A DIGITAL METHOD FOR STAR SIGHTS

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1. INTRODUCTION

Although it is now possible in certain areas to fix a ship's position with such electronic systems as Loran, Decca, etc., as well as, given certain conditions, to navigate by satellite, yet celestial navigation still remains the most usual method in spite of the fact that, in the case of stars, only the short twilight period can be used.

However, Raytheon in the U.S.A. (*) have developed a low light TV scope [1] which when coupled with a sextant can be used for star observations. It will enable the horizon at sea to be clearly seen by the night sky illumination provided it is not masked by fog. This equipment weighs 2.5 pounds, is about 28 cm long with a diameter of 7.6 cm and has a field of about 10° with fourfold magnification. Instruments of smaller dimensions will become available in the future.

If the observer is thus not obliged to take his altitudes during the twilight period he can select those observation zones on the celestial sphere which will minimize altitude errors.

The method described is based on the same spherical triangles as that advocated by S. KOTLARIČ [2], but it uses digital arithmetic simple to resolve with a calculator.

KOTLARIČ's method imposes a restriction — that of not being able to observe the second star at the meridian — but this is not sufficient, as we shall shortly see. More severe restrictions must be imposed.

As for the digital processing, it becomes viable once the vessel possesses a small calculating machine.

(*) Editor's note : and others.

2. NOTATIONS USED

E,E′	:	Observed stars
T	:	Chronometer time
M	:	Chronometer rate
φ	:	Latitude
λ	:	Longitude
P	:	Pole
Z	:	Zenith
B	:	Arc EE′
R	:	Angle PEE′
S	:	Angle ZEE′
α	:	Right ascension of Star E
δ	:	Declination of Star E
q	:	Parallactic angle of Star E
z	:	Zenith distance of Star E
t	:	Hour angle of Star E

3. METHOD OF STAR FIXING

Let us take the case when we observe two stars E and E′ at instants T and T′, given by the ship's chronometer, and measure the zenith distances z and z' (assuming that these have already been corrected for refraction, instrumental errors, etc.)

Let (α, δ) and (α', δ') be the coordinates of the stars E and E′ at the instants of observation, M the chronometer rate, and λ the longitude. We then have :

$$\lambda = T' - T + M(T' - T) - (\alpha' - \alpha)$$

We must now consider the celestial triangles for the two stars E and E′ (figure 1). For computing B we shall have :

$$\cos B = \sin \delta \sin \delta' + \cos \delta \cos \delta' \cos \lambda$$

and for computing R :

$$\sin B \cos R = \cos \delta \sin \delta' - \sin \delta \cos \delta' \cos \lambda$$
$$\sin B \sin R = \cos \delta' \sin \lambda$$

From triangle ZEE′ we can compute angle S :

$$\cos z' = \cos z \cos B + \sin z \sin B \cos S$$

and we shall then obtain the parallactic angle q at star E :

$$q = S - R$$

Given that in the triangle PZE three elements are already known,

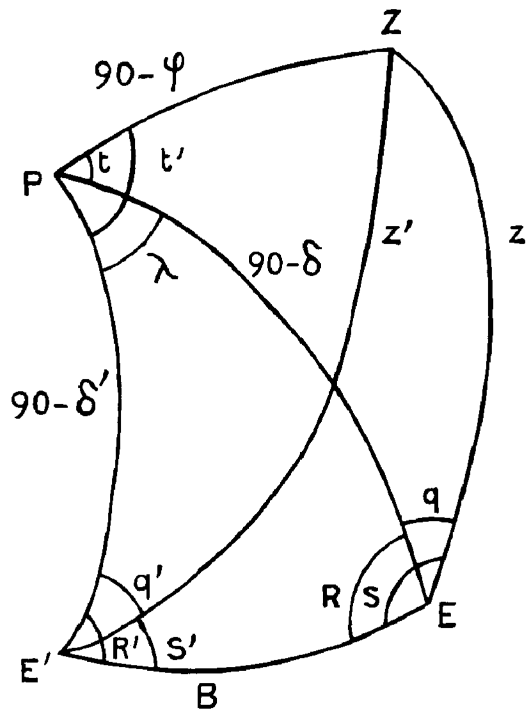

FIG. 1

we can determine not only the latitude but also the hour angle t :

$$\sin \varphi = \cos z \sin \delta + \sin z \cos \delta \cos q$$
$$\cos \varphi \cos t = \cos \delta \cos z - \sin \delta \sin z \cos q$$
$$\cos \varphi \sin t = \sin z \sin q$$

Proceeding likewise we can compute R′ from the equations :

$$\sin B \cos R' = \cos \delta' \sin \delta - \sin \delta' \cos \delta \cos \lambda$$
$$\sin B \sin R' = \cos \delta \sin \lambda$$

and obtain angle S′ from triangle ZEE′ :

$$\cos z = \cos z' \cos B + \sin z' \sin B \cos S'$$

We shall then have the parallactic angle for star E′ :

$$q' = R' - S'$$

and from triangle PZE′ we may compute the latitude and the hour angle t' :

$$\sin \varphi = \cos z' \sin \delta' + \sin z' \cos \delta' \cos q'$$
$$\cos \varphi \cos t' = \cos \delta' \cos z' - \sin \delta' \sin z' \cos q'$$
$$\cos \varphi \sin t' = \sin z' \sin q'$$

We thus see that the latitude is computed in two separate steps, and thus the verification of the bulk of the computations is possible. Furthermore as t and t' are known we can compute an exact value for the longitude :

$$\lambda = t' - t$$

The computations are valid even if a certain time has elapsed between the observations of each star, since their coordinates for the instant of observation can be ascertained from the Nautical Almanac; the precaution must, however, be taken of correcting the observed altitude of the second star in terms of the distance the ship has run between the first and second observation.

4. ZONES SUITABLE FOR STAR OBSERVATIONS

Before proceeding we must resolve the equation :

$$\cos z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos t$$

On the assumption that no error has been made in the coordinates taken from the Nautical Almanac; we have :

$$-\sin z \, dz = -d\varphi \,(\sin \varphi \cos \delta \cos t - \cos \varphi \sin \delta) - \\ - \cos \varphi \cos \delta \sin t \, dt$$

Now, given that :

$$\cos A \sin z = \sin \varphi \cos \delta \cos t - \cos \varphi \sin \delta$$

and

$$\cos \delta \sin t = \sin z \sin A$$

for the equation just resolved, we shall have :

$$dz = \cos A \; d\varphi + \cos \varphi \sin A \; dt$$

Similarly, for the second star we shall have :

$$dz' = \cos A' \; d\varphi + \cos \varphi \sin A' \; dt'$$

Since the chronometer rate is usually known accurately we shall assume that it is error free.

We will now study two cases :

a) When the chronometer readings T and T' are error-free.

We shall have :

$$d\lambda = dt' - dt = 0$$

and consequently :

$$dz' = \cos A' \; d\varphi + \cos \varphi \sin A' \; dt$$

and thus values are obtained for $d\varphi$ and $\cos \varphi dt$ of :

$$d\varphi = \frac{\sin A' \, dz - \sin A \, dz'}{\sin (A' - A)}$$

$$\cos \varphi \, dt = \frac{\cos A \, dz' - \cos A' \, dz}{\sin (A' - A)}$$

We can thus conclude that errors dz and dz' will have the least possible influence on $d\varphi$ and dt when $A' - A = \pm 90°$.

It is seen that we must observe the two stars in such a way as to ensure that $A' - A$ is not in the neighbourhood of either 0° or 180°, for the influence of the dz and dz' errors is considerable. As far as possible we should respect the general condition : $A' - A = \pm 90°$.

b) When the chronometer readings T and T' contain errors dT and dT'.

Once T contains the error dT the exact instant of the first observation is T — dT, and at that moment the zenith distance is z. Thus, at instant T the zenith distance that we shall be measuring will be $z + dz$. T can then be considered as accurate as soon as we use $z + dz$ instead of z. This means that we can express the chronometer reading errors in terms of errors of zenith distance. And this in fact brings us back to the first case (a).

5. COMMENT

From the equation :

$$dz = \cos A \; d\varphi + \cos\varphi \; \sin A \; dt$$

we can see that in the position line method of Marcq St. Hilaire stars of altitude near 0° or 90° must not be used. So far as possible we must choose to observe them in the neighbourhood of the bisectors of the meridian and the prime vertical.

6. CONCLUSION

1. This method makes it possible to observe stars both during twilight periods and throughout the night.

2. We are consequently able to choose our zones of observation on the celestial sphere, since with this method the influence of altitude measurement errors can be reduced.

3. With this method only two steps are needed to compute latitude.

4. Its method of resolution is a simple digital one, using a calculator.

5. This method complements the Kotlarič method and also the position line method of Marq St. Hilaire; the latter can also be resolved digitally (by the least squares method).

6. Provided that the angles are taken by theodolite the method can be used without modification on land both for hydrographic uses and in geodetic astronomy.

REFERENCES

[1] DUNLAP, G., EDWARDS, O. : La navigation visuelle de nuit, *Revue Technique de navigation maritime, aérienne et spatiale*, No. 76, October 1971.

[2] KOTLARIČ, S. : Two-star fix without use of altitude difference method. *Int. Hydr. Rev.* XLVIII (2), July 1971.

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