

THE ATLAS ALPHA DOPPLER NAVIGATOR SYSTEM

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PRINCIPLE

Rapid progress in the fields of electronic and digital computer technology has proved a strong stimulant to doppler navigation. A prerequisite for the application of the doppler technique is the knowledge and mastery of numerous physical parameters both in water — the propagation medium — and onboard the vehicle itself, more particularly the parameters required for the correct setting of the gyroscope. This subject will be dealt with in the second part of this paper. The ability to interpret both in mathematical and statistical terms the processes involved in doppler measurement is essential if the accuracy of the course made good by a vessel employing doppler is to be predicted.

The Alpha doppler system is based on the acoustic doppler effect, and primarily measures the ship's speed *relative to the ground*, for speeds ranging between — 5 and 36 knots, with a resolution of 0.01 knot (5 mm/sec). The track can then be calculated on the basis of this speed. Determination of the track is independent of the following factors :

- | | | |
|--|---|------------------------------------|
| — water temperature | } | When using special Alpha Doppler |
| — salinity | | |
| — degree of water pollution | | |
| — water depths within the range 0.5 to 600 m | | |
| — settings or adjustments of equipment | | |
| — speed up to 36 knots | | |
| — trim of the ship to within $\pm 3^\circ$ | } | When using double beam arrangement |
| — pitch and roll movements | | |

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- other ships within the area of operation
 - land stations
 - currents
 - wind drift
 - environmental conditions (e.g. mountains, ships, or large buildings, affecting radio navigation)
 - ground structure, but with certain restrictions
- } When using any doppler technique

Speed is determined by assessing the difference in frequency between the transmitted and the received signal. In doppler measurement travel time does not have to be assessed, the doppler frequency being independent of water depth. A doppler frequency shift occurs during transition from one medium to another if there is relative movement between the two.

Thus, doppler frequency shifts may occur :

1. During transition from the transducer to the surrounding water;
2. In the water, if there are layers with different sound velocities;
3. During reflection at the bottom, if there is a current over the bottom.

These shifts taken together total the value that the Sonar Doppler measures. As long as bottom contact is possible, this value will in all cases be proportional to the speed made good over the ground.

The intensity of reflection is not important. Fluctuations can be compensated to a large extent by AGC amplifiers and filters. Measurement above a muddy bottom is, therefore, as feasible and as accurate as measurement above rocky ground.

Three factors influence the measurement of speed, these being the acoustic conditions, the finite travel time of sound, and the influence of sound velocity.

1. Acoustic conditions.

Undisturbed acoustic conditions for transmission and reception are a prerequisite for an accurate Doppler speed and track measurement. The flat shaped Alpha Doppler transducer lends itself to an ideal flush mounting in the ship's hull. Nevertheless the installation position must be carefully selected in fast ships in order to prevent acoustic disturbance from air bubbles. Where speeds exceed 30 knots, the hull of the vessel forward of the transducer array must meet certain hydrodynamic requirements as cavitation must not be allowed to occur in this area.

2. Finite travel time of sound.

Sound takes a certain time to travel from the ship to the sea bottom and back. If the ship is moving at very high speed it will have covered a certain distance in the time between transmission and reception.

However, there is only a minor shift in the radiation angle (below 1° at 50 knots) which is completely compensated by a double-beam arrangement.

3. Influence of sound velocity.

Works of reference usually give the doppler formula expressing the relation between doppler frequency and ship's speed as :

$$\Delta f = f_s \frac{2V}{c} \cos \alpha \tag{1}$$

Δf being the doppler frequency, f_s the transmitter frequency, V the ship's speed, c the sound velocity (dependent on temperature and salinity), and α the radiation angle. $c/f_s = \lambda =$ acoustic wavelength in water.

Formula (1) shows, *inter alia*, that doppler frequency is not only a function of the ship's speed, V , but also of the transmitter frequency, sound velocity and radiation angle. However what is required is merely the proportional relationship to the ship's speed.

The Alpha Doppler System, too, works on this principle. However, in this system due to the special geometry of the transducer the radiation angle α is itself a function of sound velocity (see figs. 1 and 2) :

$$\cos \alpha = \frac{\lambda}{3a} \tag{2}$$

a being the spacing between transducer elements. Substituting this last expression in (1) :

$$\Delta f = \frac{2V}{3a} \tag{3}$$

The doppler frequency is now only a function of the variable speed of the ship and a constant mechanical quantity a .

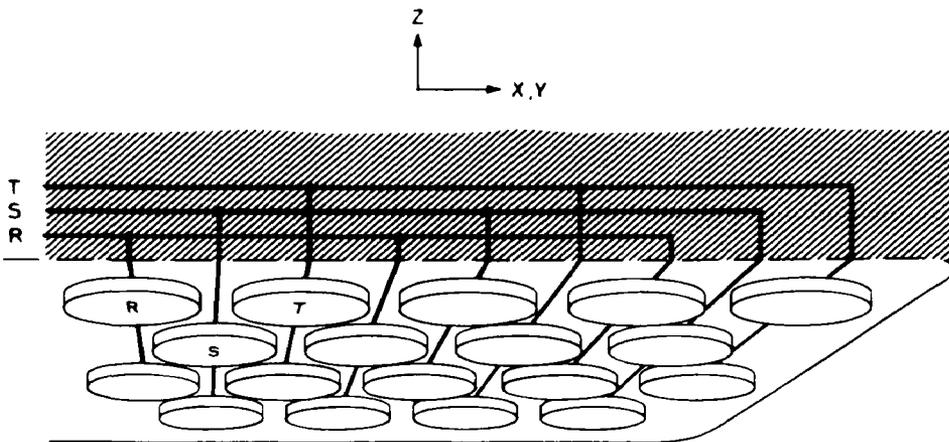


FIG. 1. — Schematic arrangement of a bottom mounted transducer array showing the cable leads.

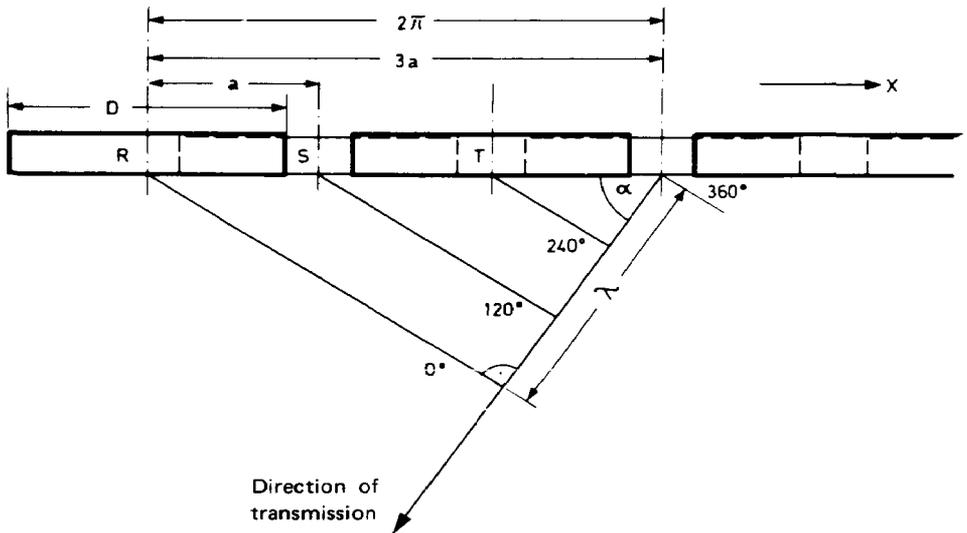


FIG. 2. — Three adjacent transducer elements showing geometry of transmission wave length λ . Radiation angle α .
 $\lambda = 3a \cos \alpha$

There are two pairs of transducer arrays, each pair set up in the two principal axes of the ship. Each array (see fig. 1) consists of 18 elements, and the fact that there are more transducer elements in one direction than in the other, results in two different beam widths, i.e. $\pm 3^\circ$ and $\pm 8^\circ$. In developing the transducer array a compromise has been made between the amount of transducer material and the fluctuation of the doppler values measured. The statistical features of the doppler signal are characteristically dependent on the width of the sound beams. The second part of this paper describes in detail how these features are reflected in the measurement results.

Compounds dissolved in the water affect sound velocity, but their influence on the doppler frequency can be compensated by automatic changes in the beam depression angle.

Particles floating in the water reflect sound, and this gives the erroneous impression that it is a doppler frequency induced by flowing water. However, the sound reflected by the water reaches the receiving transducer somewhat earlier than the sound reflected by the ground. Time separation is however feasible by adopting the pulse mode of operation.

A time filter, which adjusts automatically according to depth, separates the water echoes from the ground returns. If required it is therefore also possible to measure relative speed in relation to the water.

It has been proved statistically that fluctuations of the doppler signal in the pairs of beams are correlated. By means of special circuitry in the frequency evaluation circuit it is possible to obtain a large measure of compensation for equipment drift. During pulse operation such drift is less than 5 metres per hour.

In this connection attention should be drawn to an effect leading to disturbance. Sea-water is saturated with gas, in particular after a storm. When a ship is underway the water immediately around the hull is warmed by the ship, particularly in cold weather. With increasing warmth, the gas solubility decreases and the water begins to effervesce. This may induce loss of the sonar contact, and the drift rate will increase abruptly. Similar phenomena can sometimes be observed in calm waters of harbour basins and, at times, with the ship at anchor when methane gas from sewage rises from the bottom.

Loss of sonar signal due to ship-induced bubbles never occurs in the case of a stationary ship, at anchor or moored in flowing water, or if the vessel is moving very slowly in calm water. The very low relative speeds are sufficient to prevent the formation of air bubbles.

It is obvious that turbidity due to the ship's movement astern can also be a cause of interrupted sonar contact on account of the formation of a substantial amount of air bubbles. Precision measurements are in this case always subject to disturbance.

Doppler navigation is a dead reckoning method, the ship's position being calculated at any time from exact measurements of track length and direction since departure from a known starting point. The track length is derived from the measurement of speed relative to the ground.

The basic track length is the resultant of the track made good during a given time interval at speed v_x in the ship's longitudinal axis and at speed v_y in the transversal axis. The overall track and the corresponding position of the ship, P, can be computed by summing these two track elements vectorially.

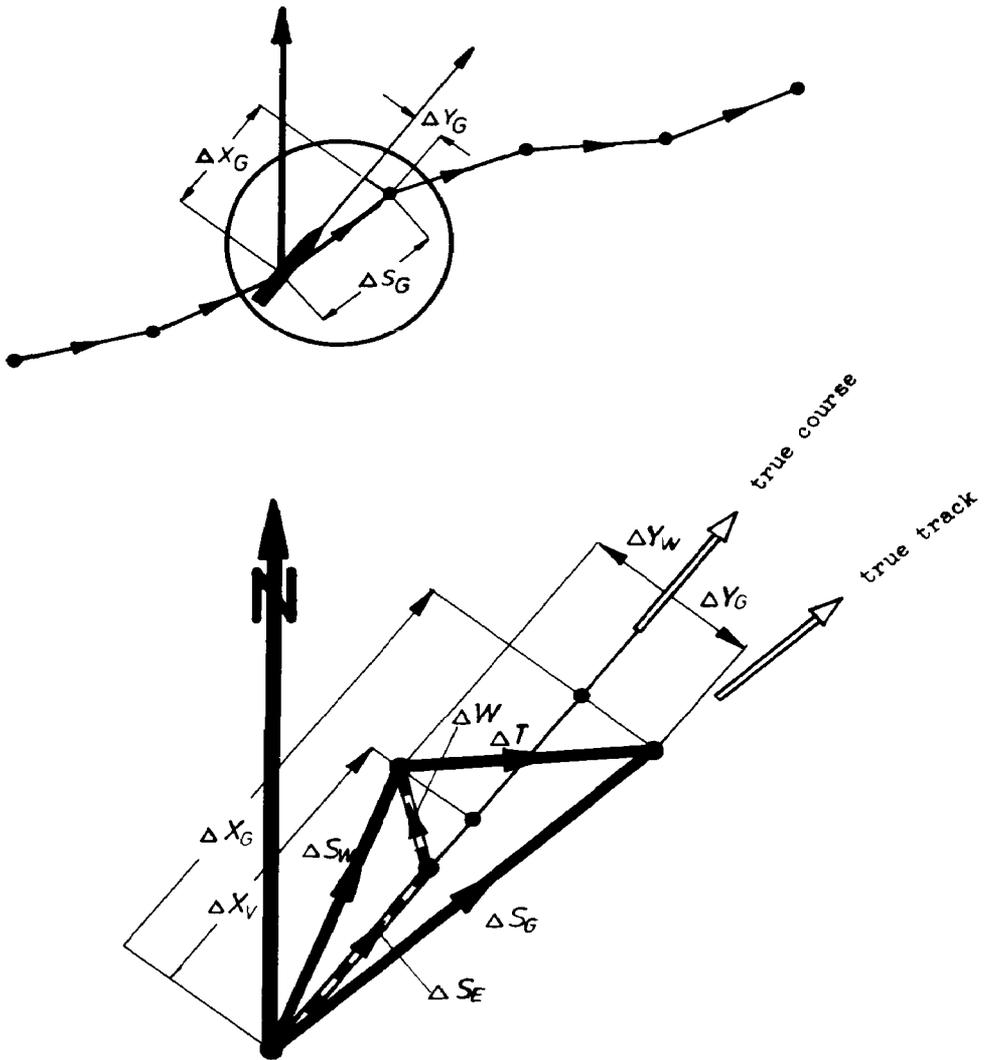
As is well known, doppler measurement also takes account of current and wind influences.

The track made good through the water, ΔS_w (see fig. 3) in a given time interval Δt is the vectorial sum of the ship's steered track ΔS_s , and the wind set ΔW . When the drift due to current ΔT , is added to ΔS_w vectorially, the track made good over the ground ΔS_g , is finally obtained.

The track made good can also be divided into two ship-referenced components by means of the true course indicated by the gyroscope, or supplied by a computer. These two components are the one in the ship's longitudinal axis, ΔX_g , and the other in its transversal axis, ΔY_g . Both are computed from the speeds measured by the doppler system.

In cases where water depths exceed 600 m and consequently no echo contact with the ground is possible, measurements can only be carried out by reflection against volume reverberation from the water. In this case the doppler system measures the relative tracks made good with respect to the water, ΔX_w and ΔY_w , and from these the resultant track element ΔS_w can be derived. This allows wind drift to be taken into account, but not drift due to currents.

The Alpha Doppler System uses four independent sound beams for the doppler measurements, with a fifth beam for depth measurements. Three frequency bands are used for separation — a band each for doppler



- ΔS_G Track made good over the ground
- ΔS_W Track made good through the water
- ΔT Drift (current)
- ΔW Wind set
- ΔS_E Ship's steered track
- ΔX_G Longitudinal track made good measured by doppler
- ΔY_G Transversal track made good measured by doppler
- ΔX_W Relative longitudinal track measured by doppler against water reverberations
- ΔY_W Relative transversal track measured against water reverberations

FIG. 3. — Composition of the course vectors.

measurement in the ship's two principal axes and a third for independent depth measurement.

The transmissions are time-locked in order to maintain separation of the information in each pair of beams. A distinction is made in terms of time between :

Main pulse	ahead — starboard
Main pulse	astern — port
Reverberating echo	ahead — starboard
Ground return	ahead — starboard
Reverberating echo	astern — port
Ground return	astern — port

The pulse time pattern may be changed, depending on whether the reverberating echo or the ground return is used, in order to obtain optimum measuring conditions for each case.

To this effect both manual and automatic selection of the operating mode have been incorporated. The duration of the main pulse can be adjusted according to the depth. The product of pulse length and repetition frequency is, however, maintained constant.

For cost reasons the transmitter circuitry has been simplified so that it is only possible to transmit one beam at a time in each main axis. However, the two doppler measuring beams and the depth beam can be received simultaneously. In this case, the channels for the doppler beams are separated according to direction and time, and the depth beam according to direction, time and frequency.

Either of two modes of operation may be selected :

Ground track (the normal mode).

Two ultrasonic beams, one pointing forward and the other astern, are transmitted to the ground, one shortly after the other. As the ship moves, the ground-reflected sound is received with a frequency shifted by the doppler frequency. The generator is clock-controlled so that the main pulse stops as soon as the return echo is received.

Water track.

In this mode of operation the ship's speed is measured relative to the water. A special window control for time ensures that only the echoes from the water mass (volume reverberation) are used for doppler measurement, whatever the depth.

This mode of operation is of particular value in the following cases :

— The depth of the water is such that reception of the ground returns is impossible.

— When the ship's engines are to be checked or adjusted, the speed is then required to be measured in relation to the water, and a ground return is deliberately avoided.

APPLICATIONS

The Atlas Alpha Doppler System has been developed as a modular system; four of the modules being as follows :

— **The Atlas Alpha Doppler Log** measures a ship's speed in the longitudinal axis only. The track made good is determined by a single integration of speed. The Atlas Survey System Susy may be connected to this log system to obtain more accurate readings.

— **The Atlas Alpha Doppler System** for satellite navigation measures the *longitudinal and transversal components of a vessel's speed*. The measured values are fed into the ship's satellite navigation system, are processed, and provide more accurate positioning, the ship's speed being an important input when making satellite fixes.

— **The Atlas Alpha Doppler Docking System** is used for monitoring the very lowest speeds or smallest movements of a vessel, in particular during docking operations. For this purpose three speed components are measured — the longitudinal speed, the transversal speed at the bow, and the transversal speed at the stern. The last digit of the indicator reads in hundredths of a knot.

— **The Atlas Alpha Doppler Navigator** is the most comprehensive module in the Alpha Doppler System, being able to accomplish all the navigational functions of a vessel. It measures both components of the ship's speed.

All other navigational data are determined by the computer, utilizing compass information, position and correction data, as well as constants. For this purpose the computer carries out the following functions :

— Interrogation of the counters in order to store the longitudinal and transversal track elements for one position-computing cycle.

— Computation of the drift angle from the longitudinal and transversal speed values. Output of this angle to an indicator and, optionally, to a tape puncher or teleprinter.

— Interrogation of the gyro to obtain the steered track angle for one position-computing cycle.

— Computation of the track made good over the bottom, and output of the distance run to an indicator, tape puncher or teleprinter.

— Computation of the gyro error from ship's speed, gyro track angle and the latitude of the ship's position.

— Computation of the true track angle from the gyro steered track angle, gyro error and ship's drift. Output of the true track angle to an indicator, tape puncher or teleprinter.

— Computation of the elements of the track made good in the North-South/East-West geodetic reference system from the elements of the ship's track made good in the longitudinal and transversal directions and from the true track angle.

— Integration of the elements of the track made good into the geodetic reference system.

— Computation of the ship's actual position in latitude and longitude taking account of the reference ellipsoid and the initial coordinates. Output to an indicator, tape puncher, or teleprinter.

— Computation on a plotter of the rectangular coordinates for the ship's actual position in the projection used, taking into account scale, the reference coordinates for the plotter and those for the projection. Output of these coordinates to the plotter.

— Interrogation of the digital clock and output of the time.

All the reference and correction values, as well as all constants — the scale factor of the chart, the ship's initial position at the start of a voyage, and comparative values from other navigational aids, for example — necessary for these computations may be fed into the computer via the teleprinter.

Most of the information required for the solution of the undermentioned problems is therefore available in this system.

— Storage of target coordinates. Computation of the most suitable course to the destination point, and the automatic holding of this course with an automatic pilot. Continuous monitoring of the course and, if necessary, its correction.

— Integration of the doppler navigation system with other navigation systems, for example with Hi-Fix, or satellite or inertial navigation systems, in order to increase accuracy, by using the data obtained from these systems to the best advantage.

— Prevention of collisions, by determining the courses and speeds of vessels in the vicinity, and thus forecasting the time and distance of the closest point of approach (CPA).

— Coordination of position and depth values in the case of surveys whose object is computation of depth contours (two-dimensional surveying).

TEST RESULTS AND ACCURACY OF POSITIONING

This part of the article is an extract from a comprehensive report on tests of the Atlas Alpha Doppler Navigator system for accuracy of positioning. The general experience gained from the tests carried out between November 1971 and March 1972 will be described. The problem of navigational accuracy will also be considered, and the results will be given.

A major problem in testing a navigational system is the determination of the accuracy of position finding under the prevailing environmental conditions. This is a problem that obviously cannot be solved in the laboratory; the system has to be operated in the field. This requires a considerable amount of time and money because with the doppler principle — a dead reckoning technique — numerous measurements have to be carried out often over long distances and with the complete system

operating in two dimensions. This is the only way in which one can be assured of acquiring valid comparative statistics for positional errors. In this connection uncertainty in the determination of the course by gyro compass — an uncertainty which is mainly due to the vessel's acceleration — itself proves to be a great problem.

To provide a reference system for the doppler system tests a Hi-Fix chain was available, thus permitting almost continuous comparative determinations of position.

Another problem during accuracy measurements results from the various environmental conditions each of which can lead to errors affecting the doppler navigation system. Figure 4 lists these influences and also gives information regarding the effect of these errors and the possibilities for their elimination.

It is important to realize that only a part of the statistical speed and course error is due to external effects, what remains of these two errors forms the true positional error of the system. All the other errors can be assessed and eliminated by either physical measurements or mathematic computation. The tests took into account most of the existing environmental conditions.

Only those test data of general interest will be given. They illustrate the scope of the tests and the efficiency of the system.

Test period : November 1971 to March 1972
 Test area : Western Baltic Sea, Skagerak
 Test vessel : *Hans Bürkner*
 E 71, Eckernförde

Test runs to be evaluated	Number of runs
Of less than 20 n.m.	5
Of 20-40 n.m.	7
Of 40-60 n.m.	10
Of 60-80 n.m.	9
Of 80-110 n.m.	3
Of more than 110 n.m.	1
	—
	TOTAL : 35
Total distance of test runs :	2000 n.m. approx.
Total distance covered :	4000 n.m. approx.
Total duration of operations :	> 700 hours
Speed range :	— 5 23 knots
Effective acceleration of vessel :	Translation : 0 $6.5 \cdot 10^{-3} g$ Rotation : 0 $1.5 \cdot 10^{-2} g$
Maximum depths (measured in the Skagerak) :	500 600 m (depending on bottom topography)
Environmental conditions :	Wind velocity : 0 11 knots Sea state : 0 7 (without system failure) Currents : 0 2.5 knots Temperatures : — 1 + 14 °C

	<i>Influence factors (measurement parameters)</i>	<i>Range of measure- ments during test period</i>	<i>Effect of errors</i>	<i>Error elimination</i>
Installation influences	Installation		Error of reference Constant course error and the speed factor	Errors can be measured and eliminated by computer
Vessel's motion	Speed	- 5 ... + 23 kt	Inclinations are dependent on ship type and speed error ----- Bubble effects interfere with Doppler spectrum ; statistical error	Vessel's inclination eliminated by computer ----- Partly eliminated by com- puter or electronic unit Residual statistical error
	Effective acceleration	Translation : 0 ... 6.5 . 10 ⁻³ g Rotation : 0 ... 1.5 . 10 ⁻² g	Inclination of vessel and speed errors ----- Course error dependent on acceleration intensity. Duration and repetition dependent on gyro type	Vessel's inclination eliminated by computer ----- Partly eliminated by computer. Partly residual statistical dependent on gyro type
Environmental influences	Wind Sea Current	0 ... 11 kt 0 ... 7 0 ... 2.5 kt	Static and dynamic vessel inclinations ; speed error ----- Compass oscillations, gimbal error	Vessel's inclination eliminated by computer ----- Vessel inclination
	Temperature Salinity Pressure	- 1° ... + 14°C	Change of sound velocity and Doppler frequency ; speed error	Eliminated by automatic radiation angle correction ; Therefore α -Doppler
	Air Impurities		Interference of Doppler spectrum, possibly echo failure ; statistical speed error	Error eliminated partly by computer and partly by electronic unit. Residual statistical speed error
	Depth Continuous wave Pulse operation I } Pulse operation II } Ground and water return	4 ... 600 m	Ground return disturbance by reverberation ----- Weak echo amplitudes, ampl. disturbed by reverberation, current error ; statistical speed error	Error eliminated partly by computer and partly by electronic unit. Residual statistical speed error
	Depth change per unit of time		May influence Doppler spectrum (should not normally occur), echo disturbed by depth readjustment ; statistical speed error	Error eliminated partly by computer, partly by electronic unit. Residual statistical speed error
	Nature of the ground		Disturbance of the Doppler spectrum by inhomogeneities (material & structure) ; statistical speed error	Error eliminated partly by computer, partly by electronic unit. Residual statistical speed error

Fig. 4. — External influences affecting the accuracy of position determination by sonar doppler.

Table of measurement parameters to be taken into consideration.

Nature of seabed :

Sandy (in western Baltic)
Rocky (in the Skagerak)

Different methods were used for fixing reference positions as follows :

- (a) bearings of the following landmarks - Eckernförde pier, beacons marking the Langhöft two mile measured distance, and Kiel Light-house;
- (b) bearings of seamarks including various buoys and Fehmarn light vessel;
- (c) using the Eckernförder Bucht-Fehrman-Olpenitz Hi-Fix chain;
- (d) using dead reckoning when speed and course were constant.

During the test phase the doppler speed sensors operated without any hardware failure. Fortunately, no echo disturbances, such as air bubbles, were experienced up to the maximum test speed of 23 knots. However, during the vessel's movements astern there were echo failures of fairly long duration due to air bubbles. These resulted in measurable positional errors. At present this problem can only be solved by additional updating of positions used for position correction. To what extent air bubbles on the ship's bottom can be prevented from coming into contact with the surface of the transducer by hydrodynamic measures remains to be investigated.

Before discussing the results of these tests the possible errors of the doppler navigation system in general must briefly be mentioned.

Basically, three types of errors can be distinguished :

- a) Systematic static errors
- b) Systematic dynamic errors
- c) Statistical errors.

Permanent statistical errors are in fact the limiting factor for positional accuracy.

When possible the systematic static errors of the speed sensors, which are due to tolerances accepted by the transducers themselves and their installation, should be determined by taking several measurements with each system as soon as it is installed. These errors can then be eliminated in the computer program by corresponding constant correction factors for the speeds.

The systematic dynamic errors of speed are caused by deviations of the transducers from the horizontal position as a result of pitching or rolling or the trim of the vessel. Considerable reduction of error is achieved by using a double-beam configuration. Figure 5 shows which speed errors remain despite the use of this configuration. At pitching amplitudes of $< 3^\circ$, for example, a negative relative longitudinal track error of $< 0.1\%$ will occur. For the major part of the trials this condition was satisfied, i.e. these inclination errors were negligible. If the doppler system is to operate accurately at even greater deviations from the horizontal the vessel's inclinations will have to be measured, and the speed measurements corrected accordingly.

Among the statistical speed errors which cannot be determined by measurements are the doppler fluctuation errors caused by the finite beam width for emission and reception of sonar energy — these errors have

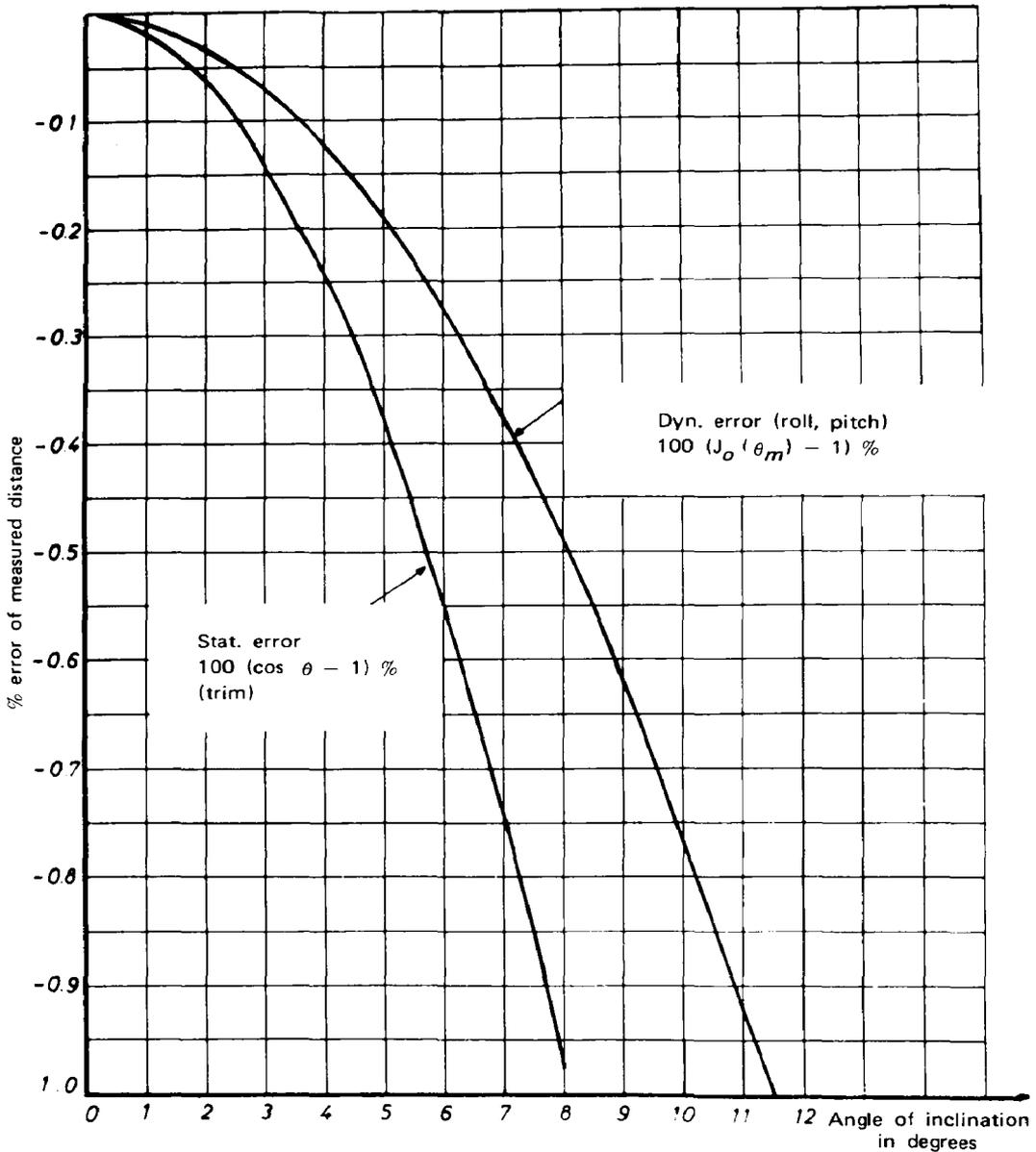


FIG. 5. — Inclination errors for the Alpha Doppler Sonar beam arrangement.

very short correlation periods — added to which there is the effect of the structure and nature of the seabed, the depth variations and impurities in the water, for which the correlation periods may be short or long.

Track errors due to statistical speed errors show a certain smoothing which increases with the distance covered. This implies that absolute statistical track errors do not increase in proportion to the distance covered, but approximately in proportion to the square root of that distance.

These static, dynamic and statistical speed errors, taken together,

form position errors whose magnitude is dependent on the distance covered, which is usually substantially greater in the longitudinal direction than in the transversal direction. The effect of the transverse speed errors, although also depending on the distance run, is thus smaller, being approximately proportional to the arc subtended by the angle of the vessel's drift. This angle of drift is given by the ratio between the transversal and the longitudinal speeds (in this case the angle of drift is the angle between the true course and the true track).

The systematic static error of the course sensor depends largely upon the installation's tolerances. The magnitude of this error depends on the deviation between the gyro compass azimuth reference and the reference axis of the longitudinal speed sensor. This error should also be determined, if possible, by making several test runs with each system after installation. In order to obtain accurate position calculations it will then be necessary to feed the correction to be applied to the course sensor into the computer.

A commercial gyro compass (Anschütz Standard IV type) with liquid damping is utilized as course sensor in the doppler system. The values transmitted from the gyroscopic compass to the computer have an accuracy of better than 0.1° . The static compass error amounts to only $\leq 0.1^\circ$.

Since it is possible to calculate the so-called speed error with sufficient accuracy, it is the dynamic errors caused by the vessel's acceleration which constitute the major problem in course determination.

Liquid damping is provided for the gyroscopic sphere which is suspended like a pendulum so that it may freely accede to the northerly directional moment. However, accelerations of the vessel impart precession moments causing the gyroscope sphere to oscillate.

The phenomenon for a vessel accelerating from 0 to 30 kt in 5 minutes in a northerly direction has been described in technical literature elsewhere. Figure 6 is a teleprinter printout for a fictitious example of this phenomenon.

The maximum amplitude of the resulting azimuth deviation amounts to approximately 1.13° . Even with a constant northerly course there would be a consequent error of position. Although the angular error diminishes after some hours to zero, the total position error (resulting from the integration of all the errors) remains of considerable magnitude. In practice horizontal accelerations in north-south and east-west directions are generally a fairly continual occurrence during manoeuvring operations. As a result there are coupled oscillations in the gyroscopic sphere that are no longer controllable, and accordingly errors in azimuth, and thus position, are produced.

Most acceleration oscillations can only be avoided by using other methods of suspension for the compass — for example gyroscopes suspended by their centre of gravity. The horizontal stabilization and orientation to north of the gyroscope is then effected by accelerometers via electronic control-loops having special low-pass characteristics. There already exist such compass systems entailing very small azimuth errors

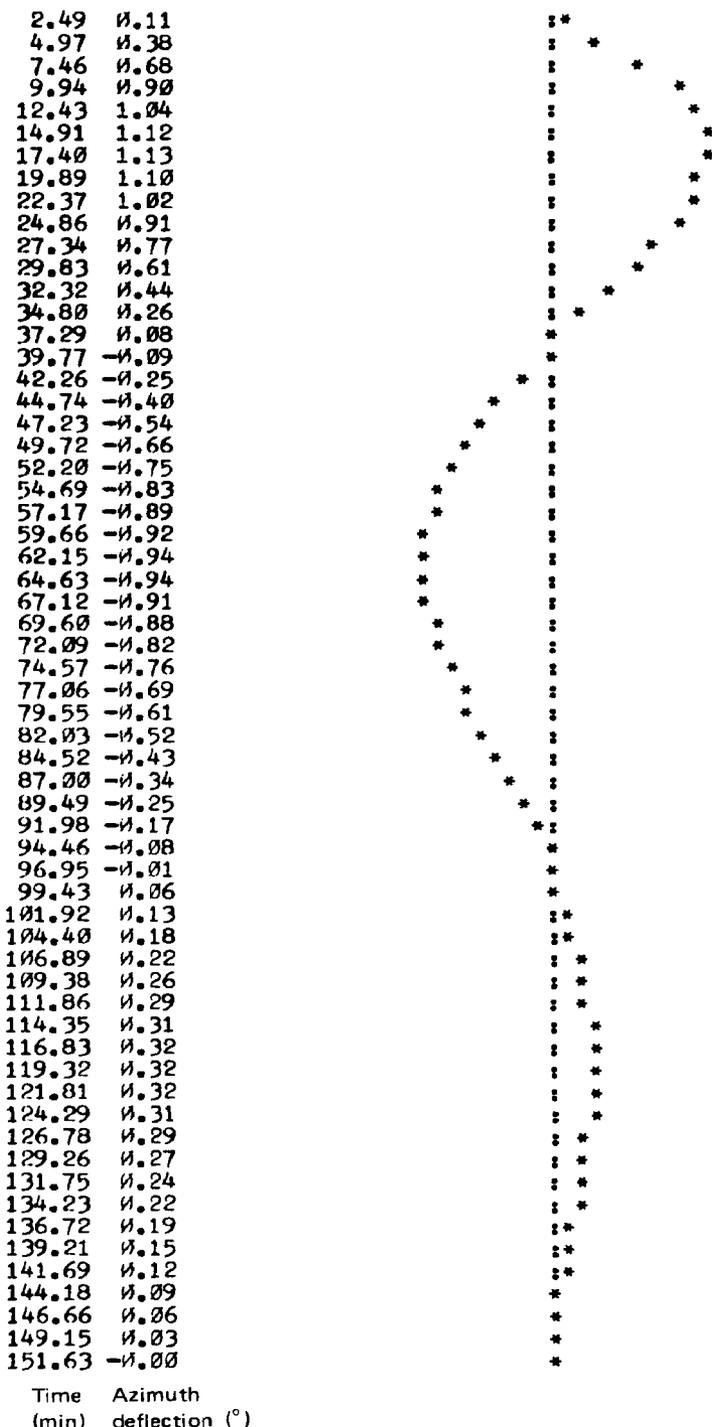


FIG. 6. — Simulation of azimuth oscillation in the gyroscopic sphere of an Anschütz Mark IV gyro compass for vessel acceleration from 0-30 kt in 5 min in a northerly direction.

even under extreme acceleration conditions, but they are very expensive and at present they can hardly have commercial applications.

In order to reduce considerably the effects of such azimuth errors on position determination another method was successfully tried. A computer program based on a mathematical compass model was prepared for use in the processing computer used in the sonar doppler navigator. The oscillations of the gyroscopic sphere and accordingly its azimuth error were thus calculated continuously during the tests by means of this program. This was also done for the vessel's accelerations which can be derived from the doppler measurements. The accuracy of this azimuth error calculation naturally depends on the accuracy of the mathematic gyroscopic model used. Although only an incomplete mathematical model has hitherto been employed, the success obtained — in the form of considerable increase in positional accuracy — should encourage the completion of this mathematic model. The advantage of this method essentially lies in the fact that for a reduction of the azimuth error by, say, a factor of 5 the position error depending on it is reduced in the same proportion. Values of this order were obtained during the tests. The important fact is that the influence of each acceleration disappears asymptotically in the course of time both in the mathematical model calculations and by damping in the gyroscope itself.

The measurements carried out with the doppler system were examined for short term fluctuations of the sensor, for long term track errors, and for overall errors. Short term fluctuations of the speed sensor, for example, are revealed by unsteadiness in the speed indications.

Uncorrected sensor data, recorded every 1.6 sec on the punched tape during some of the test runs were used for this investigation. The resolution of the speed data amounted to 0.01 kt, and that of the course data to 1.3'. It was decided to evaluate only those sections with course and speed conditions as constant as possible, and speed and course variations were thus largely avoided.

Figures 7, 8 and 9 show the results of these statistical investigations in the form of frequency distributions as determined by the computer. In addition, both the arithmetic means and the standard deviations for the various measured quantities were determined. The frequency distributions reveal clearly noticeable maxima for the speed and course values that in all probability existed in fact.

Figures 7 and 8 show longitudinal and transversal speed values respectively. The measurement interval for the distribution was 0.02 kt. The effects of the doppler fluctuation are included in the short-term fluctuations, but in this case undesired, though small, true acceleration processes have a minor effect.

The transversal speed is subject to additional fluctuations as a result of course unsteadiness, i.e. if the vessel yaws about its vertical axis, as this does not necessarily pass through the point at which the transducer is installed. It can be clearly seen that the fluctuation of the transversal speed values is almost twice as large as that of the longitudinal values. In the example given, the short-term fluctuations in longitudinal and

1.510	0		
1.512	0		
1.514	0		
1.516	0		
1.518	0		
1.520	0		
1.522	1 +		
1.524	0		
1.526	0		
1.528	1 +		
1.530	2 ++		
1.532	1 +		
1.534	3 +++		
1.536	5 +++++		
1.538	12 ++++++		
1.540	12 ++++++		
1.542	6 +++++		
1.544	11 ++++++		
1.546	21 ++++++		
1.548	28 ++++++		
1.550	20 ++++++		
1.552	35 ++++++		
1.554	21 ++++++		
1.556	28 ++++++		
1.558	25 ++++++		
1.560	25 ++++++		
1.562	15 ++++++		
1.564	12 ++++++		
1.566	10 ++++++		
1.568	10 ++++++		
1.570	4 +++++	Date of voyage	1 March 1972
1.572	6 +++++	Record No.	70...86
1.574	4 +++++	V_{Im}	15.54 knots
1.576	1 +	σ_{vI}	0.10 kt
1.578	1 +	Measurement	
1.580	0	period	8.7 min
1.582	0	Number of values	
1.584	0	measured	320
1.586	0		
1.588	0		
Longi- tudinal speed in knots (1/10)	Number of values within the speed interval		

FIG. 7. — Statistical short-period evaluation of the longitudinal speed. (Frequency distribution).

transversal speed are respectively 0.65 % and 1.15 % in relation to the longitudinal speed. Figure 9 shows short term fluctuations in course (track angle) values. The interval chosen for the frequency distribution is 0.05°. It is mainly the actual unsteadiness of the vessel on its course that is here reflected.

Hi-Fix positions were used to test route accuracy. In order to be able to differentiate between longitudinal and transverse errors, and also

-0.062	0		
-0.060	2 ++		
-0.058	0		
-0.056	0		
-0.054	0		
-0.052	1 +		
-0.050	1 +		
-0.048	1 +		
-0.046	3 +++		
-0.044	0		
-0.042	4 ++++		
-0.040	3 +++		
-0.038	2 ++		
-0.036	2 ++		
-0.034	2 ++		
-0.032	9 ++++++++		
-0.030	11 ++++++++		
-0.028	6 ++++++		
-0.026	8 ++++++++		
-0.024	5 ++++++		
-0.022	13 ++++++++		
-0.020	9 ++++++++		
-0.018	12 ++++++++		
-0.016	9 ++++++++		
-0.014	10 ++++++++		
-0.012	12 ++++++++		
-0.010	10 ++++++++		
-0.008	18 ++++++++		
-0.006	16 ++++++++		
-0.004	16 ++++++++		
-0.002	11 ++++++++		
0.000	18 ++++++++		
0.002	18 ++++++++		
0.004	6 ++++++		
0.006	7 ++++++		
0.008	9 ++++++++		
0.010	8 ++++++++		
0.012	8 ++++++++		
0.014	13 ++++++++		
0.016	6 ++++++		
0.018	6 ++++++		
0.020	8 ++++++++		
0.022	3 +++		
0.024	6 ++++++		
0.026	1 +		
0.028	0		
0.030	4 ++++		
0.032	1 +		
0.034	0		
0.036	0		
0.038	0		
0.040	2 ++		
0.042	0		
Transver- sal speed in knots (1/10)	Number of values within the speed interval	Date of voyage	1 March 1972
		Record No.	70 . . . 86
		V_{qm}	- 0.06 kt
		σ_{vq}	0.18 kt
		Measurement period	8.7 min
		Number of values measured	320
		(V_{Im})	15.54 knots)

FIG. 8. — Statistical short-period evaluation of the transversal speed.
(Frequency distribution).

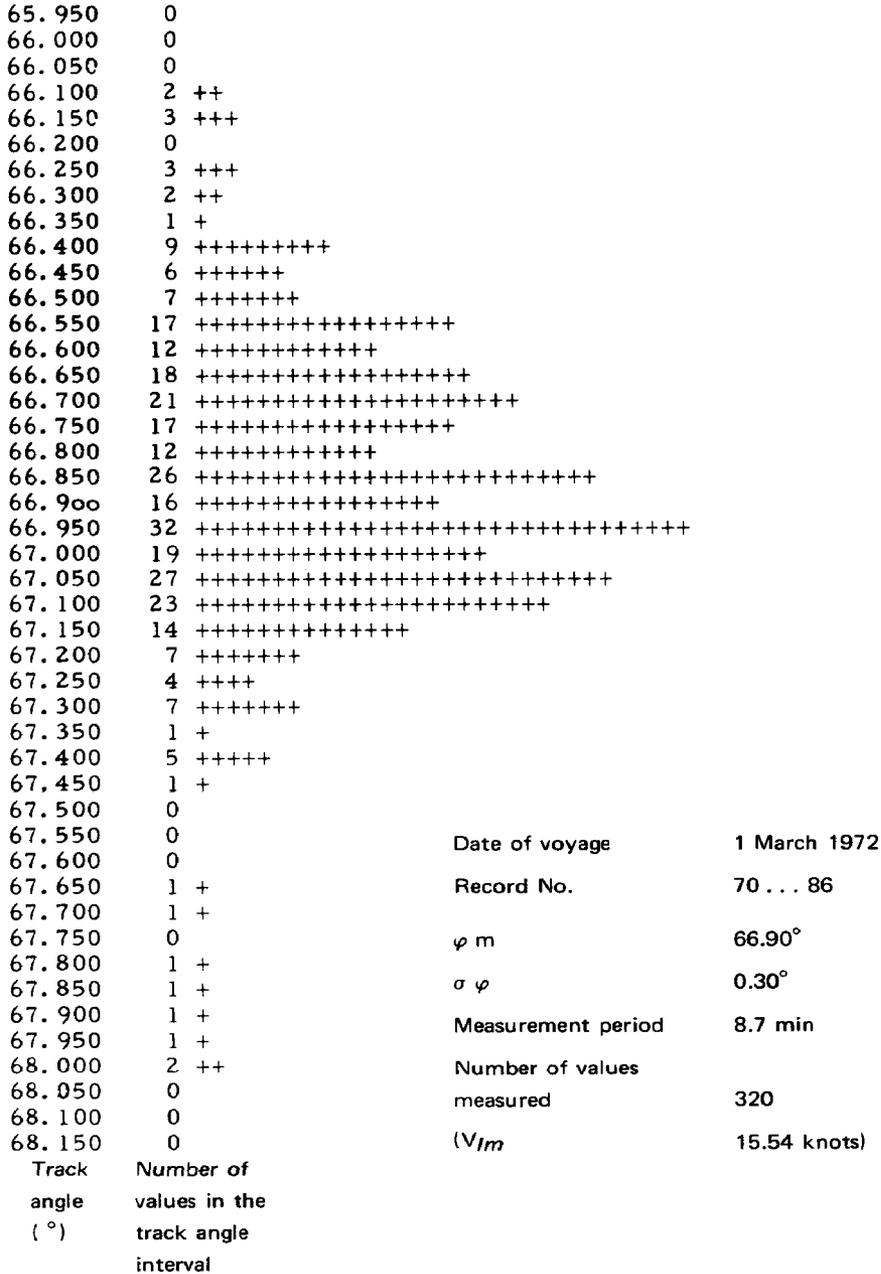


FIG. 9. — Statistical short-period evaluation of the track angle. (Frequency distribution).

between the systematic and the statistical errors, as well as to determine the dependence of all errors on the distance covered, sections of route with course and speed as constant as possible were utilized for those tests.

Judging the reliability of the Hi-Fix measurements proved a difficult matter. It was most important to avoid including in the evaluations either any errors due to the Hi-Fix itself — as these would give the impression of jumps in position — or any Hi-Fix values from unfavourable areas,

i.e. in coastal regions, or where the angles of intersection of the hyperbolae are small. As the Hi-Fix values were used for comparison purposes, a great deal of importance was attached to this matter.

There was a further means of comparison. The vessel proceeded successively along most of the sections of route on a constant course and with engine power also constant. As external effects, such as wind and current, were slight the vessel was able in fact to run at a fairly constant ground speed.

In principle, it cannot be expected that in the doppler measuring system (which is almost without inertia) the statistical position errors will compensate for the small variations of speed invariably occurring in successive route sections to the extent that the resulting speed value will appear more constant than it really is.

As these measurements were carried out with constant time intervals, the route sections should also be constant in length and taken at a constant speed.

All the results obtained were analyzed to determine the fluctuations in a series of successive route sections measured with the doppler and the Hi-Fix systems.

Figures 10, 11 and 12 show some of the results of this analysis.

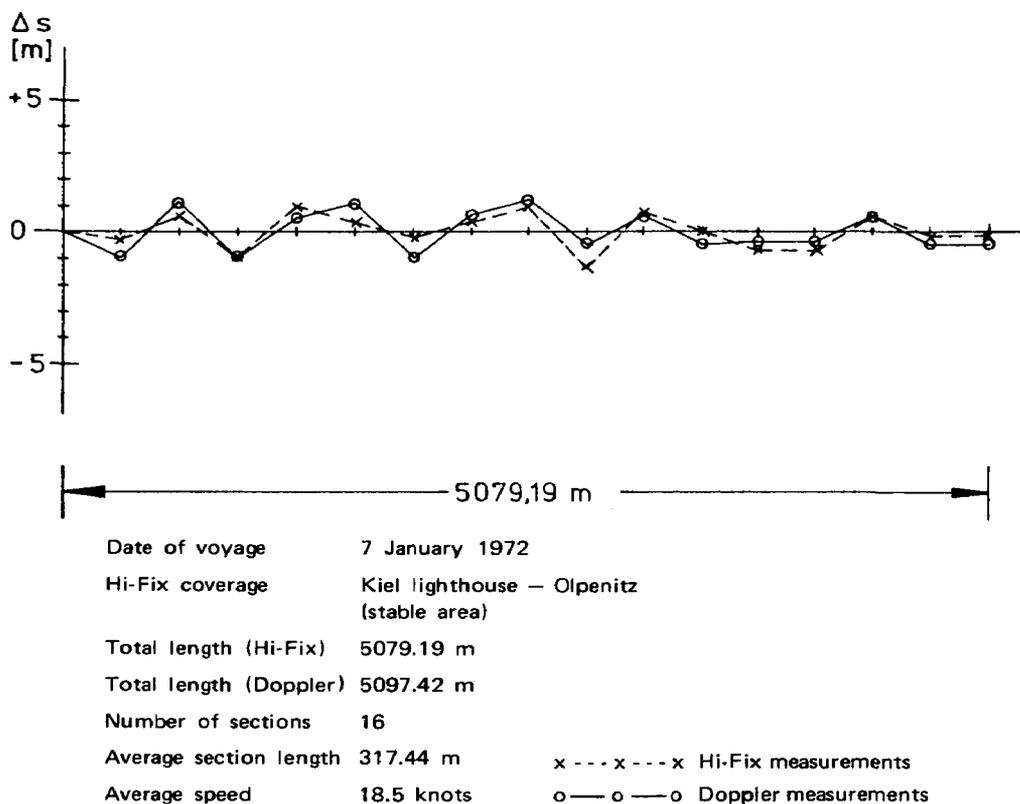


FIG. 10. — Fluctuations of Hi-Fix and doppler measurements along track sections using stable Hi-Fix values.

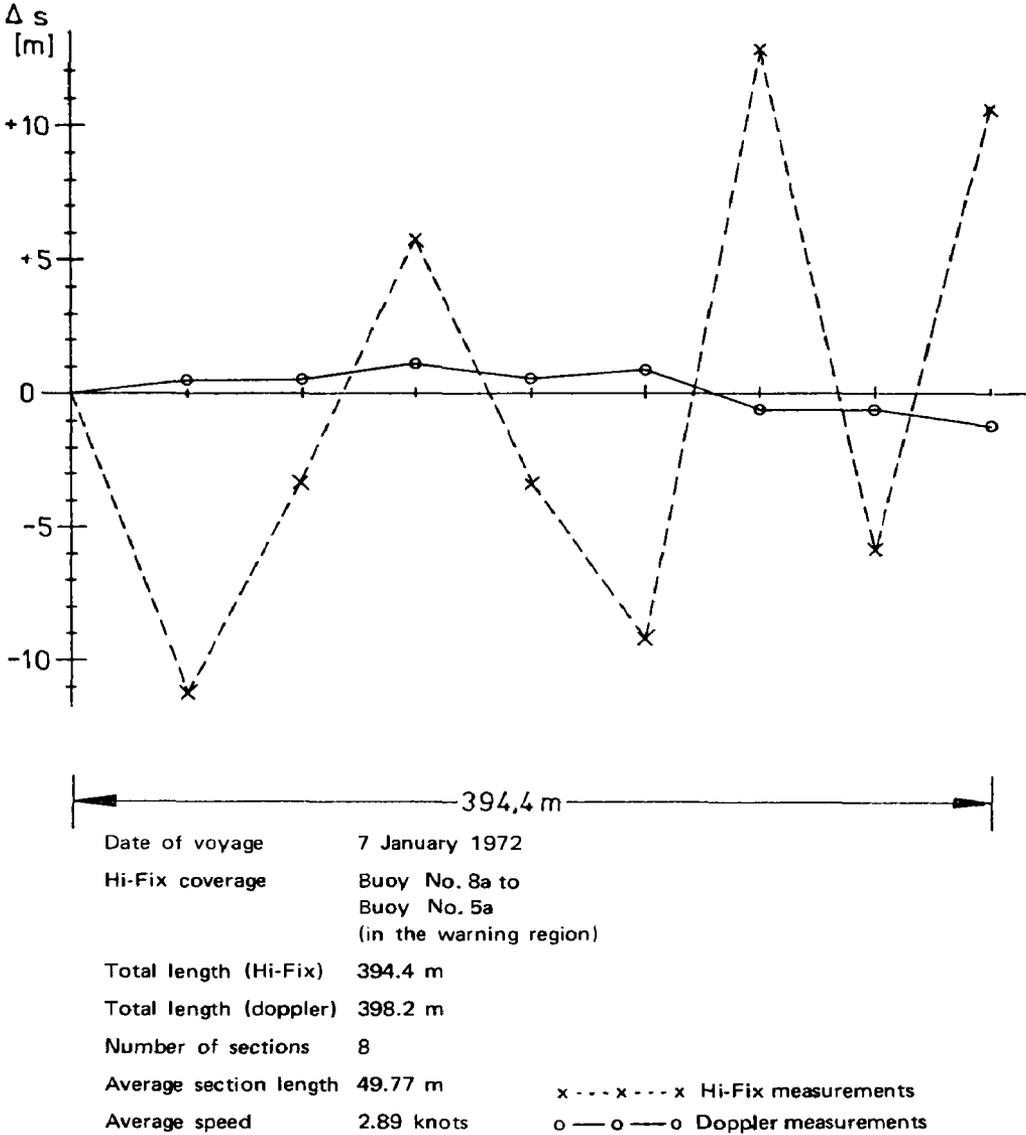


Fig. 11. — Fluctuations of Hi-Fix and doppler measurements along track sections using unstable Hi-Fix values.

In the examples given, the overall lengths of route were determined from their series of measured sections. Assuming a constant speed for each overall route length, each of these was divided by its number of sections to obtain an average length for its sections. The values determined from the Hi-Fix data were plotted on the abscissae in metres. The ordinates represent the deviations of the Hi-Fix and the doppler lengths for each successive route section and are also given in metres.

Figure 10 shows the results of measurements with stable Hi-Fix values. The measurements were made over a route from Kiel Lighthouse to the No. 3 Olpenitz buoy. The fluctuations in the values obtained with Hi-Fix and with doppler were negligible, being only about ± 1 m (± 0.3 %).

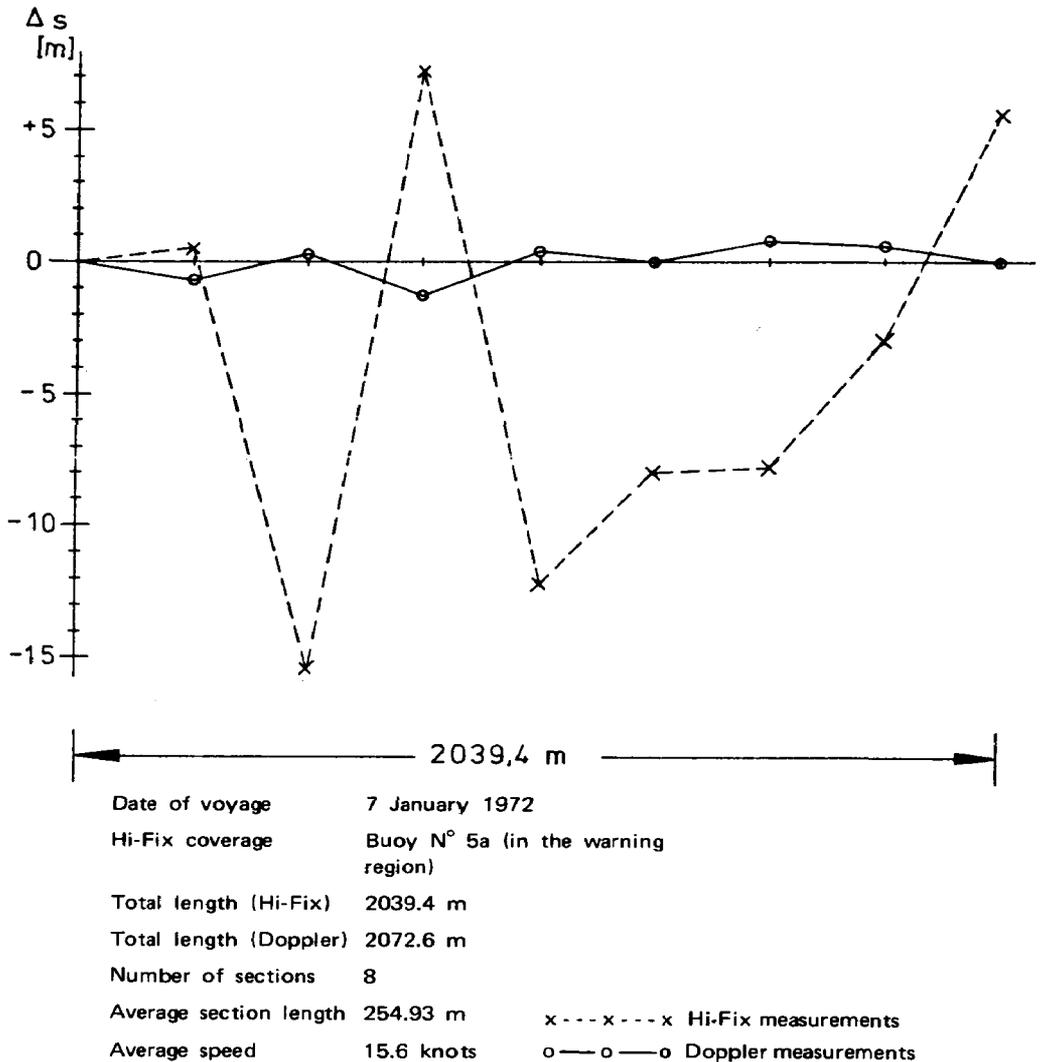


FIG. 12. — Fluctuations of Hi-Fix and doppler measurements along track sections using unstable Hi-Fix values.

What is interesting about this example is that it showed that there is a strong and obvious correlation between the doppler and the Hi-Fix measurements, indicating the fluctuations of true speed which occurred over the overall length of the route, and in consequence the actual fluctuations in measurements which remain when this has been allowed for were considerably smaller.

Figures 11 and 12 show examples of unstable Hi-Fix values. The measurements were taken over the route from Buoy No. 8a to Eckernförde — that is, well within the Eckernförder Bucht. The fluctuations in Hi-Fix values were clearly larger than those of the doppler values which were once more in the range of ± 1 m. No correlation exists between the deviation values in the two systems. Hi-Fix measurements of this kind cannot be used as comparative data, and were not used for the analysis.

A computer program was used to calculate the length of each section between the starting and end points of the route and their north-referenced course, in order to determine the route errors from the doppler and Hi-Fix positions.

Then the differences between the doppler and the Hi-Fix route lengths (hereinafter called "longitudinal route differences") were determined, together with the course differences. The "transversal route differences" could then be calculated from the course differences by being referenced to the Hi-Fix route length. Systematic and statistical components were determined from the longitudinal and transversal differences for route sections with approximately constant length. Finally, the mean arithmetic errors and the mean square errors were calculated as a function of the length of the section of route. About 460 sections of route were analysed from three typical test runs which it should be noted included manoeuvres involving great changes in course as well as of speed (up to ± 22 knots). This fact is of importance, since the acceleration error of the gyro compass has larger effects under such conditions.

The longest route sections with constant course and speed were of 5 nautical miles, and a number of them were monitored with the Hi-Fix system and then analysed. It was established that most of the course errors that had been assumed to be statistical, resulting from gyro compass acceleration errors over successive sections of the route, are in fact correlated. This fact must be taken into consideration when analyzing the results.

Figures 13 to 16 show the results, and take account of the errors for both the doppler system and the Hi-Fix system.

Figure 13 gives information about the characteristics of the systematic longitudinal error which to begin with had been deliberately left uncorrected. With increasing distance, the error increases in a nearly linear way. The relative systematic error for the route is approximately $+0.3\%$. On the basis of these measurements a correction factor of 0.97 was fed into the computer in order to eliminate this systematic error of the system.

Figure 14 demonstrates the standard deviation of the overall longitudinal route error, that is to say, the statistical error.

The dashed line indicates approximately the curve of the theoretical function. What is important is that the increase in the accidental error for the longitudinal route is not proportional to the distance covered but only to the square root of this distance. According to this figure the relative standard deviation is approximately of ± 1 to $\pm 0.8\%$ over short distances, ± 0.4 to $\pm 0.3\%$ over medium distances and $\pm 0.18\%$ over the longest section of route. The relative error would decrease even further over longer distances.

In those sections of route of up to 5 nautical miles in length the curve flattens out and this demonstrates that the errors occurring in the successive sections are remarkably independent from a statistical point of view.

The curve that may be theoretically expected will not pass through the origin of coordinates, but will cut the axis of the ordinate at a point

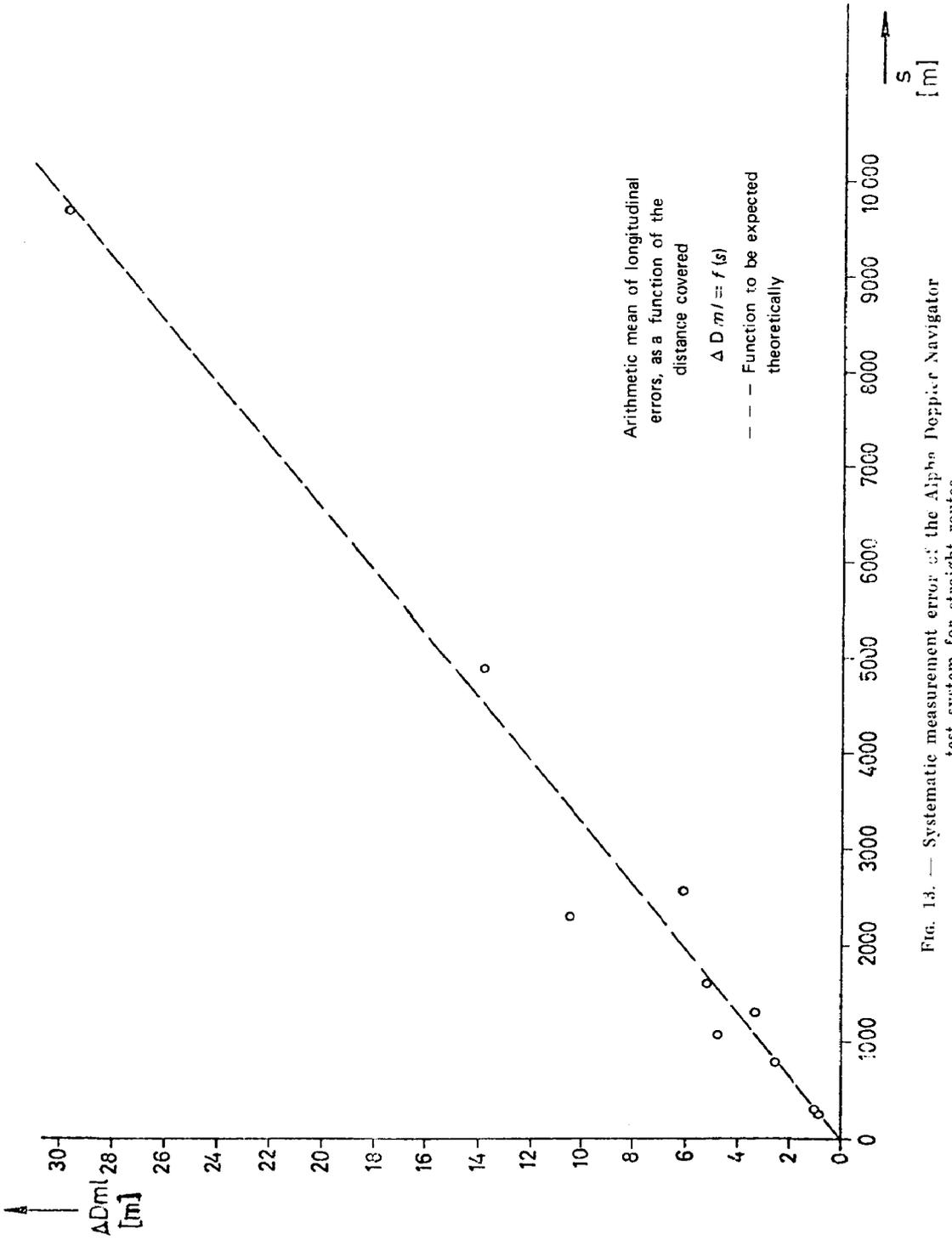


FIG. 13. — Systematic measurement error of the Alpha Doppler Navigator test system for straight routes.

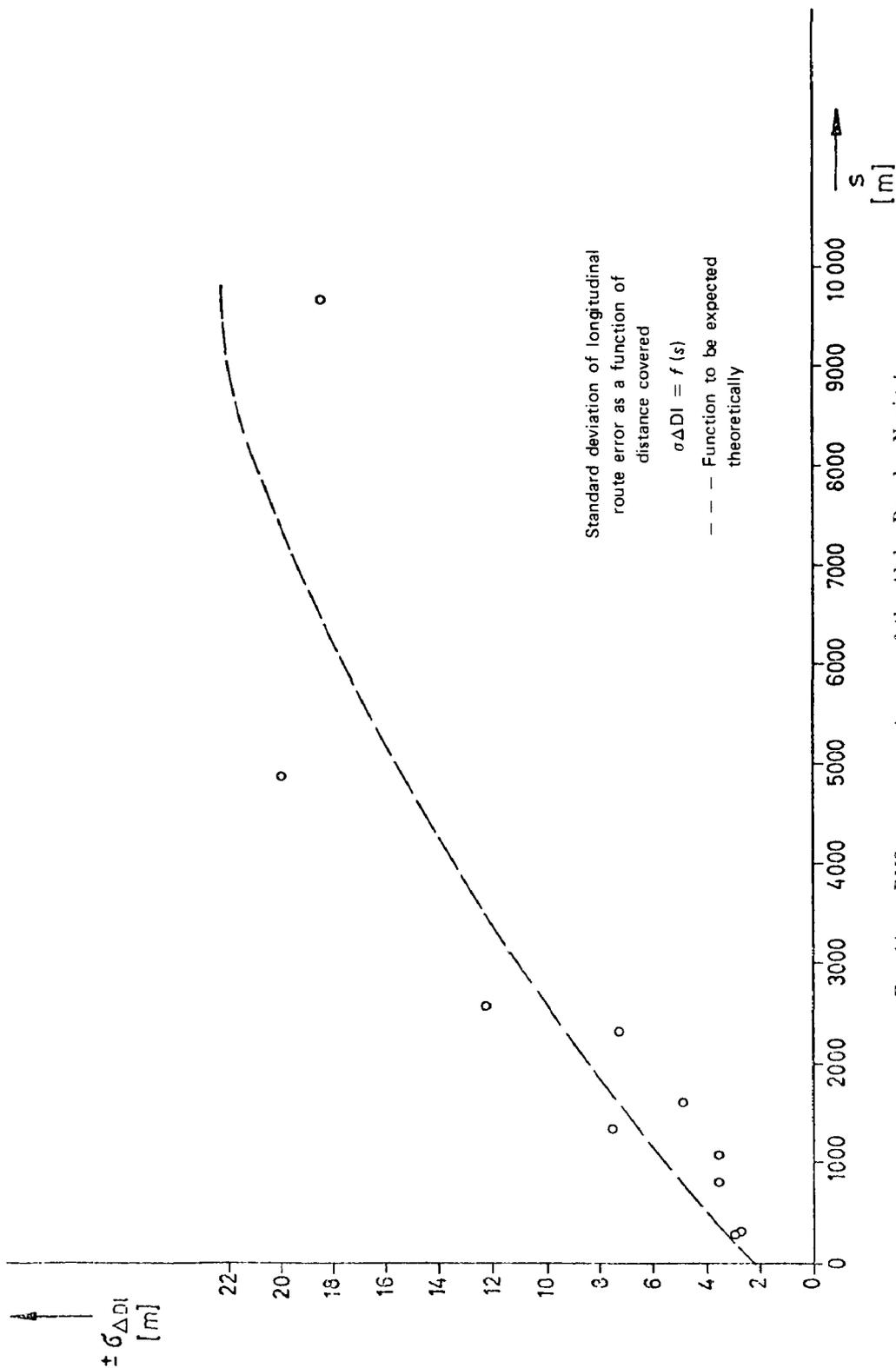


FIG. 14. — RMS measurement error of the Alpha Doppler Navigator on straight routes.

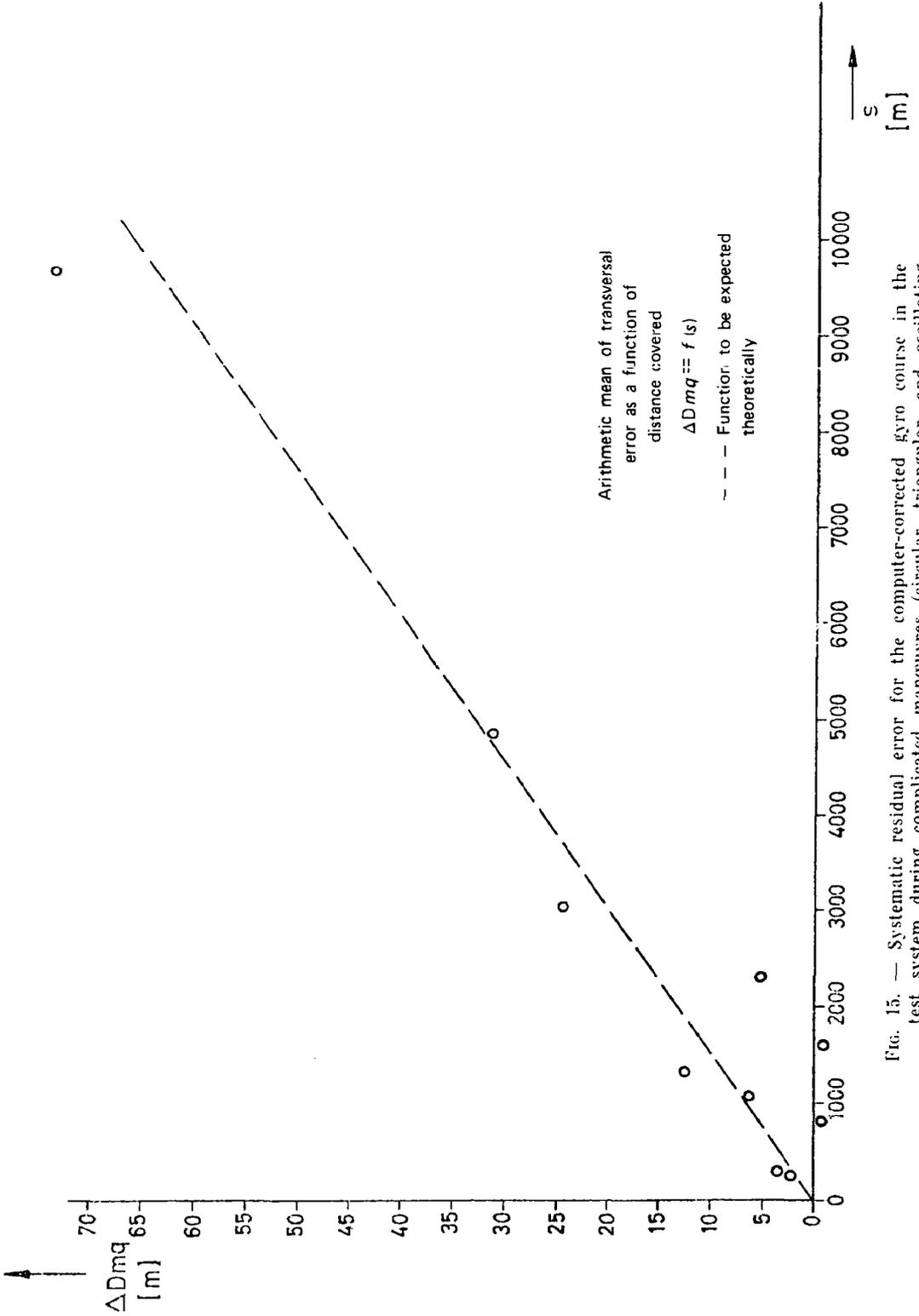


Fig. 15. — Systematic residual error for the computer-corrected gyro course in the test system during complicated manoeuvres (circular, triangular, and oscillating runs in North-South and East-West directions with speeds varying up to ± 22 knots).

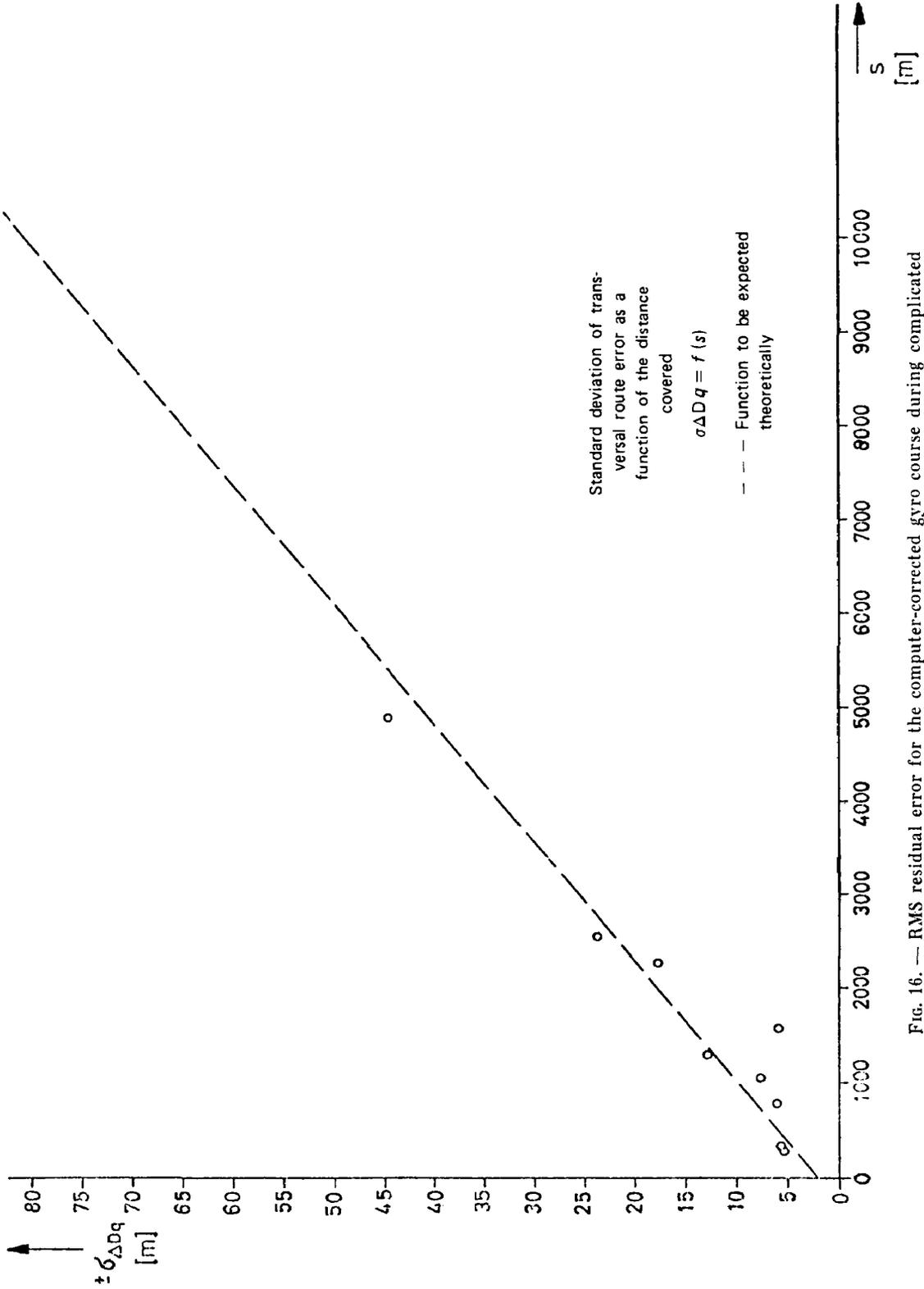


Fig. 16. — RMS residual error for the computer-corrected gyro course during complicated manoeuvres (circular, triangular and oscillating runs in North-South and East-West directions with speeds varying up to ± 22 knots).

approximately ± 2 m. This value corresponds approximately to the Hi-Fix system error, which is independent of the route sections and is superimposed on all the measured values.

Figure 15 shows the systematic transversal error versus the distance covered. The relative error amounts to approximately 0.65 %, and corresponds to a systematic course error of 0.37° . The course was corrected in the computer program by -0.37° and thus the error was on the average eliminated.

Figure 16 shows the standard deviation of the statistical transversal route error measured during the ship's most complicated manœuvres. For this range of short sections, unlike the longitudinal error, the transversal error shows an increase proportional to the distance covered. In actual fact the course errors in the successive sections — errors which were assumed to be accidental — are still very closely correlated since the period of the oscillations of the acceleration error exceeds 1 hour. The relative error amounts to approximately ± 0.85 %, thus corresponding to 0.5° . This being so, the overall accuracy of the system depends essentially on the accuracy of the course. It can be expected that this acceleration error will decrease considerably for slower speeds than those employed during the test, and for less extreme changes of course. Even here a smoothing effect will manifest itself after the run has been in progress for more than several hours, that is to say the absolute transversal error will increase by approximately the square root of the distance covered, and the relative transversal error will decrease by the inverse of this square root. The graph in figure 16 is for the area immediately surrounding the origin of coordinates shown in figure 14. Here, too, the expected theoretic curve will not pass through the origin of coordinates but will cut the axis of the ordinates at a point about ± 2 m from the origin. This will constitute the approximate standard deviation — which will be superimposed on all measurements — of the statistical error which is independent of distance. This error is not caused by the doppler system.

The overall accuracy of the system can finally only be determined by means of these statistical errors of route and course. For a 50 % mean circular error probability (CEP) a value of 0.41 % was determined by analyzing 22 test runs, all performed during complicated manœuvres, each test varying between 20 and 90 nautical miles, the total length being 1200 nautical miles. This relative value refers to the distance covered, and the percentage of error is certain to be less for longer routes and lower speeds, as well as for less frequent alterations of course and speed.

The determination of the correction values for longitudinal speed and for the course, i.e. the actual corrections to the system, may in practice be performed as follows :

The ship covers a distance l between two geographically known points A and B. This distance should be as long as possible (see fig. 17).

The uncorrected doppler measurement leads to a point B'. Deviation Δl for the longitudinal direction of the route and deviation Δq for the transversal direction can then be determined.

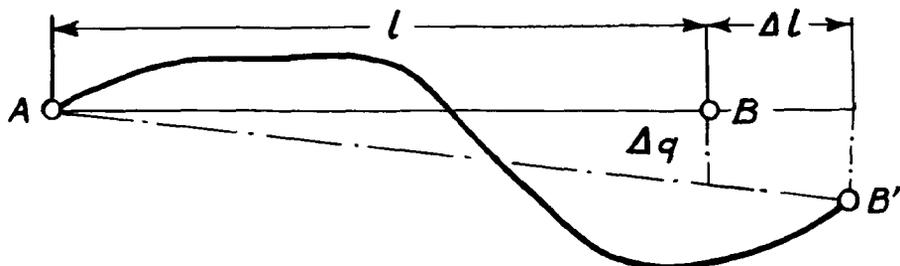


FIG. 17. — Determination of correction values for longitudinal speed and course.

Thus $(1 \pm \Delta l/l)$ is the correction factor for the distance and speed measurements, and $\Delta\phi = \arctan \Delta q/l$ the correction factor for the compass error.

Since in the case of a single measurement the statistical errors are included in the measurement results the correction values obtained must be regarded as first approximations. They may be improved by further test runs with any known starting and finishing points, thus without the necessity of following a particular route.

In conclusion, and to summarize, a few remarks should be added.

This article has attempted to show the complexity of the problem of environmental effects on the accuracy of position fixing with the sonar doppler system.

In this connection, it is important to have the possibility both of knowing the effects of a large number of possible errors and of eliminating them by physical or mathematical means. Only a part of the statistical errors of course and speed then remain as position errors. Analysis of these remaining position errors reveals the problem of the accuracy of the reference system. It is only by spending much time (for many long distances have to be covered) and by the use of the most accurate known surveying methods that it is possible to give firm evidence of the errors attributable to the sonar doppler system.

When discussing the results obtained from measurements account must be taken of the fact that the test runs were planned so as to represent nearly the worst possible conditions for the gyro compass. The resulting errors are remarkably small. Furthermore, the relative errors show a tendency to decrease with the length of the route.

As relative error values are bound to continue to decrease with lower speeds (with the ship *Bürkner* this was not possible), and with less frequent changes of course and speed, it can be asserted that this navigation system consisting of two doppler components and a commercial compass opens up new possibilities of activity for hydrographic surveying as well as for the accurate navigation of commercial vessels in coastal waters.