KRIGING : A METHOD
FOR CARTOGRAPHY OF THE SEA FLOOR

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ABSTRACT

Geostatistics is a method of analysis and spatial data treatment developed at the Mathematical Morphology Centre, directed by Professor G. MATHERON. It has been applied to fields as varied as mining, gravimetry, meteorology, biology and oceanography to solve problems such as ore deposit valuation, contouring, simulation, map numbering, etc.

One of the most used techniques in geostatistics is Kriging, and in this paper it is presented as applied to the Iroise Sea bathymetry, from sounding records of the French Naval Hydrographic and Oceanographic Service. From the same data not just a single map but several different maps can be drawn according to the desired objective; for instance the navigator will pay particular attention to shoals and reefs, while oceanographers need maps showing only the main features of the relief.

By taking relief structure into account Kriging gives a specific solution to each aim. The influence of data density on the quality of the map is also stressed; for comprehensive (small-scale) maps, the sounding record is much too rich, whereas detailed (large-scale) charts have relatively too few data.

INTRODUCTION

The Centre de Morphologie Mathématique has developed a method of optimum estimation which permits the best possible representation of a terrain from available data. This method is called the Kriging method after D.G. KRIGE, a mining specialist, and was originally developed for assessment problems in the mining industry. The theory on which it is based, which we owe to G. MATHERON, proves however to be a very general one and to have varied applications : for forestry assessments, topographic depiction, meteorology, gravimetry, and so on. The present article deals with the application of Kriging to bathymetric charting.

THE PROBLEMS DEFINED

Cartography is a huge subject, stretching from the survey itself right up to the printing of the chart. When dealing with the sea floor a host of problems arise; among them such questions as automated methods for collecting new data, retrieval and verification of earlier data, quality and accuracy of soundings, reliability of the contour lines, magnitude of shoals and reefs, storage and retrieval of random soundings and contour lines, up-dating, scale changes, international coordination, etc. It is obvious that the Kriging technique does not resolve all these questions; its use is in the field of estimation, which alone raises a number of aspects. For a hand-drawn chart the cartographer deals with these while plotting the contours, sometimes even sub-consciously. If the plotting of contour lines is to be automated, all these points must be foreseen and solutions found before any computing can be started. The problems can be divided into groups — those concerning the spatial structure of the terrain, the chart scale, the purposes and the accuracy of the estimation. These will now be considered one by one.

The spatial structure of the variable

When plotting contour lines an interpolation has to be made between the experimental data — here the soundings. When this work is done manually some interpretation is a necessity, varying according to the density of the soundings and to the degree of regularity of the variable under study. Contour drawing is also a matter of experience. For instance in marine cartography the cartographer will adopt different approaches for an abyssal plain, a ridge, or a fracture zone. Although only few data may be available, it is nevertheless possible to establish a reasonable working hypothesis based on similar and already known structures. If the object is to develop an automated method for plotting contour lines, it is essential to take the characteristics of the structure of the phenomenon into account.

Chart scale

A single automated method must be suitable for two extreme cases :
— for plotting of a general aspect chart based on ample data;
— for plotting a large scale chart incorporating all known detail.

It goes without saying that the evaluation of sounding density will depend on the scale adopted. If the soundings have a spacing of 1 000 m and the finished chart is to be at 1/1 000 000 then there will be abundant information. For a chart at 1/10 000 it would be less adequate.

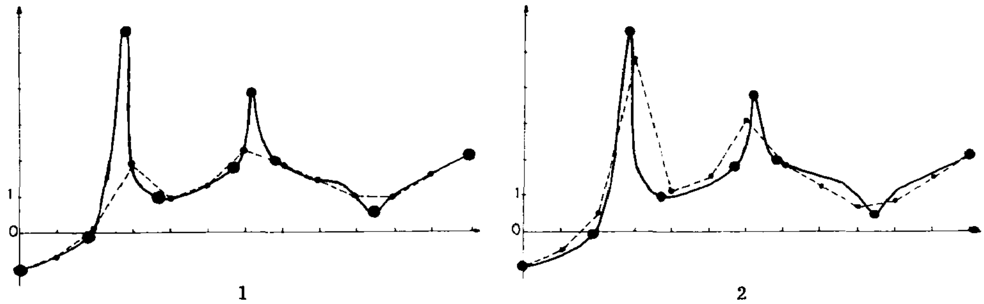
It is also quite obvious that even large scale charts can only feature already known details. In bathymetry, for instance, if there is a shoal still undiscovered because sounding lines have passed to one side of it, there is positively no method in automated cartography capable of locating this shoal — unless by mere chance, and this in itself would be most disquieting. Good cartographic methods cannot actually create information — they can only put existing information to the best use.

Theoretical aims

When working out a method for automatic contouring we are in practice obliged to use a regular grid as basis. Provided this is sufficiently fine, the contouring proper can be carried out by a series of simple interpolations. Thus, it must be possible at each grid intersection to estimate the value of the variable we wish to chart. However, before seeking a method of estimating, the alternatives in mind must be clearly defined. At least four of these can be discerned.

1. *Computation of the unknown, real value* of the variable under study; and use of the best possible estimator.

2. *Establishment of a chart which "best fits" the experimental data.* The problem occurs when one is dealing with a phenomenon that has a high variability at small scale. The difficulty arises from the discretisation of the domain where the estimation is to be performed. Assuming that it will be possible to find the best estimator in the absolute sense, i.e. that yielding the real value at each point — and obviously we are a long way from this — one might think the estimation problem solved. This would indeed be so for a sufficiently regular variable. However, for an irregular variable it would certainly not, since the grid intersections would generally miss the extreme values. If the variable is very chaotic at small scales, and for example exhibits "peaks", then most of these peaks will disappear. If they represent measurement errors or are of no significance this does not matter much; but it would still be better to employ an estimator that will filter them. If, however, (as is often the case in bathymetry) the main interest lies precisely in these peaks then an estimator providing optimum restitution of the peaks is preferable to this filtering estimator which remains nevertheless the best from the absolute point of view. The peaks are likely to be somewhat magnified, but this is only a by-issue; what is important is that they should be brought out. Such an estimator will be referred to as a "security estimator". Figure 1 gives a one-dimensional illustration. Here it was assumed that the whole curve was known. But in fact only selected characteristic values were utilised as experimental data, and the estimation curve was then plotted by joining up the grid intersection points. It is seen



1 2

FIG. 1. — Optimum estimator and Security estimator.

1. *Optimum estimator* giving exact value : experimental variance for estimation $\sigma^2 = 0.35$

—— curve of the reality

● experimental data

• grid intersection points

- - - - estimation curve deduced from intersection points.

2. *Security estimator*; experimental variance for estimation $\sigma^2 = 0.70$

that the optimum estimator follows the curve well except where an extreme value is concerned. The security estimator provides a better representation of the peaks, although the variance of the discrepancy between exact and estimated values is twice as large in the case of the optimum estimator. If we are interested in very small fluctuations — and our knowledge of these is poor except along a sounding profile — a security estimator will be used; if not, the optimum estimator.

3. *Establishing a chart of the drift*, or trend, of the variable under study. To do this the variable must first be rid both of small and medium fluctuations (see figure 2). Such a trend chart is often what is needed for a physical interpretation (gravimetric assessment for instance).

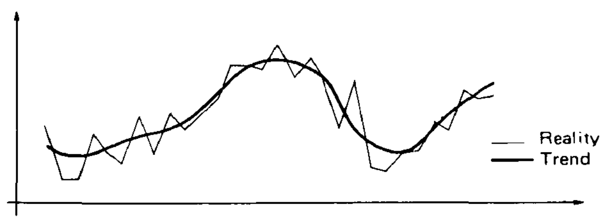


FIG. 2. — Trend.

4. *Simulation of the reality*. Drawing a chart from point data means adapting a surface to these experimental points; this surface is then usually more even than the actual seabed since it cannot reproduce details not revealed. However, for certain problems it may be preferable to have a chart somewhat removed from the reality, but which — since it still honours the experimental data — will present local variations resembling those of reality itself. This will give an idea of the chaotic aspect of the phenomena. Thus, the aim is to obtain one or more variants of the chart of the unknown reality — in other words to simulate the reality rather than to estimate it (see figure 3).

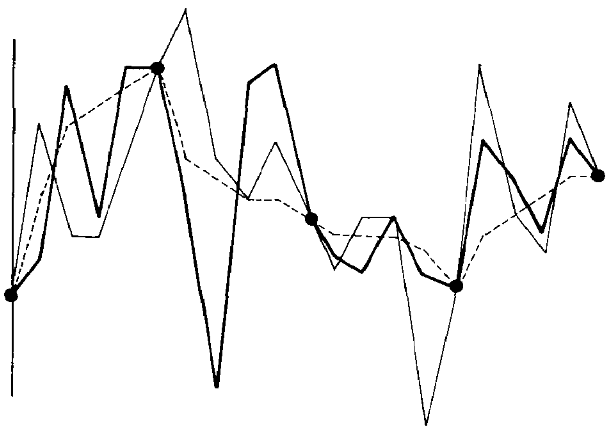


FIG. 3. — Reality — Estimation — Simulation.
● experimental data
—— reality
- - - - estimation
—— simulation

Accuracy of the estimation

We may plot a chart, but this is not enough; we must also know how much confidence can be placed in it. This is particularly important for interpreting a chart where the data are poorly distributed (widely spaced sounding lines, for instance). If a "dome" is shown up, does this really exist, or is it perhaps merely due to lack of data at that particular locality ? A similar problem is often encountered at the edges of the chart where we can no longer interpolate, but are obliged to extrapolate. Hence a parameter is needed that will characterize the error arising from utilizing an estimated value instead of the unknown real value.

Also connected with this problem is how best to position additional individual soundings or lines of soundings. The aim is here to quantify the additional information, i.e. improved density of soundings, which a further survey would provide.

PRESENTATION OF KRIGING

The problems of estimation and plotting of isolines are not new, and various methods exist for attempting to resolve them : least squares, spline functions, or polygons of influence, for instance. But almost all these give solutions taking no account of the spatial structure of the variable under study; they are moreover not capable of analysing this structure. The same estimators will thus be obtained whether the variable is fairly regular (for gravimetry or the bathymetry of an abyssal plain) or on the other hand very chaotic (for geochemistry or the bathymetry of a fracture zone). In addition we have just seen that several very different objectives can be pursued. Naturally, methods which are appropriate for a single type of chart cannot at the same time be suitable for all the other aims. In general, each will meet only one purpose without, moreover, providing the best solution since it takes no account of the structure of the phenomenon. Finally, since this structure remains unknown the accuracy of the estimation cannot be precisely quantified with any of these methods. We should note that the residual variance in the least squares method is completely different from a variance of estimation; it is smaller, and usually considerably smaller, than the variance of estimation.

The Kriging theory, however, does take account of the structure of the phenomenon by employing a structural function — the semi-variogram — which is determined on the basis of the available experimental data. The Kriging theory thus finds the best solution for each of the various alternatives, defining them precisely and also quantifying the chart's degree of accuracy — since all estimation is necessarily accompanied by a variance of estimation.

It is not intended here to go deeply into the theory of Kriging; an outline will suffice. Interested readers may refer to the works of G. MATHERON cited in the bibliography.

The basis of the Kriging concept

The phenomenon under study can be considered as a function $Z(x)$ whose values are known only at certain points $x_1, x_2 \dots x_n$. The first problem encountered is that of "punctual" kriging, i.e. estimation of the value $Z(x_0)$ of the variable Z at a point x_0 by a linear combination

$$Z^* = \sum_{\alpha} \lambda^{\alpha} Z(x_{\alpha})$$

of the data. For the best possible estimator, i.e. the most appropriate coefficients λ^{α}, it will be necessary to take into account both the spatial distribution of the data and the degree of regularity of the variable. The random and yet structured character of the $Z(x)$ functions encountered in the earth sciences suggests probabilistic approach. The variable here is considered as a realization of a random function, i.e. as the outcome of a chance draw from a set of similar functions. Two essential characteristics of this random function can be defined :

Its mean : $m(x) = E[Z(x)]$
(symbol E = Mathematical expectation)

Its covariance : $C(x, y) = E[Z(x) - m(x)]\,[Z(y) - m(y)]$

However, the determination of functions $m(x)$ and $C(x, y)$ from a single realization requires additional assumptions. It can often be assumed that $Z(x)$ is stationary in the wide sense, i.e. that $m(x)$ is constant and $C(x, y)$ depends merely on the $y-x$ vector.

$$E[Z(x)] = m$$

$$E[Z(x+h) - m]\,[Z(x) - m] = C(h)$$

In many applications though, these assumptions are not borne out. Take the case of coastal regions where the depth of the sea bottom increases progressively with the distance from the shore. The mean $m(x)$ of the depth will show a trend and can no longer be considered as a constant. Furthermore it becomes apparent that optimizing procedures do not demand that the variable itself should have a stationary covariance, but only that its increments should have.

In view of this we shall make the following assumptions :

1. The trend $m(x) = \mathrm{E}[Z(x)]$ is not necessarily constant, but will be sufficiently regular to have a local representation in the form :

$$m(x) = \sum_{l=0}^{k} a_l \; f^l(x)$$

The f^l are "given functions" (in general monomials); the a_l are unknown coefficients. In cases where the mean is constant the only basic function is $f^0 = 1$: then indeed $m(x) = a_0 f^0(x) = a_0$.

2. The variance of the increment $Z(x+h) - Z(x)$ depends entirely on the vector h. We then define the semi-variogram $\gamma(h)$ as :

$$\gamma(h) = \tfrac{1}{2}\, \mathrm{D}^2[Z(x+h) - Z(x)]$$

(D^2 representing the variance).

This function of vector h reveals the isotropic or anisotropic character of the variable. For a fixed direction it shows how the mean quadratic difference of values at any two points x and $x + h$ is related to distance $|h|$. A very regular variable has a semi-variogram correspondingly continuous. Conversely, for a variable displaying very strong and localized irregularities, the semi-variogram will show a discontinuity at the origin (the so-called "nugget" effect).

Kriging

Having defined the basis of Kriging it is now possible to resolve the various problems. The first aim is to achieve optimum estimation of $Z(x_0)$ from the available experimental data. Among the linear estimators in these n known points

$$Z^* = \sum_{\alpha} \lambda^\alpha \; Z(x_\alpha)$$

we must find the one fulfilling the two conditions :

— the mean error is nil : $\mathrm{E}[Z^* - Z(x_0)] = 0$;

— the variance of the error is minimum : $\mathrm{D}^2\,[Z^* - Z(x_0)]$ minimum.

Following the assumption about the form of the trend the first condition may be written as :

$$\sum_{\alpha} \lambda^\alpha \sum_{l} a_l\, f^l(x_\alpha) - \sum_{l} a_l\, f^l(x_0) = 0$$

or

$$\sum_{l} a_l \; [\sum_{\alpha} \lambda^\alpha f^l \; (x_\alpha) - f^l\,(x_0)] = 0$$

As the a_l are unknown coefficients, this expression must be nil whatever the values of the various a_l. This requires :

$$\sum_\alpha \lambda^\alpha f^l(x_\alpha) = f^l(x_o) \qquad l = 0, 1, \dots k$$

The variance of the error can be related to the semi-variogram :

$$D^2 [Z^* - Z(x_o)] = -\sum_\alpha \sum_\beta \lambda^\alpha \lambda^\beta \gamma(x_\beta - x_\alpha) + 2 \sum_\alpha \lambda^\alpha \gamma(x_\alpha - x_o)$$

Minimizing this variance under the $k+1$ conditions

$$\sum_\alpha \lambda^\alpha f^l(x_\alpha) = f^l(x_o) \qquad l = 0, 1, \dots k$$

we obtain the Kriging system with $k+1$ Lagrangian multipliers μ_l

$$\begin{cases} \sum_\beta \lambda^\beta \gamma(x_\beta - x_\alpha) + \sum_l \mu_l f^l(x_\alpha) = \gamma(x_\alpha - x_o) & \alpha = 1, 2, \dots n \\ \sum_\alpha \lambda^\alpha f^l(x_\alpha) = f^l(x_o) & l = 0, 1, \dots k \end{cases}$$

This is a regular system admitting of a single solution, except where the various $f^l(x_\alpha)$ are not linearly independent over the whole set of n data points. At the optimum the estimation variance is :

$$D^2 [Z^* - Z(x_o)] = \sum_\alpha \lambda^\alpha \gamma(x_\alpha - x_o) + \sum_l \mu_l f^l(x_o)$$

It will be noted that this variance depends only on the semi-variogram and on the λ^α and μ_l solutions of the Kriging system, i.e. solely on the structure of the phenomenon and on the pattern of the sample points.

We have seen how to solve this problem for the case of optimum estimation (aim 1). If, however, a security estimator is preferred then an estimator computed with a continuous semi-variogram — e.g. a linear semi-variogram with $|h|^\alpha (0 < \alpha < 2)$ — will be sufficient. For aim 3 — estimation of the trend — the method can be exactly similar to that used for optimum estimation. Aim 4 will be simple to resolve, given a procedure for the construction of simulations of random functions to be achieved with a given semi-variogram. This problem has been solved, but will not be discussed further here.

APPLICATION OF KRIGING TO CARTOGRAPHY OF THE SEA FLOOR

A rectangular area of the sea floor in the Mer d'Iroise off Brest was studied using a 1/20 000 plotting sheet supplied by the French Hydrographic Office. In this area of 14 000 × 18 000 metres there were a total of 6 265 point depths from sounding lines approximately 150 or 300 m apart, according to the area. The profiles had been arranged so that the distance between adjacent soundings was about the same as that between the sounding lines. Chart A0 shows these sounding profiles and also the contour lines hand-drawn as on the original plotting sheet.

From an analysis of the structure one might deduce that there was in fact no trend. However, the region is not homogeneous. It was therefore divided into 2 000 m squares and in each square the semi-variogram was adjusted. The adjustment is always of a linear type with the nugget effect. The linear semi-variogram is consistent with a continuous component which is nevertheless fairly irregular. The nugget effect is due to measurement errors, but even more to structures of smaller size than the distance between two adjacent experimental points (i.e. 100 - 200 m according to circumstances). It should be noted that in general the nugget effect is very considerable, often forming a large part of the estimation variance.

This outline of the possibilities of Kriging is confined to the following three aspects :

— Kriging of the whole area on a 500 m grid for a small scale chart;
— Kriging at a 40 m grid of two special 2 000 × 2 000 m squares for obtaining charts of the detail. Here Kriging is used for the first two aims : each time two charts were drawn up — one for the optimum estimator, based on the chosen criterion for the variance of estimation, the other for a security estimator;
— To evaluate the influence of sounding line density, various possible line spacings — approximately 150 m (i.e. employing all data), 300 m, 600 m and 1 200 m were considered. Charts showing the estimation standard deviation will allow the influence of the sounding density on the chart's accuracy to be quantified.

Most of these charts are reproduced here. They were plotted from a program adapted by M. Albuisson.

Small scale charts

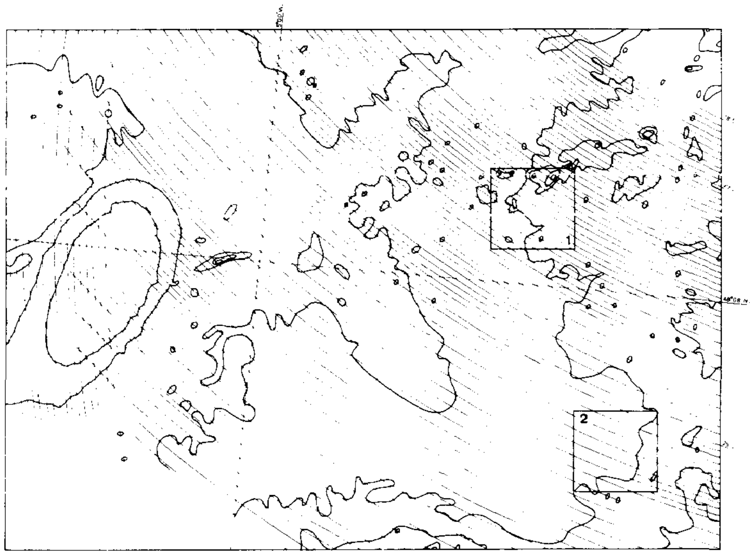
The aim was a chart giving a good idea of the general aspect without reproducing small details; depths were thus, in plotting, only estimated at the node points of a 500 m grid. Therefore an estimator minimizing the estimation variance was used.

Four charts were drawn up for the different sounding profile densities (charts A1 to A4) :

— Profiles about 150 m apart (in some cases 300 m), i.e. all profiles :
6 265 points;
— Profiles about 300 m apart (starting from profile No. 18 200) :
4 290 points;
— Profiles about 600 m apart (starting from No. 18 500) :
2 088 points;
— Profiles about 1 200 m apart (starting from No. 18 500) :
1 053 points.

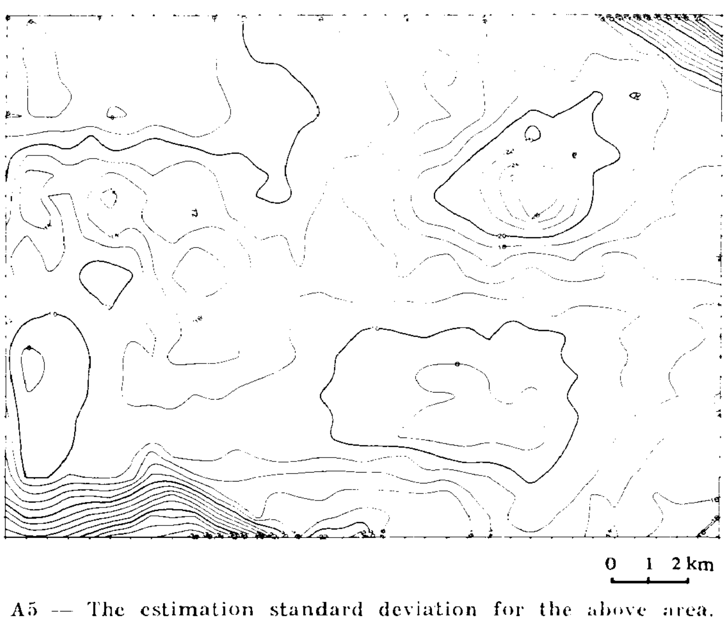
In addition to the chart showing depths one showing the estimation standard deviation was also drawn up (Chart A5 is the first of these). These charts demonstrate that :

— The charted area is not at all homogeneous; thus strong spatial variations exist in the accuracy of the estimation. On the chart incorporating all the data the estimation standard deviation varies between 0.60 m and 2.80 m (neglecting the border zones where the standard deviation is large due to lack of data). Note the highly uneven area in the NE corner of the chart (Chart A5).



0 1 2 km

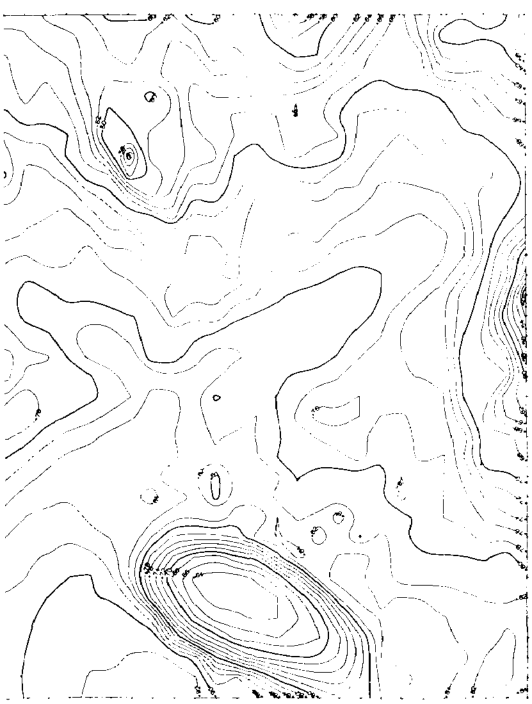
A0 — The area under study with the original sounding lines and contour lines from the original plotting sheets. The limits of squares 1 and 2 are shown.



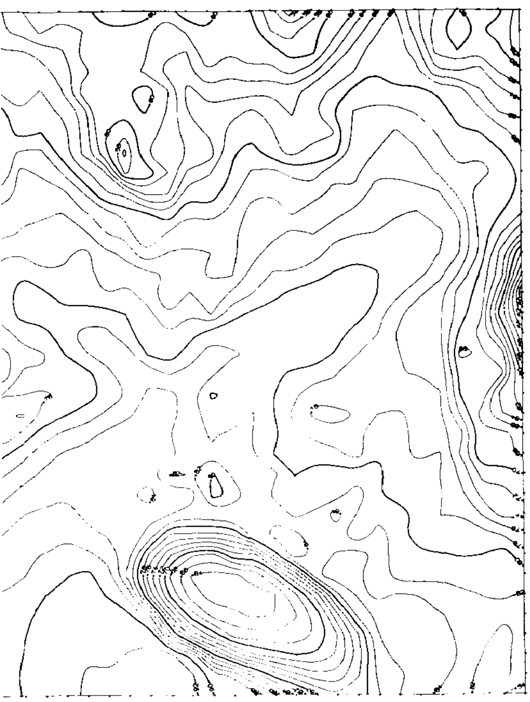
0 1 2 km

A5 — The estimation standard deviation for the above area.

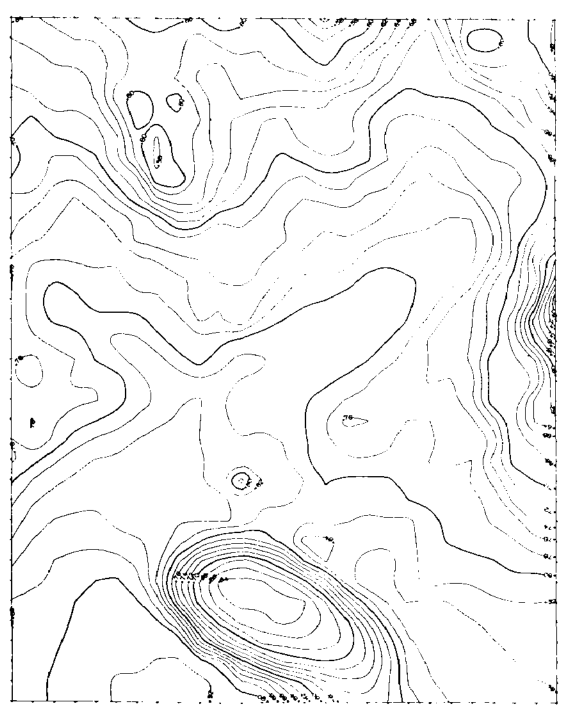
Sounding interval 150 m. (Standard deviation in decimetres. Isoline interval 0.2 m).



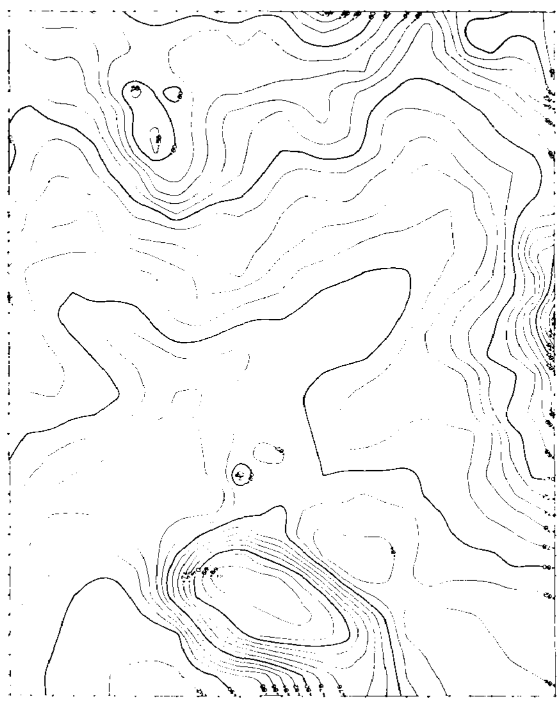
A1 — The whole area. Sounding line interval 150 m.



A2 — The whole area. Sounding line interval 300 m.



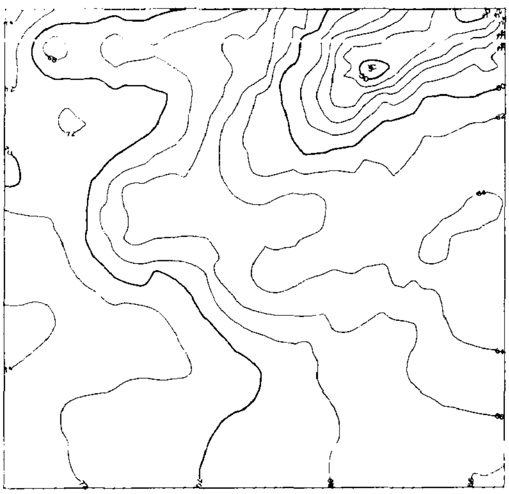
A3 — The whole area. Sounding line interval 600 m.



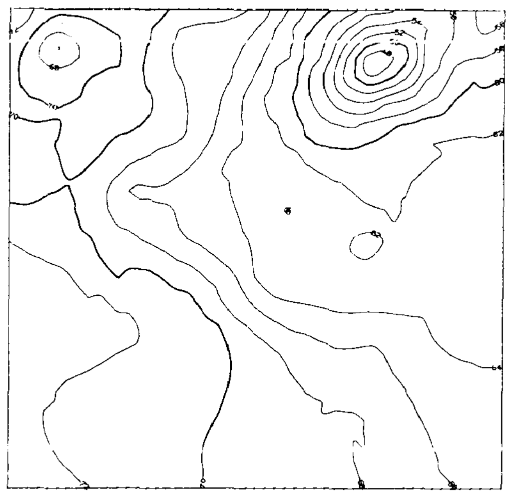
A4 — The whole area. Sounding line interval 1 200 m.

(Depths in metres; isobath interval 2 m)

— The general aspect shows up well, even when only a few sounding lines are selected (Charts A1-A4). Thus the chart based on a 600 m spacing is very nearly as good as the one containing all the data. However, the picture starts to deteriorate as soon as the chart is based on profiles 1 200 m apart, especially in the areas where the estimation variance is strongest (particularly in the highly uneven area in the NE and close to the Armen shoal in the South West). This is confirmed by the charts of the standard deviation : using as reference value the standard deviation for the chart incorporating all data, it is seen that on the charts with 300, 600 and 1 200 m spacing the standard deviation increases by respectively 3%, 15%

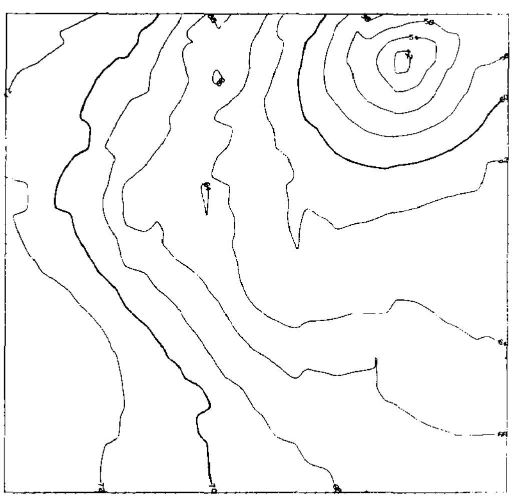


B1 — Square 1. Optimum chart. Sounding line interval 150 m.

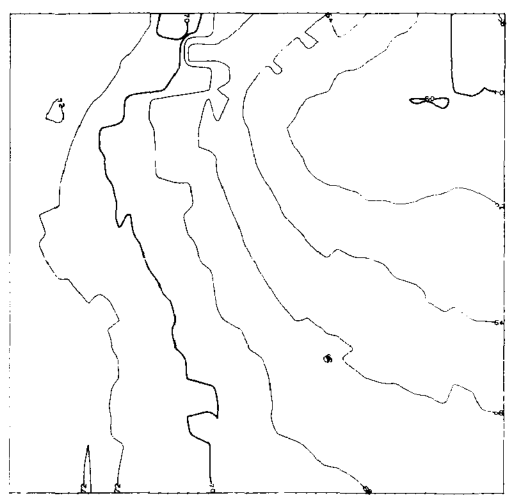


B2 — Square 1. Optimum chart. Sounding line interval 300 m.

0 200 400 m

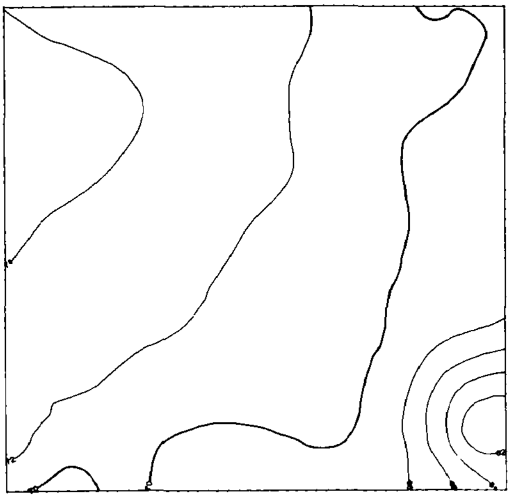


B3 Square 1. Optimum chart. Sounding line interval 600 m.

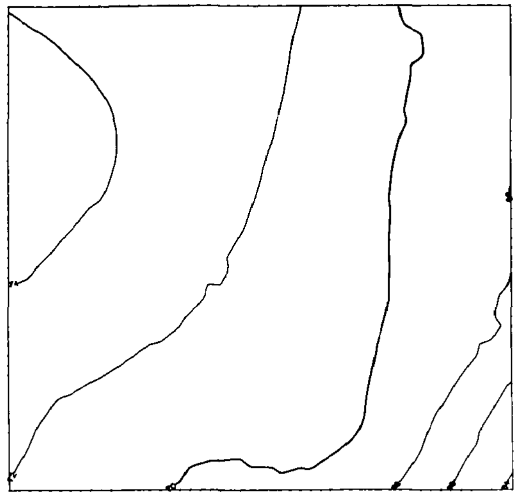


B4 Square 1. Optimum chart. Sounding line interval 1 200 m.

(Depths in metres; isobath interval 2 m)

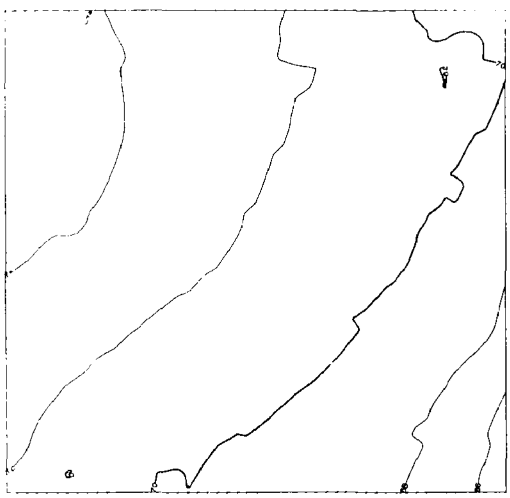


C1 — Square 2. Optimum chart.
Sounding line interval 300 m.

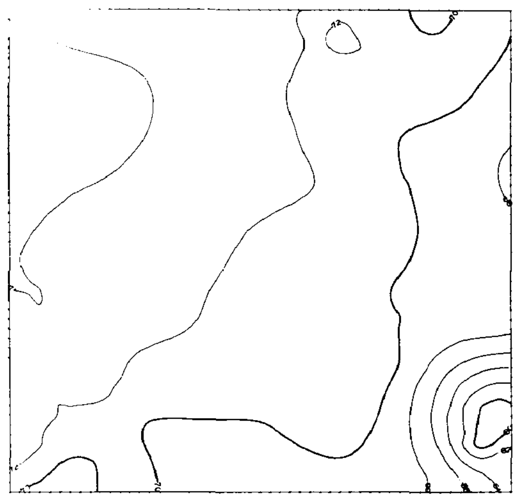


C2 — Square 2. Optimum chart.
Sounding line interval 600 m.

0 200 400 m



C3 — Square 2. Optimum chart.
Sounding line interval 1 200 m.



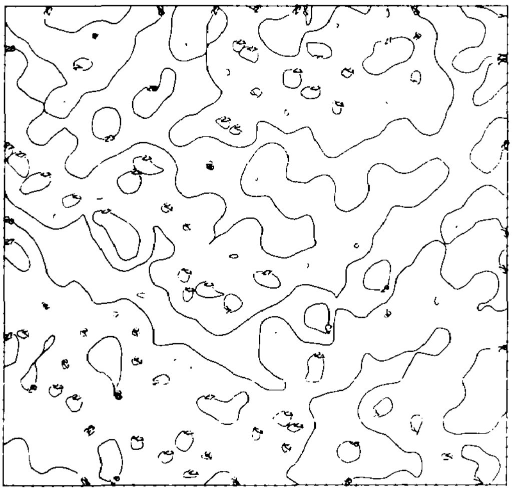
C4 — Square 2. Security chart.
Sounding line interval 300 m.

(Depths in metres; isobath interval 2 m)

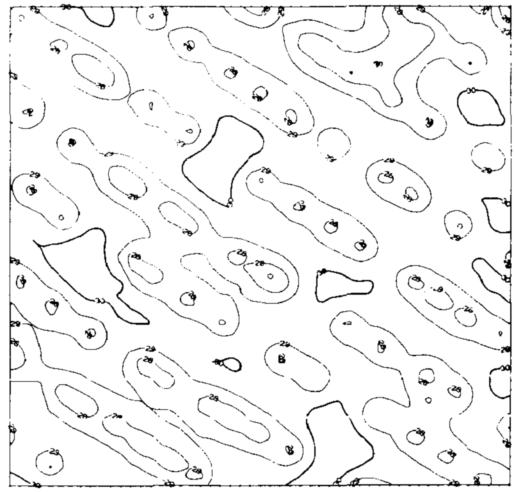
and 35 % approximately; the increase is small in the first two cases but significant for the 1 200 m spacing.

Large scale charts

The objective here is a chart with the maximum degree of accuracy incorporating all available information. The chart has thus been drawn up using estimations at the nodes of a 40 m grid. For the purposes of our

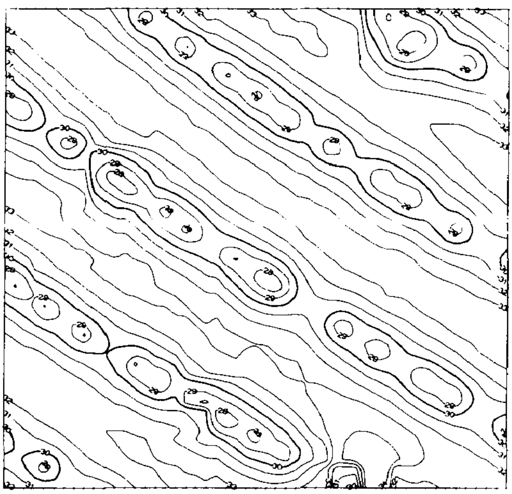


B5 — Square 1. Estimation standard deviation. Sounding line interval 150 m.

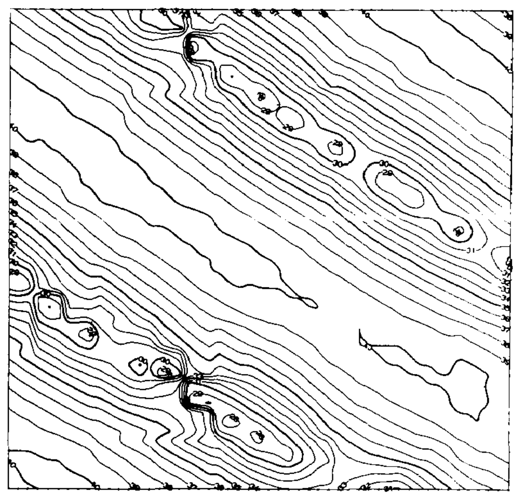


B6 — Square 1. Estimation standard deviation. Sounding line interval 300 m.

0 200 400 m



B7 — Square 1. Estimation standard deviation. Sounding line interval 600 m.

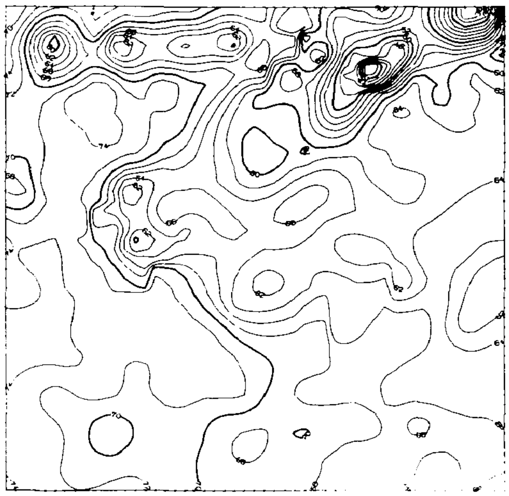


B8 — Square 1. Estimation standard deviation. Sounding line interval 1200 m.

(Standard deviation in decimetres; isoline interval 0.1 m).

study a chart of the whole plotting sheet area was not necessary, and thus we confined ourselves to two 2 000 m squares — one the most highly irregular square, for it is precisely in such areas that the various cartographic methods are best differentiated; and the other more typical of the average topography.

First of all optimum charts were drawn up for the estimation variance (aim 1). Here again charts were established for the different sounding densities, i.e. for lines 150, 300, 600 and 1 200 m apart (Charts B1 - B4 for the first square and C1 - C3 only for the second square, as the profiles there were run 300 m apart). It is particularly apparent that for these



B9 — Square 1. Security chart. Sounding line interval 150 m.
(Depths in metres; isobath interval 2 m)

charts of detail a lack of information immediately leads to large errors, even in the relatively regular area of the second square (see charts C1 and C2). On charts B1 to B4 of the first square it is seen that the principal shoal is increasingly badly positioned and takes a progessively more rounded form as fewer profiles are taken into account, and on Chart B4 where one in eight sounding lines were used this shoal has completely disappeared. On these same charts the 70 m isobath, for instance, changes aspect completely from chart to chart. On Chart B2 in particular a slight shoal in the NW corner is artificially given prominence. Thus it is clear that for charts of detail one cannot have too many soundings. As was feared from the outset, the strong nugget effect conveys the existence of fluctuations of lesser dimension than the grid.

Charts of the estimation standard deviation were also drawn up for the first square (Charts B5 to B8). The first is obviously fairly unreadable, since the distribution of the data is regular and the standard deviation

Sounding line spacing (m)	Square 1		Square 2	
	mean (m)	maximum (m)	mean (m)	maximum (m)
150	2.78	2.95	–	–
300	2.91	3.07	1.17	1.26
600	3.15	3.46	1.31	1.46
1200	3.60	4.09	1.59	1.85

FIG. 4. — *Estimation standard deviation relatied to sounding line spacing.* (Sounding lines in Square 2 started at 300 m spacing).

is practically constant (between 2.70 and 2.90 m); on the other charts the experimental data used for the estimation show up clearly. Indeed in the immediate vicinity of each point the estimation standard deviation becomes minimum, indicated by two or three concentric lines around these points. These charts are shown merely by way of illustration; what should be noted is the magnitude of the standard deviation (cf. figure 4). Relatively small increases in the maximum standard deviation (i.e. that of points falling between two profiles) are nevertheless associated with very disparate charts.

As already noted, these optimum charts filter out topographic irregularities of smaller dimension than the spacing of the data, i.e. less than about 100 m. Moreover, it would usually be misleading to attempt to represent small fluctuations when the soundings are not very dense; the few peaks that are known will show up, but it is obvious that the chart cannot display the far greater number of so far unlocated peaks. Nevertheless in bathymetric cartography of the sea floor it is often important to draw up a chart showing all known shoals, even if others exist in fact. Security charts are then required (aim 2), and these have been established for both squares utilizing all available soundings (Charts B9 and C4). The charts are entirely different in aspect from the optimum charts, for here particular attention is paid to encircling each known shoal with a series of concentric lines. The shoals are thus certainly extended, but they are nevertheless depicted. The principal shoal in the first square therefore stands out very clearly. At this particular spot the optimum chart and the security chart differ by 10 m. However, it should be borne in mind that such a chart merely represents known shoals; and as a result the cartographer when he is dealing with areas of irregular topography simply shows characteristic soundings and does not aspire to a faithful representation of the bottom topography by means of contour lines.

CONCLUSION

Kriging techniques, since they take acount of the structure of the phenomenon under study, enable a specific solution for each particular problem to be found. Here we have described the technique employed for drawing up both optimum charts and security charts. The possibility of drawing up trend charts must not, however, be forgotten, i.e. charts depicting only fairly large structures of over a kilometre in size. At the other end of the scale it would be possible to make conditional simulations which, without being the actual reality, would nevertheless give an excellent idea of the small irregularities of the sea floor.

Let it be remembered too that every estimation has its own particular estimation standard deviation characterizing its accuracy. This is most important when it comes to assessing what degree of confidence to place in the chart — or to deciding whether to carry out a new survey campaign, since the standard deviation chart will indicate which areas are the least well known. Furthermore, the sites for new soundings can be worked out, and the resulting standard deviation calculated. The potential yield in extra information from such new soundings can thus now be quantified.

Finally, we may observe that the implementation of this theory does not demand an excessive computing time; and thus a chart drawn up from programs developed at the Centre de Morphologie Mathématique is no more costly than the classical automated scribing of contour lines.

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