A METHOD OF PRESENTING THE PREDICTED TIDES OF A MIXED TYPE WITH STRONG DIURNAL INFLUENCE

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IHB Note. — The Canadian Hydrographic Service has adopted this method for tidal predictions for ports with large diurnal inequalities in the Canadian Tide and Current Tables of 1975 onward.

Canada has used this method for predictions sent to other Member States through the international exchange of tidal predictions.

ABSTRACT

The data most widely used in presenting the variation of tides are the high and low tides. The curve between the high tide and the low tide can be satisfactorily described by a cosine function if the shallow water distortion is negligible; otherwise the cosine function can be replaced by a polynomial series with coefficients determined by regression analysis. For mixed tides with large diurnal influence, however, the shape of the curve between the low tide and the high tide changes from day to day.

In this report, a method has been developed on how to divide the curve into segments so that each can be described by a cosine function. The following constraints are applied to the method to achieve a uniformity of the format in tidal publication:

1. The sequence of the high and low tide must not be altered by the inclusion of additional points between them.
2. The total number of points, including the high tide and the low tide in a day, must not be more than four (4).

I. INTRODUCTION

The variation of the water level due to tidal forces has for many years been presented in the form of high and low tides. For diurnal or semi-diurnal tides, the time and height of high and low tide can be computed from the change of sign of the derivative.

The tidal elevation at points between high and low tides can be computed by the cosine function:

$$X(T_i + t) = \frac{1}{2}(X(T_{i+1}) - X(T_i))\left(1-\cos\left(\pi\frac{t}{T_{i+1}-T_i}\right)\right) + X(T_i) \qquad (1)$$

where T_i and $X(T_i)$ represent the time and elevation of the i^{th} tide.

When shallow water distortion becomes significant, equation (1) could be replaced by the following more general equation:

$$X(T_i + t) = (X(T_{i+1}) - X(T_i))\sum_{j=1}^{N} A_j\left(\frac{t}{T_{i+1}-T_1}\right)^j + X(T_i) \qquad (2)$$

where the coefficients A_j can be determined by regression analysis (Ku, 1968).

For the tides of the mixed type with a strong diurnal influence as shown in figure 1, the use of either equation (1) or equation (2) would produce a significant error due to the irregularity of the shape of the tidal curve. Therefore, additional information must be provided to assist the user of the Tide Table to carry out the above computation.

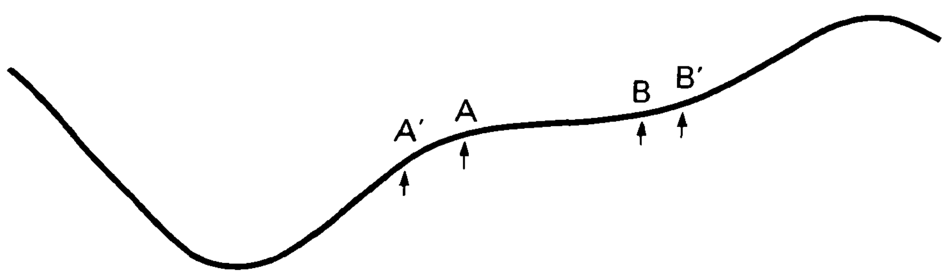

FIG. 1. — Turning points.

The best way of presenting this additional information is probably by *adding the elevation and time of the tide at points as shown in figure 1*, these points A and B are called "turning points" in the following discussion.

The problem seems to be simple if it is to be done manually. However, manual extraction lacks consistency and uniformity and, in addition, is very time consuming. The interpretation by different individuals is not the same. Since computers are used for the predictions, a computer program to overcome these problems was required.

II. METHOD

The computation is divided into the following stages:

1. Compute the tidal elevations X_i at 15-minute intervals.
2. Compute the high and low tides.
3. If the time difference between high and low tides is larger than a value T, then compute two turning points (a suggested value of T is 14 hours).
4. When all the points for the desired period have been selected, check that the order of the tide is in proper high and low sequence. If not, insert an appropriate turning point.
5. Select a set of high and low tides and turning points in every 24-hour period. If these are more than four, delete the least significant points.

Many problems arose in selecting the computational scheme for the different stages of the procedure. Several approaches to these problems were tried and evaluated. Some features of the method used in the final program are as follows:

1. The high and low tides are identified by the change of sign of the first derivative. Those originating from high frequency oscillation with $f > 1$ cycle/hour are ignored. Also, two consecutive tides must have a time difference of at least two hours and a range difference of at least 0.05 feet.
2. Turning points are identified by the change in sign of the rate of change for the forward difference. Two turning points are searched for between two tides, one by proceeding forward from one tide, the other by proceeding backward from the other tide.

The method used for proceeding forward is now described. The method for proceeding backward is the same except for different indices. Let X_m be the height of either the high or low tide, and m be the time sequence of the 15-minute prediction. The slope at $m + 4$ is computed by:

$$S_{m+4} = (X_{m+4} - X_m)$$

The sign of S indicates the direction of the curve. When S is positive, the tide is rising and vice versa. Then compute:

$$d_n = \frac{X_{n+1} - X_m}{n + 1 - m} - \frac{X_n - X_m}{n - m}$$

for $n = m + 4$, $m + 5$, ... A turning point is defined at $n = N$, when:

$$S_{m+4} \cdot d_N < 0$$

In figure 2, the two turning points are indicated by A′ and B′. These two points are moved closer to the flat part of the curve by insisting that:

$$|d_n| \geqslant \epsilon \, |S_N|$$

where S_N is the slope at the turning point. The values of ε used in the final program are $\varepsilon_L = 0.1$ for the point closer to the low tide and $\varepsilon_H = 0.02$ for the point closer to the high tide. This produces the points A and B.

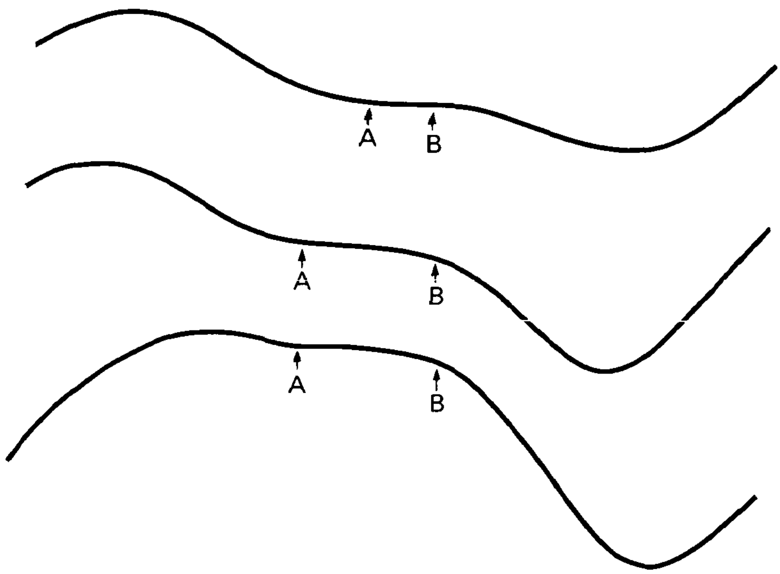

FIG. 2. — Typical examples of turning points.

3. The points A and B must be at least 2.5 hours from the low tide and high tide respectively, and must be at least one hour from each other.
4. If the method fails to find any turning points between high and low tides, then the interval between these two tides is divided into thirds and turning points are inserted at these points. If the method finds only one turning point, say A, then the other point B is inserted halfway between A and the other tide.
5. The sequence of high and low tides is checked. For example let X_1, X_2 and X_3 denote the height of three consecutive tides. If $X_1 < X_2 < X_3$, then X_2 is eliminated and an attempt to find two turning points between X_1 and X_3 is made. If this fails, X_2 is retained and one turning point is inserted between X_2 and X_1.
6. The following procedure is used to ensure that the total number of points including the high tide and low tide in a day is not exceeding four. Each point is given a rating, r_i, which is defined as:

$$r_i = |X(T_i) - Y(T_i)|$$

where $X(T_i)$ is the elevation of the i^{th} tide (it could be either a high or low or a turning point), and $Y(T_i)$ is the elevation interpolated from the two adjacent tides, $X(T_{i+1})$ and $X(T_{i-1})$. The points eliminated are those with the smallest r value. When one point is eliminated the r_i's are recalculated before another is eliminated.

This is an insurance feature which is rarely used. For example, in predicting the tides for the whole year of 1974 at Victoria, British Columbia, the method produced only one day with five tides.

III. EXAMPLES

A few examples of the turning points that are selected are illustrated in figure 2. These examples are from the 1974 predicted tide at Victoria, British Columbia. Examples (a), (b) and (c) are typical cases of the turning points found by the method described. Figure 3 illustrates a case where the procedure could find only one turning point A. Therefore, to preserve the proper sequence of high and low tides, another point, B, is inserted.

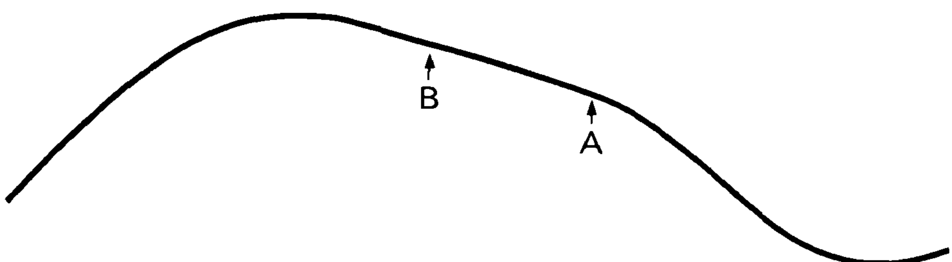

FIG. 3. — Turning point A and inserted point B.

The program was written for the CDC 6400. It also produced the hourly predictions, and the execution time for one year of predictions is 2.5 minutes.

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