International Hydrographic Review, Monaco, LII (2), July 1975

SIMPLIFICATION IN OBSERVATION AND COMPUTATION OF A TWO-STAR FIX
WITHOUT USE OF THE ALTITUDE DIFFERENCE METHOD

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INTRODUCTION

This paper explains the changes introduced in the procedure for observation and computation of a two-star fix using KOTLARIĆ's Tables K_{11} [1], published by the Yugoslav Hydrographic Institute (*). A practical example is given.

These changes eliminate the need to take the compass azimuth of the first star, and to correct the first star's altitude for the time elapsed between non-simultaneous observations of two stars. Instead, a shorter, simpler and more accurate procedure for correcting a fix for elapsed time is introduced. This is done directly by applying corrections from the Main Tables to the tabulated Latitude and LHA Aries; from the latter the Longitude is determined by adding the tabulated SHA of the second star and subtracting its GHA.

Details of the Direct Method for computation of the coordinates of a ship's position from observations of two selected stars without the need to use the altitude difference method (Marcq St. Hilaire method) were given by the author in the July 1971 issue of the *International Hydrographic Review* [2]. It was there shown that the computation of altitude and azimuths — in order to obtain intercepts for plotting position lines, the intersection of which gives the observer's position — is not necessary if Tables K_{11} are used, since the fix (the observer's Latitude and Longitude) is obtained directly from the observed altitudes of two selected stars. Also given in that article were the formulae used in the construction of Tables K_{11}, together with excerpts from the Tables and instructions for their use. A practical example was included. The way in which corrections have

(*) Volume V N (Latitude 40° to 49°30′ North), published 1971.
Volume IV N (Latitude 30° to 39°30′ North), published 1972.
Volume III N (Latitude 20° to 29°30′ North), published 1974.
Volumes II N & I N, covering 20° North to the Equator, are in press.

to be applied for non-simultaneous observations in order to obtain a fix was also demonstrated, the principle being the same as if simultaneous observations were taken.

As regards non-simultaneous observations, there is still a need to correct for the ship's run between the first and second star sights; Tables III A and B remain in force, no difficulties having been discovered or criticisms received from mariners. Even if the heading angle of the first star (i.e. the relative azimuth) is out by 5° the resulting error can not exceed 0'.3.

In the case of the time elapsed between the first and the second sight (i.e. the increase of the Greenwich Hour Angle of the First Point of Aries) it was demonstrated that a correction has to be applied to the observed altitude of the first star using the well-known formula : diff. Alt = diff. LHA × cos Lat × sin AZ. This gives the first star's altitude as if it were taken simultaneously with the altitude of the second star. In order to compute this correction it was necessary to take the compass azimuth of the first star and to transform it into true azimuth; the compass azimuth had to be taken immediately after the second star sight. For simplification, i.e. to do away with interpolations, sign rules, etc., the author constructed a special auxiliary table (Tables IV A and B) which rendered this computation considerably shorter and simpler than if the formula given above were solved step by step. However, in spite of this simplified table for correction of non-simultaneous observations, one difficulty has remained. This is the taking of a correct compass azimuth of the first star. The new procedure described in the present paper eliminates this drawback, and thus both the computation of the fix and the observations of the two stars are shortened and simplified.

EXPLANATION OF THE NEW PROCEDURE

In the Direct Method, the need to take the compass azimuth of the first star was an extra step in the non-simultaneous observations of two stars, one not required in the St. Hilaire Method. Moreover navigators have complained of the difficulties in taking the bearing of a star at high altitudes. The accuracy of such an azimuth is thus liable to error. Considering that the non-simultaneous observation of stars is the most common procedure in celestial navigation, one must realise that in the procedure of correcting the altitude of the first star with the above formula (using Tables IV A and B) an error of 1° in the observed azimuth can produce an error in the corrected altitude of 1', leading to a possible position error of greater than 1 nautical mile. This is because the error in altitude of the first star (dHo_1) is reflected both in angle B and in the parallactic angle *ni*; and the error in the parallatic angle (d*ni*) is further reflected both in the computed Latitude (Lat) and in the Local Hour Angle of the second star (LHA_2), i.e. in the Longitude. The following formulae explain this :

$dB = - dHo_1 . \sec Ho_2 . \operatorname{cosec} (AZ_1 - AZ_2)$

$ni = A+B,\ ni = A - B,\ ni = B - A,\ ni = 360° - (A+B)$

$dLat = - dni . \cos Ho_2 . \sin AZ_2$

$dLHA_2 = - dni . \cos Ho_2 . \cos AZ_2 . \sec Lat$

These formulae were derived in the present author's earlier work "New Methods of Ship Position Finding from Celestial Observations" (1955) [3]. As was noted on pages 158-159 (in Serbo-Croat language) of the second edition (1973), an error of + 1′ in Ho_1 leads to an error in Latitude of — 1′.1, and in LHA_2 to an error of + 0′.2 in Longitude.

It is important to note that with the new procedure here described for direct computation there is no possibility of such azimuth errors, since the need to take the first star's compass azimuth and to make the corresponding correction to its altitude based on this azimuth have both been eliminated. Thus the auxiliary Tables IV A and B are no longer necessary. Instead, the new procedure introduces corrections for the coordinates of the observer's zenith (the Latitude and LHA Aries) taken directly from the Main Tables in K_{11}. This correction is based on a single argument, i.e. the elapsed time between sights of the first and the second star. It is obtained from Table IV C as a correction for the change in difference of sidereal hour angles (diff. SHA) of the two stars with its sign, which is multiplied (through Table V - Multiplication Table) by the correction indices (given in the Main Tables) to allow for a change of sidereal hour angles (ISU).

Table IV C and an example of this method of computing a fix are shown below in detail. Firstly, however, it must be shown why the correction of the first star's altitude for the elapsed time is identical with the correction to the Latitude and LHA Aries, both representing the sidereal hour angle difference for the time elapsed between the two star sights.

As is well known, the difference between the sidereal hour angles of two stars is numerically equal to the difference between their Greenwich hour angles ($SHA_1 — SHA_2 = GHA_1 — GHA_2$). Accordingly, if 2 m 4 s has elapsed between observations of the first and the second star then an increment of 31′.1 of GHA Aries is obtained from the Nautical Almanac. This indicates that the Greenwich hour angle of the first star has been similarly increased; consequently the difference between Greenwich hour angles for these two stars amounts to the same value. The result would be the same if the difference between *sidereal* hour angles of these stars were changed by the same amount of 31′.1. Thus corrections to the coordinates of the observer's zenith, i.e. corrections to the Latitude and LHA Aries (Longitude), can be computed to allow for the change in LHA of the first star in the same manner as for the change in its sidereal hour angle. It will however be necessary to pay attention to the *signs* of the Lat. and LHA Aries corrections since in Tables K_{11} the indices of the corrections for changes in difference of sidereal hour angles (the ISU indices) are tabulated for + 1′ increments of tabulated difference of sidereal hour angles ($SHA_1 — SHA_2$), always provided that declinations and altitudes of both stars remain unchanged. Furthermore, the elapsed time between sights of the first and the second star transformed with the help of the

Nautical Almanac into an increment of GHA Aries — this is seen in the sky as apparent motion of the star from East to West — can be expressed as an increase or decrease in the meridian angle of the first star, depending on its azimuth. This means an increase or decrease in the first star's altitude, and consequently the conditions employed when tabulating the sign for ISU indices are not the same. Hence, Latitude and LHA Aries corrections for increase of Greenwich hour angle of the first star will not always have the same sign as that of the increase in the difference of tabulated sidereal hour angles $SHA_1 - SHA_2$. If ISU indices for Lat and LHA corr., with their signs, are employed then the sign for the correction to the difference of sidereal hour angles must also be known, so it is now given above the tabulations in Table IV C. With the elapsed time as the entering argument, the numerical value is found. From a study of the celestial sphere a very simple rule for this sign has been drawn up. This is : the sign for corr. diff. SHA is always +, except when the first star is west of the second one, when it will be —. Compass azimuths of stars are not necessary for this purpose because in the Main Tables K_{11} the azimuths are tabulated below the star names. Thus at a glance it can be seen if AZ_1 is greater than AZ_2; it is evident also from the star observations whether the first star is west or eastward of the second one.

Table IV C therefore yields corr. diff. SHA with its sign, and this has to be algebraically added to the value diff. SHA which has been obtained by subtracting the $(SHA_1 - SHA_2)$ value shown at the top of the pages in Main Tables K_{11} from the $(SHA_1 - SHA_2)$ value read from the Nautical Almanac, in order to determine a final diff. SHA. By application of the ISU indices from the Main Tables for Lat. corr. and LHA corr., this final diff. SHA yields corrections to both Latitude and LHA Aries, with their signs; the multiplication Table V is used for this operation.

The above explanations may seem complicated to a mariner, but they are intended primarily for those interested in making a deeper study of navigational methods. In fact, the advantage of the improved method is that the mariner has no need to go into the theory because the Instructions for Use together with the solution of an example printed at the end of the present paper enable an easy understanding of this computational method for a fix. The use of Tables K_{11} is further facilitated by a cardboard insert of the Instructions for Use, including a sample solution printed in five colours.

The example given on page 105 of my 1971 paper [2] shows that for the time elapsed between observations of Antares and Vega the correction of the first star's altitude was 21'.1 (see Table IV A and B), with as entering arguments the true azimuth of Antares (136°) and the dead reckoning latitude (12°.2). This correction (diff. $Ho_1 = +21'.1$) when multiplied by the tabulated indices IV_1 (Indices of corrections to the Latitude and Local Hour Angle of Aries for a change of + 1′ in the first star's altitude) yields the following corrections to the Latitude and LHA Aries :

	diff.	Index LAT.	LAT. corr.		diff.	Index LHA	LHA♈ corr.
Ho_1 (V_1)	+ 21′.1 ×	− 80 =	− 16′.9	Ho_1 (V_1)	+ 21′.1 ×	+ 62 =	+ 13′.1

Meanwhile, by the new procedure and the elapsed time of 2 m 4 s taken from the new Table IV C (see page 164) we obtain the correction to the difference of sidereal hour angles as corr. diff. SHA = — 31'.1. If this correction is multipled by tabulated ISU correction indices the following corrections to the Latitude and LHA Aries are obtained :

	diff.	IND. LAT.	LAT. corr.		diff.	IND. LHA	LHAγ corr.
SHA (SU)	− 31'.1	× + 54	= − 16'.8	SHA (SU)	− 31'.1	× − 42	= + 13'.0

This example worked by the two different methods, (a) the auxiliary Tables IV A and B, and (b) the new auxiliary Table IV C, shows that the corrections to the fix for elapsed time are very nearly identical in both cases. With Table IV C there is a difference of 0'.1 for each correction, compared with the Table IV A and B results, but the former represents more accurate values since Table IV C applies elapsed time corrections directly to the coordinates of the observer's zenith found in the Main Tables K_{11}. This is better than the indirect procedure using Tables IV A and B to find an approximate solution for diff. Ho_1 = diff. LHA × cos Lat. × sin AZ, where the azimuth is liable to error. The new procedure using Table IV C is furthermore simpler both for observations and for computation.

Stationary observer

It is now considered useful to explain why there is a need to correct the Latitude and LHA Aries (i.e. computed observer's position) if the ship does not move during the elapsed time between the observation of the two stars. It must be remembered that Tables K_{11} are entered with the two star altitudes not taken simultaneously, and thus the fix obtained from Tables K_{11} is erroneous since the change in difference of their sidereal hour angles for the time elapsed between the observations has been neglected. Thus the corrections to Lat. and LHA Aries obtained by use of Table IV C and ISU Indices from the Main Tables in fact represent corrections for this neglected error rather than for movement of the actual position of the observer.

This new procedure for correcting a two-star fix for the time elapsed between the observations has necessitated changes in the current Instructions for Use for Tables K_{11}, and alterations have accordingly been made in paragraphs 3, 4, 5, 6, 9, 10, 11, 14 and 16. The new shortened Instructions for Use are given in Serbo-Croat and in English (reprinted overleaf) accompanied by a Work Form printed in five corresponding colours, and show the solution of the following example of the two-star fix by the Direct Method using Tables K_{11}.

Example

On June 12, 1968, the zone time $19^h 50^m$ dead reckoning position of a ship is Lat. 41°50' N, Long. 17°10' E. The ship is under way, the speed

SHORTENED INSTRUCTIONS FOR USE

Preliminary work and work with Nautical Almanac

1. Using the Approx. GMT for the intended observation and DR Long. compute the Approx. LHA♈.
2. With the approx. DR Latitude and LHA♈ found in Step 1 turn to the pages in Tables K11.
3. Choose one of the four pairs of stars given either on the left or the right page and take a sight on the first and then on the second star.
4. Observations of the two stars can be made with any time interval up to 4 minutes. The second sight can be taken within a short interval if the altitude of the second star is measured approximately in advance. If the 4-minute interval is exceeded, and the observations cannot be repeated, the first star's altitude should first be corrected from Table III for 4 minutes and then for the remainder of the time (See Step 11). In that case Table IV C in Step 14 will be used in the same way.
5. For non simultaneous observations take, for the first star, azimuth relative to course (heading angle, 5° accuracy satisfies). In addition take note of the speed of the ship.
6. Solve the problem by inserting the values in their respective places on the work form. (The work form booklet can be purchased).
7. The position is being solved for the time of the second sight. Using the GMT for the second sight find the GHA♈ and the coordinates for both stars (D1, D2, SHA1 and SHA2).
8. Find the value SHA1 – SHA2 by subtracting the smaller value. If the result exceeds 180° subtract it from 360°.
9. Find the GHA of the second star (GHA2).
10. Determine the difference of GMT between the two sights (d. GMT).
11. Correct the sextant altitude of the first star (hs1) by Table I (or by Table II if a bubble sextant was used) to obtain the observed altitude Ho1. If the two sights were taken at different times (see Step 4), the value Ho1 should now be corrected by Table III to obtain Ho1 cortd. The signs are given in the Table and no interpolations are needed.
12. Correct the sextant altitude of the second star (hs2) by Table I (or by Table II) to obtain the observed altitude Ho2.

Use of the Main Tables and Multiplication Table

13. Reenter the Main Tables on the same page and stars, but in columns Ho1 and Ho2, using the line showing figures nearest to the observed altitudes Ho1 (or Ho1 cortd.) found in Step 11, and the Ho2 found in Step 12.
    From this line take out: (a) the tabulated values Ho1 and Ho2; (b) LHA Aries; (c) the correction indices IV1, IV2, ID1, ID2 and ISU, with their signs, from the columns "Index LAT. Corr." and "Index LHA Corr." (all the values printed without a sign are +). From the top of the Table take: Lat., D1, D2 and SHA1–SHA2.
14. Find the differences, together with their signs:
    Ho1 (cortd.) – Ho1 tabulated = diff. Ho1;
    Ho2 – Ho2 tabulated = diff. Ho2;
    D1 – D1 tabulated = diff. D1;
    D2 – D2 tabulated = diff. D2;
    (SHA1 – SHA2) – (SHA1 – SHA2) tabulated = = diff. SHA, then add corr. from Table IV C.
15. Insert the signs of the LAT. and LHA corrections in the work form after determining them by means of the signs for the differences obtained in Step 14 and of the signs of the corresponding correction indices taken from the Main Table, in Step 13 (c). When these differences are positive the signs for the corrections are the same as those for the correction indices in the Main Table. For the negative differences the signs are opposite.
16. Enter Table V with the correction index IV1 on the left and diff. Ho1 at the top to find the required correction. The correction to the tabulated Latitude is obtained by using the IV1 of the Index LAT. Corr. Then using the IV1 of the Index LHA Corr. the correction to LHA Aries is obtained.
    Do the same for the IV2 and diff. Ho2, ID1 and diff. D1, ID2 and diff. D2, ISU and diff. SHA. No mental interpolation is required.
17. Add the positive corrections and subtract the sum of the negative corrections, in order to obtain the true values for LATITUDE and LHA♈.
18. To the LHA♈ add the tabulated SHA2 from the top of the Main Table to obtain the LHA2. Deduct 360° if necessary.
19. Subtract the GHA2 (Step 9) from the LHA2 to obtain the LONGITUDE with its sign (– for W and + for E). If the result is greater than 180° subtract it from 360°, changing the sign.

WORK FORM FOR DIRECT FINDING OF A TWO – STAR FIX WITH KOTLARIĆ'S TABLES K11

Date 12 June 1968, zone time $19^h\ 50^m$, zone description – 1^h,
dead reckoning Lat. 41° 50' N, Long. 17° 10' E,
1st star's AZ relative to course 16°, speed 16 kn, h. eye 11 m, IC 0'.

Approx. LAT.	42° N
Greenwich date	12. VI – 1968
Approx. GMT	$19^h\ 00^m$
Approx. GHA♈	171.1°
corr.	15.0
Approx. GHA♈	186.1
+ DR Long.	+ 17.2 E
Approx. LHA♈	203.3°
GMT 2	$18^h\ 59^m\ 46.0^s$
– GMT 1	18 58 05.0
d. GMT	$1^m\ 41.0^s$
1st star's Relative AZ	16°

1. SPICA		2. REGULUS	
CT	$18^h\ 55^m\ 45.5^s$	CT	$18^h\ 57^m\ 26.5^s$
CE	+ 2 19.5	CE	+ 2 19.5
GMT 1	$18^h\ 58^m\ 05.0^s$	GMT 2	$18^h\ 59^m\ 46.0^s$
hs 1	37° 19.1'	hs 2	36° 30.7'
IC	0	IC	0
Tab. I ~~or II~~	– 7.2	Tab. I ~~or II~~	– 7.2
Ho 1	37° 11.9'		
Tab. III	+ 0.4		
Ho 1 cortd.	37° 12.3'	Ho 2	36° 23.5'
– tabulated	– 36 57.5	– tabulated	– 36 29.5
diff. Ho 1	+ 14.8'	diff. Ho 2	– 6.0'
Dec. 1	– 10° 59.9'	Dec. 2	+ 12° 07.5'
– tabulated	∓ 11 00.4	– tabulated	± 12 06.8
diff. D1	+ 0.5'	diff. D2	+ 0.7'
SHA 1	159° 07.0'	GHA♈	171° 07.2'
– SHA 2	208 19.8	corr.	14 59.0
SHA 1 – SHA 2	49° 12.8'	+ SHA 2	208 19.8
– tabulated	– 49 12.2	GHA 2	394° 26.0'
diff. SHA	+ 0.6'		
Tab. IV C	+ 25.3		
diff. SHA	+ 25.9'		

- ● Main Tables
- ● Tab. I, II, III and IV
- ● Tab. V
- + diff × ± IND = ± corr.
- – diff × ± IND = ∓ corr.

diff.		IND. LAT.	LAT. + corr.	LAT. – corr.	IND. LHA	LHA Aries + corr.	LHA Aries – corr.
			42° 00.0' N			203° 00.0'	
Ho 1 (V 1)	+ 14.8'	– 102		– 15.1'	+ 45	+ 6.7	
Ho 2 (V 2)	– 6.0	+ 5		– 0.3	– 144	+ 8.6	
D 1	+ 0.5	+ 102	+ 0.5		– 46	·	– 0.2'
D 2	+ 0.7	– 4		– 0	+ 99	+ 0.7	
SHA (SU)	+ 25.9	+ 4	+ 1.0		– 2		– 0.5
			42° 01.5'	– 15.4'		203° 16.0'	– 0.7'
		LAT.	41° 46.1' N		LHA♈	203° 15.3'	
				+ tab.	SHA 2	208 18.1	
					LHA 2	411 33.4	
					– GHA 2	– 394 26.0	
					LONG. +	17° 07.4' E	

EXCERPT FROM TABLES K_{11} — MAIN TABLES

42 N FI (LAT.)	SPICA*	REGULUS*	SU1 - SU2 = 49 12.2 159 05.9 - 208 18.1 = (SHA1-SHA2)								
ARIES	AZ 178-202 D1 -11 00.4	AZ 249-264 D2 +12 06.8	(INDEX LAT. CORR.) INDEKSI POPRAVAKA FI					(INDEX LHA CORR.) INDEKSI POPRAVAKA MS			
MS (LHA)	V1 (HO1)	V2 (HO2)	IV1	IV2	ID1	ID2	ISU	IV1 IV2	ID1	ID2	ISU
° '	° '	° '									
200	36 59'2	38 35'5	-99	-1	99	1	-1	48-143	-52	100	1
202	36 59'0	37 11'7	-101	3	101	-2	2	46-144	-48	100	-1
202 30	36 58'4	36 50'6	-101	4	101	-3	3	46-144	-47	99	-1
203	36 57'5	36 29'5	-102	5	102	-4	4	45-144	-46	99	-2
203 30	36 56'4	36 8'3	-102	7	102	-4	5	44-144	-45	99	-2
220	34 9'9	24 8'7	-113	45	108	-29	33	14-140	-14	91	-4

TABL. IV C

CORRECTION TO THE diff.SHA FOR THE TIME ELAPSED UNTIL SECOND STAR SIGHT

(corr. diff.SHA always "+", except "—" when the first star is west of the second)

m 0	popr. (corr.)	m 0	popr. (corr.)	m 1	popr. (corr.)	m 1	popr. (corr.)	m 2	popr. (corr.)	m 2	popr. (corr.)	m 3	popr. (corr.)	m 3	popr. (corr.)
s	'	s	'	s	'	s	'	s	'	s	'	s	'	s	'
0	0.0	30	7.5	0	15.0	30	22.6	0	30.1	30	37.6	0	45.1	30	52.6
1	0.3	31	7.8	1	15.3	31	22.8	1	30.3	31	37.9	1	45.4	31	52.9
2	0.5	32	8.0	2	15.5	32	23.1	2	30.6	32	38.1	2	45.6	32	53.1
3	0.8	33	8.3	3	15.8	33	23.3	3	30.8	33	38.4	3	45.9	33	53.4
4	1.0	34	8.5	4	16.0	34	23.6	4	31.1	34	38.6	4	46.1	34	53.6
5	1.3	35	8.8	5	16.3	35	23.8	5	31.3	35	38.9	5	46.4	35	53.9
6	1.5	36	9.0	6	16.5	36	24.1	6	31.6	36	39.1	6	46.6	36	54.1
7	1.8	37	9.3	7	16.8	37	24.3	7	31.8	37	39.4	7	46.9	37	54.4
8	2.0	38	9.5	8	17.0	38	24.6	8	32.1	38	39.6	8	47.1	38	54.6
9	2.3	39	9.8	9	17.3	39	24.8	9	32.3	39	39.9	9	47.4	39	54.9
10	2.5	40	10.0	10	17.5	40	25.1	10	32.6	40	40.1	10	47.6	40	55.2
11	2.8	41	10.3	11	17.8	41	25.3	11	32.8	41	40.4	11	47.9	41	55.4
12	3.0	42	10.5	12	18.0	42	25.6	12	33.1	42	40.6	12	48.1	42	55.7
13	3.3	43	10.8	13	18.3	43	25.8	13	33.3	43	40.9	13	48.4	43	55.9
14	3.5	44	11.0	14	18.6	44	26.1	14	33.6	44	41.1	14	48.6	44	56.2
15	3.8	45	11.3	15	18.8	45	26.3	15	33.8	45	41.4	15	48.9	45	56.4
16	4.0	46	11.5	16	19.1	46	26.6	16	34.1	46	41.6	16	49.1	46	56.7
17	4.3	47	11.8	17	19.3	47	26.8	17	34.3	47	41.9	17	49.4	47	56.9
18	4.5	48	12.0	18	19.6	48	27.1	18	34.6	48	42.1	18	49.6	48	57.2
19	4.8	49	12.3	19	19.8	49	27.3	19	34.8	49	42.4	19	49.9	49	57.4
20	5.0	50	12.5	20	20.1	50	27.6	20	35.1	50	42.6	20	50.1	50	57.7
21	5.3	51	12.8	21	20.3	51	27.8	21	35.3	51	42.9	21	50.4	51	57.9
22	5.5	52	13.0	22	20.6	52	28.1	22	35.6	52	43.1	22	50.6	52	58.2
23	5.8	53	13.3	23	20.8	53	28.3	23	35.8	53	43.4	23	50.9	53	58.4
24	6.0	54	13.5	24	21.1	54	28.6	24	36.1	54	43.6	24	51.1	54	58.7
25	6.3	55	13.8	25	21.3	55	28.8	25	36.3	55	43.9	25	51.4	55	58.9
26	6.5	56	14.0	26	21.6	56	29.1	26	36.6	56	44.1	26	51.6	56	59.2
27	6.8	57	14.3	27	21.8	57	29.3	27	36.9	57	44.4	27	51.9	57	59.4
28	7.0	58	14.5	28	22.1	58	29.6	28	37.1	58	44.6	28	52.1	58	59.7
29	7.3	59	14.8	29	22.3	59	29.8	29	37.4	59	44.9	29	52.4	59	59.9
30	7.5	60	15.0	30	22.6	60	30.1	30	37.6	60	45.1	30	52.6	60	60.2

Note: Whether the first star is west of the second it is also shown in Main Tables by tabulated azimuths, i. e. whether AZ1 is greater than AZ2.

16 knots, zone description — 1^h. The true position is wanted by observations of two stars at approx. ZT 20^h (GMT 19^h), and solved by Tables K_{11}. From the Nautical Almanac with approx. GMT and D.R. Longitude the approx. LHA♈ 203° is obtained. With this and approx. Lat. 42° N the Tables K_{11} are opened and the pair of stars SPICA - REGULUS chosen for observation. The sights are taken, firstly on Spica hs_1 37°19'.1, chronometer time CT_1 $18^h\,55^m\,45.5^s$, then on Regulus hs_2 36°30'.7, CT_2 $18^h\,57^m\,26.5^s$. Chronometer error on GMT is $+\,2^m\,19.5^s$. Spica's heading angle (azimuth relative to course) is 16°. Index correction of the sextant is 0, height of eye 11 m. Find a Fix at the time of the second sight.

Note: The necessary excerpts from Main Tables K_{11} and Table IV C are given opposite.

The same example by the previous method, i.e. using the Table IV A and B, was shown in Volume V of Tables K_{11}, page 0.29.

PRINTED AMENDMENTS

The new procedure for computation of a two-star fix from non-simultaneous observations, as here described, will be included not only in Volumes I, II and III of Tables K_{11} now in press, but also in Volumes IV and V already issued.

The latter already contain the special five-colour cardboard inserts in English and Serbo-Croat; owners of these volumes may now obtain the revised inserts free of charge from : The Hydrographic Institute, 58000 SPLIT, Yugoslavia, to enable them to use this new and shorter procedure for the observation and computation of a two-star fix using any volume of Tables K_{11}.

CONCLUSION

The essential difference in the new procedure is the elimination of the need to take the compass azimuth of the first star; this answers the main criticism of Tables K_{11}. Also, the new procedure, besides being shorter and simpler, is more accurate. What now remains to be done is the process of familiarizing navigators with this new method and of breaking down the barrier of conservatism which is a fairly common hazard for all innovations.

However, it must be remembered that Tables K_{11} enable calculation of the fix coordinates direct from the Tables themselves, and eliminate the precise graphical work on the plotting sheet or an approximate sketch, and the mathematical process of finding the coordinates for the intersection of the position lines required in the St. Hilaire method. This mathematical process becomes more complicated when position lines are plotted from different assumed positions as is usual in navigational practice. In this respect the Direct Method is preferable to the indirect Altitude Difference Method.

THE FUTURE

We live in an era of ever increasing applications for computers, and the efforts being made all over the world to introduce electronic calculators and computers aboard ship cannot be ignored. Computer reduction of celestial observations to lines of position will thus be possible, and the need for the different types of existing celestial navigation tables may be diminished. In my estimation, the above described Direct Method of finding fix coordinates will be of more interest than the St. Hilaire Method for computer applications.

Many different short method tables for altitude and azimuth computation exist, and as far as I know create no difficulties for navigators. The fact that my own Tables K_1 (1958) [4] were in their third edition in 1971 testifies to their continued wide use (see reviews of these Tables [5], [6]).

Accordingly, in my opinion, there is less need to replace the short method tables by mini-calculators or computers to give altitudes and azimuths than to develop a computer method giving directly the fix coordinates (Latitude and Longitude). With this goal in mind my book "New Methods of Ship Position Finding from Celestial Observations" [3], 1973 edition, suggested the annual tabulation in the Nautical Almanac of about 50% of the data required for this Direct Method for computation of a two-star fix; the remaining calculations — one angle (B) in the spherical triangle from three known sides, and one angle (LHA) and one side (Lat.) from the other spherical triangle with two known sides and their included angle — could be carried out easily with a celestial navigation computer, or even a hand-held calculator. With Latitude and LHA computed it will not be difficult to find the Longitude since this is obtained, with its sign, from the Local and the Greenwich hour angles (Long. = LHA − GHA).

The application of this Direct Method to double observations of the Sun was also demonstrated in the same book, and the solution of a practical example given. This would be yet another use to which one of these inexpensive pocket-size calculators could be put as part of the navigator's personal equipment.

Research is at present being carried out into the applications of electronic calculators and computers for work onboard ship in several different countries; in the U.S.A., Hewlett-Packard, and the Marine Division of the Micro-Instrument Company have both made significant contributions to this new development. However, only the future will show whether the new developments will meet navigators' wishes to such an extent that they will finally be convinced of the need to change their conservative methods for sight reduction, packing away their reduction tables to be kept in reserve, and to take up the electronic box for their calculations. For the time being, short method tables remain in predominant use for solving a fix from celestial observations — even on board ships belonging to countries which lead the way in computer production and the applications of computer techniques to the various kinds of human activity. Just how long the existing shipboard situation will continue is difficult to foresee; the choice had better be left to the user; in other words let us leave navigators the major responsibility of deciding which of the accessories available — the tables or the electronic gadgets — they will use for their onboard calculations.

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(Manuscript submitted in English)