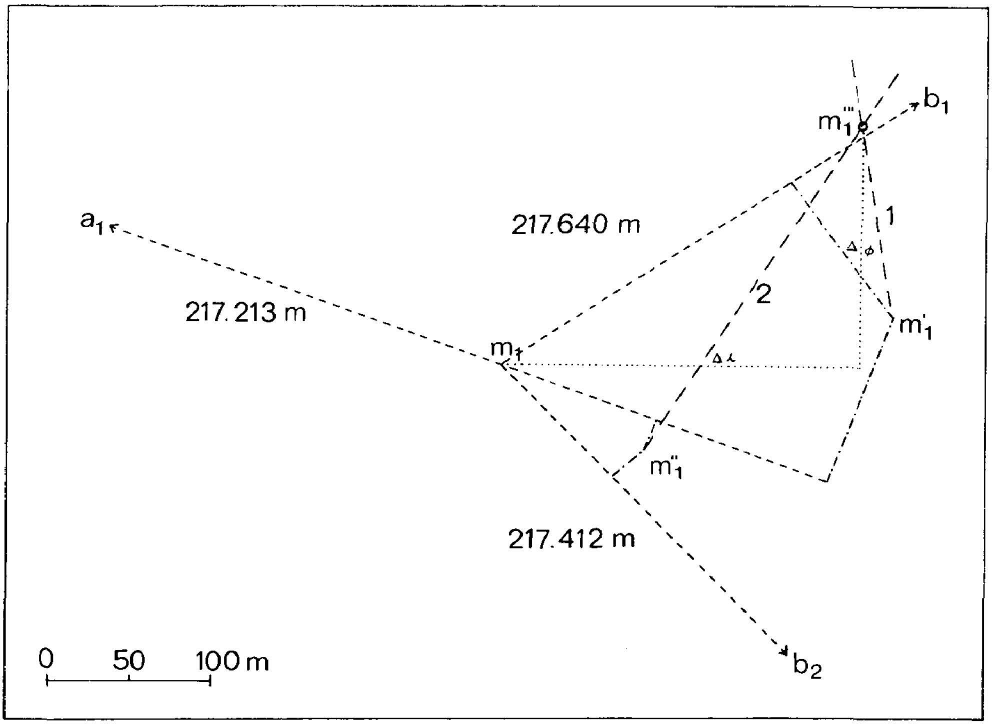

Fig. 6. — Detail of portion of figure 5. An example of a large scale Cartesian plot. m' and m'' have been plotted on the basis of mean distances of respectively 217 426 m and 217 313 m.

A suitable scale, to allow for adequate presentation of small differences in calculated distance, must be chosen. We must also keep in mind that for first order accuracy the position line at a given distance from a fixed point will be a line perpendicular to the appropriate azimuth in the given distance. Two arbitrary equidistance points m_1' and m_1'' referring respectively to a_1, b_1 and a_1, b_2 are then chosen in the vicinity of m_1 and are plotted. In the present case the mean of the distances m_1a_1, m_1b_1 and m_1a_1, m_1b_2 has been used. From the general plot we then draw lines through m_1' and m_1'' in directions parallel to the appropriate bisector legs 1 and 2 (see figure 5). The intersection of these lines is point m_1''' which is the position of the true equidistance turning point. The coordinates — which may be either UTM or geographical — for m_1''' may be easily read to the desired accuracy from the large scale plot.

If necessary, a check on the determination of point m_1''' can readily be made by computation of the geodetic distances and azimuths from this point to the control points a_1, b_1 and b_2. In cases where the desired accuracy (which must be within the limits of the accuracy of the control points) is not immediately obtained the plotting process will have to be repeated.

The above procedure is repeated for each equidistance turning point, and this gives the composite equidistance line between points m_1, m_2, ... m_9 in geographical coordinates.

As regards general accuracy, the limiting factor is the degree of reliability of the coordinates for the coastal control points since geodesics can in practice be computed without any loss of accuracy. With future precise geodetic or navigational fixing at sea in view, we can state that a definitive accuracy of a tenth of a second of arc would normally be attained from a well surveyed surrounding landmass.

APPENDIX B

SUMMARY OF ELECTRONIC COMPUTER PROGRAMS

1. Program FRV 131-1 : Spheroidal Areas.

FRV 131-1 is a program in Algol (written by F.G. STRØBECH in 1972) which will give the approximate area of any n-corner polygon on the spheroid (*). The approximation can be refined by increasing the number of points along the curves. This question will however be dealt with in more detail later.

The only limitations as to the positions and number of corner points for the polygon are the following :

(*) Cf. N.W. HUMPHRIS : International Global System for defining Maritime Boundaries. *I.H. Bulletin*, December 1971, pp. 412-416.

(1) The difference in longitude of two adjacent corner points must not exceed 180°.

(2) Neither of the Poles may be encircled by the polygon. (They may, however, well be one of the corner points).

(3) The length of the polygon's sides should be limited — preferably to not more than 100 nautical miles — in order to assure high accuracy.

(4) The polygon must not be too small. It should preferably be of more than 1 km^2.

(5) The number of corners allowed will depend on the capacity of the specific computer.

Computation is based on the very simple formula :

$$S = e\, R^2$$

S being the spherical area, R the spherical radius (here the radius of the Meussnier sphere), e the spherical excess which is equal to the sum of the polygon's angles minus $(n - 2)\pi$ where n is the number of corners. This means that in principle the computation is reduced to calculating the spherical angles and the radius of the Meussnier sphere. This is the sphere which offers the best spheric approximation to the spheroid in the vicinity of a single point as it has a radius equal to the Gaussian curvature of the spheroid at the said point.

The total area of the polygon is subsequently found by calculation of $n - 2$ spheric triangles formed by a selected combination of corner points. A fixed rotation in the listing of corner point coordinates allows for both this selection and possible concavities of the polygon. The area of each sub-triangle is calculated as if it were lying on the Meussnier sphere at the mid-latitude of the triangle. To reduce the influence of the latitudinal factor the program itself arranges the sub-division into triangles in such a way as to give each triangle the minimum latitudinal extension. The spherical angles are computed according to Napierian rules.

As indicated earlier, the approximation may be refined by insertion of extra points along the outer lines of the polygon. These points may come from geodesics, great circle arcs, loxodromes, or from any other curve. To meet this requirement sub-programs for the geodesic (FRV 133-1), the great circle arc (FRV 132-1), and the loxodrome (FRV 134-1) have been written. Where a coastline or the limits of territorial waters is one of the outer lines, the subdivision may be carried out by selecting the coordinates along this line either manually or with a pencil follower.

2. FRV 132-1. Subdivision of the Great Circle Arc.

This computer program (written by F.G. STRØBECH in 1971) will supply geographical latitude to the nearest hundredth of a second of arc for every full minute step in geographical latitude along the great circle arc between the two given positions.

The total distance (to the nearest hundredth of a metre) along the great circle arc is based on the Earth's radius at 45° Latitude.

If required, further subdivision can be made manually without loss of accuracy. Geographical tables (*) could be used to find the distances corresponding to other radii.

Table 1 shows part of the printout for the example given at the beginning of this article.

TABLE 1

Latitude N	Longitude E	Latitude N	Longitude E
....	....	....	....
59 38 07.56	03 35 00.00	59 46 55.22	03 56 00.00
59 38 32.86	03 36 00.00	59 47 20.15	03 57 00.00
59 38 58.15	03 37 00.00	59 47 45.06	03 58 00.00
59 39 23.43	03 38 00.00	59 48 09.95	03 59 00.00
59 39 48.68	03 39 00.00	59 48 34.82	04 00 00.00
59 40 13.92	03 40 00.00	59 48 59.68	04 01 00.00
59 40 39.13	03 41 00.00	59 49 24.52	04 02 00.00
59 41 04.33	03 42 00.00	59 49 49.34	04 03 00.00
59 41 29.51	03 43 00.00	59 50 14.14	04 04 00.00
59 41 54.67	03 44 00.00	59 50 38.93	04 05 00.00
59 42 19.82	03 45 00.00	59 51 03.70	04 06 00.00
59 42 44.94	03 46 00.00	59 51 28.45	04 07 00.00
59 43 10.05	03 47 00.00	59 51 53.18	04 08 00.00
59 43 35.14	03 48 00.00	59 52 17.89	04 09 00.00
59 44 00.22	03 49 00.00	59 52 42.59	04 10 00.00
59 44 25.27	03 50 00.00	59 53 07.27	04 11 00.00
59 44 50.31	03 51 00.00	59 53 31.93	04 12 00.00
59 45 15.32	03 52 00.00	59 53 56.57	04 13 00.00
59 45 40.33	03 53 00.00	59 54 21.19	04 14 00.00
59 46 05.31	03 54 00.00	59 54 45.80	04 15 00.00
59 46 30.27	03 55 00.00	59 55 10.39	04 16 00.00
....	....	....	....

D: 710.22087 km

3. FRV 133-1. Subdivision of Spheroidal Loxodrome.

This is a specific program (written by B. Rost ANDERSEN in 1972) for giving the geographical longitude to the nearest fourth decimal in seconds of arc for every full minute step in geographical latitude along the loxodrome between points A and B.

Computation of the total distance along the loxodrome (to the nearest hundredth of a metre) is on the basis of numerical integration.

A more flexible program is now under development.

Table 2 shows part of the printout for the example given at the beginning of this article.

(*) Cf. D.H.K. AMIRAN and A.P. SCHICK : Geographical Conversion Tables, Zürich, 1961.

TABLE 2

Latitude				Longitude			
59	38	0.0000	N	3	56	16.4001	Ø
59	39	0.0000	N	3	58	44.5785	Ø
59	40	0.0000	N	4	1	12.8307	Ø
59	41	0.0000	N	4	3	41.1570	Ø
59	42	0.0000	N	4	6	9.5573	Ø
59	43	0.0000	N	4	8	38.0317	Ø
59	44	0.0000	N	4	11	6.5804	Ø
59	45	0.0000	N	4	13	35.2033	Ø
59	46	0.0000	N	4	16	3.9008	Ø
59	47	0.0000	N	4	18	32.6726	Ø
59	48	0.0000	N	4	21	1.5191	Ø
59	49	0.0000	N	4	23	30.4402	Ø
59	50	0.0000	N	4	25	59.4361	Ø
59	51	0.0000	N	4	28	28.5068	Ø
59	52	0.0000	N	4	30	57.6525	Ø
59	53	0.0000	N	4	33	26.8732	Ø
59	54	0.0000	N	4	35	56.1690	Ø
59	55	0.0000	N	4	38	25.5399	Ø
59	56	0.0000	N	4	40	54.9861	Ø
59	57	0.0000	N	4	43	24.5078	Ø
59	58	0.0000	N	4	45	54.1048	Ø
59	59	0.0000	N	4	48	23.7774	Ø
60	0	0.0000	N	4	50	53.5257	Ø
60	1	0.0000	N	4	53	23.3496	Ø
60	2	0.0000	N	4	55	53.2494	Ø
60	3	0.0000	N	4	58	23.2251	Ø
60	4	0.0000	N	5	0	53.2768	Ø
60	5	0.0000	N	5	3	23.4045	Ø
60	6	0.0000	N	5	5	53.6084	Ø
60	7	0.0000	N	5	8	23.8886	Ø
60	8	0.0000	N	5	10	54.2451	Ø
60	9	0.0000	N	5	13	24.6781	Ø
60	10	0.0000	N	5	15	55.1876	Ø
60	11	0.0000	N	5	18	25.7737	Ø
60	12	0.0000	N	5	20	56.4365	Ø
60	13	0.0000	N	5	23	27.1761	Ø
60	14	0.0000	N	5	25	57.9926	Ø
60	15	0.0000	N	5	28	28.8861	Ø
60	16	0.0000	N	5	30	59.8567	Ø
60	17	0.0000	N	5	33	30.9044	Ø

D: 713.530522 km

4. FRV 134-1. Subdivision of Spheroidal Geodesic.

This is a program (written by F.G. STRØBECH in 1972) for providing intermediate points in geographical latitude and longitude to the nearest thousandth of a second of arc along the spheroidal geodesic between two given points. Both the number of subdivisions and the reference ellipsoid can be chosen at will.

Furthermore, the forward and the back azimuth at each step in then obtained, together with the cumulative distance (to the nearest hundredth of a metre) between the two extreme end points.

TABLE 3

Latitude	Longitude	back azimuth	forw. azimuth
059 38 06.284 n	003 34 54.834 e	230 11 36.295	050 11 36.295
059 38 30.858 n	003 35 53.087 e	230 12 26.558	050 12 26.558
059 38 55.425 n	003 36 51.363 e	230 13 16.846	050 13 16.845
059 39 19.984 n	003 37 49.663 e	230 14 07.157	050 14 07.157
059 39 44.536 n	003 38 47.986 e	230 14 57.492	050 14 57.492
059 40 09.081 n	003 39 46.333 e	230 15 47.851	050 15 47.851
059 40 33.619 n	003 40 44.704 e	230 16 38.234	050 16 38.234
059 40 58.149 n	003 41 43.099 e	230 17 28.641	050 17 28.641
059 41 22.673 n	003 42 41.517 e	230 18 19.072	050 18 19.072
059 41 47.189 n	003 43 39.959 e	230 19 09.527	050 19 09.527
059 42 11.697 n	003 44 38.425 e	230 20 00.006	050 20 00.006
059 42 36.199 n	003 45 36.914 e	230 20 50.509	050 20 50.509
059 43 00.693 n	003 46 35.427 e	230 21 41.036	050 21 41.036
059 43 25.180 n	003 47 33.964 e	230 22 31.587	050 22 31.587
059 43 49.660 n	003 48 32.525 e	230 23 22.162	050 23 22.162
059 44 14.132 n	003 49 31.109 e	230 24 12.761	050 24 12.761
059 44 38.597 n	003 50 29.718 e	230 25 03.384	050 25 03.384
059 45 03.055 n	003 51 28.350 e	230 25 54.031	050 25 54.031
059 45 27.505 n	003 52 27.006 e	230 26 44.702	050 26 44.702
059 45 51.949 n	003 53 25.685 e	230 27 35.398	050 27 35.398
059 46 16.385 n	003 54 24.389 e	230 28 26.117	050 28 26.117
059 46 40.813 n	003 55 23.116 e	230 29 16.861	050 29 16.860
059 47 05.234 n	003 56 21.867 e	230 30 07.628	050 30 07.628
059 47 29.648 n	003 57 20.643 e	230 30 58.420	050 30 58.420
059 47 54.055 n	003 58 19.441 e	230 31 49.236	050 31 49.236
059 48 18.454 n	003 59 18.264 e	230 32 40.076	050 32 40.076
059 48 42.846 n	004 00 17.111 e	230 33 30.940	050 33 30.940
059 49 07.231 n	004 01 15.981 e	230 34 21.828	050 34 21.828
059 49 31.608 n	004 02 14.876 e	230 35 12.740	050 35 12.740
059 49 55.978 n	004 03 13.794 e	230 36 03.677	050 36 03.677
059 50 20.341 n	004 04 12.736 e	230 36 54.637	050 36 54.637
059 50 44.696 n	004 05 11.703 e	230 37 45.622	050 37 45.622
059 51 09.044 n	004 06 10.693 e	230 38 36.632	050 38 36.631
059 51 33.385 n	004 07 09.707 e	230 39 27.665	050 39 27.665
059 51 57.718 n	004 08 08.745 e	230 40 18.722	050 40 18.722
059 52 22.044 n	004 09 07.807 e	230 41 09.804	050 41 09.804
059 52 46.362 n	004 10 06.892 e	230 42 00.910	050 42 00.910
059 53 10.673 n	004 11 06.002 e	230 42 52.040	050 42 52.040
059 53 34.977 n	004 12 05.136 e	230 43 43.194	050 43 43.194
059 53 59.273 n	004 13 04.294 e	230 44 34.373	050 44 34.373
059 54 23.562 n	004 14 03.476 e	230 45 25.576	050 45 25.576

D: 712.850930 km

Table 3 is part of the printout for the example given at the beginning of this article. The International ellipsoid is used, and the number of subdivisions is 600.

5. FRV 135-1. Bilateral Equidistance Points.

This is a program (written by F.G. STRØBECH in 1972) for computing single equidistance turning points based on straight baselines which are taken as geodesics.

Figure 7 demonstrates the different approaches to the computation of equidistance turning points for the case of coastal control points and for straight baselines.

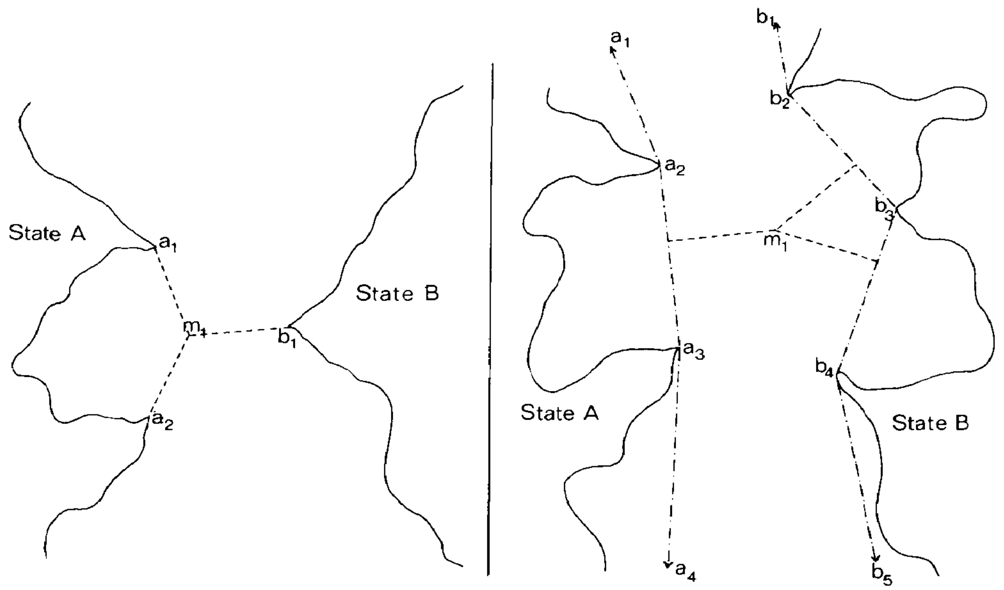

FIG. 7

6. FRV 135-2. Bilateral Equidistance Line.

Program No. 5 (FRV 135-1) is actually a sub-program of the program FRV 135-2 (written by F.G. STRØBECH in 1973) and it attempts to provide the whole extent of the equidistance line between two adjacent or opposite states.

In cases where the coastal state does not have straight baselines along either the whole or part of its coast, then the low water line is digitized discretely in a manner compatible with the coastal configuration. For instance when the coastline is highly indented the discrete steps will be short — and vice versa. This computation is based on straight baselines (geodesics) only. It can also be used for those parts of a coast where no formal baselines exist, the reasoning here being that the digitized sections are composed of short lengths of straight line which follow the low water line very closely.

7. FRV 136-1. Unilateral Equidistance Line.

As the next step a program for giving a sequence of points along a curve at a given distance from the low water line is to be written by F.G. STRØBECH. Territorial water lines and fishery and pollution zones are all based on the distance criterion, and the delimitation of the international seabed area is also likely to be based on a similar criterion. This program could thus be used for the accurate determination of all these limits.

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(Manuscript submitted in English by Danish author).