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THE IDENTIFICATION OF TIDAL CONSTITUENTS

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ABSTRACT

Shallow water effects create constituents whose frequencies may overlap those of the constituents of direct gravitational origin. Such frequencies are investigated and the problem of the unambiguous identification of tidal constituents is considered.

INTRODUCTION

An analysis yields the amplitude and phase of the tidal constituents which will be used subsequently in the preparation of predictions. In order to ensure the success of such predictions it is essential that :

a) the constituents be properly identified;

b) their amplitude and phase be virtually "constant" from year to year.

The need to assess the identity of constituents is not considered in the classical methods of analysis (Doodson, 1928; Lecolazet, 1956; Suthons, 1959) and it is impossible in fact to check on the identity of the analyzed constituents even from a one-year analysis. The stability of the constituents in time as well is seldom investigated systematically. This has slight relevance for earth tides or good oceanic tides but it becomes quite important in the study of shallow water tides where non linear combination of the fundamental constituents may at times coincide with, or fall very near, the frequencies of constituents of direct gravitational origin.

In this paper we will review the combinations of the fundamental constituents which lead to frequencies overlapping those of other constituents and we will outline a technique which can help clarify this situation. We must emphasize that this can be accomplished only if more than one year of observations is available.

FUNDAMENTAL CONSTITUENTS

We use this name to designate the constituents of direct gravitational origin which account for the bulk of tidal action at any point on the earth. These are (along with their Doodson numbers affecting the astronomical variables τ, s, h, p, N′, p′) :

$$
\begin{array}{lrrrrrr}
Q_1 & 1 & -2 & 0 & 1 & 0 & 0 \\
O_1 & 1 & -1 & 0 & 0 & 0 & 0 \\
P_1 & 1 & 1 & -2 & 0 & 0 & 0 \\
K_1 & 1 & 1 & 0 & 0 & 0 & 0 \\
N_2 & 2 & -1 & 0 & 1 & 0 & 0 \\
M_2 & 2 & 0 & 0 & 0 & 0 & 0 \\
S_2 & 2 & 2 & -2 & 0 & 0 & 0 \\
K_2 & 2 & 2 & 0 & 0 & 0 & 0
\end{array}
\qquad (1)
$$

All the shallow water constituents of significance result from their interaction, e.g. : NK_1, OP_2, NK_3, MN_4, $2MN_6$, $3MS_8$, etc. A study of the various possible combinations of (1) will therefore suffice to identify any shallow water constituent. At a given location, the order of magnitude of the shallow water constituents may be roughly estimated from the relative magnitudes of the fundamental constituents. Thus if M_2 predominates, the combination M_4 is liable to be larger than MS_4; similarly second order interaction normally yields larger constituents than those resulting from triple interaction. For instance M_4 will normally be larger than M_6, MS_4 will be larger than $3MS_4$, and so on. The relative magnitude of the fundamental constituents helps therefore in the search and identification of the shallow water constituents. The latter may fall on all frequency bands including the diurnal and semidiurnal where they may at times cause extreme perplexity in the evaluation of the results of successive analyses.

We deduce from the fundamental constituents given in (1) those combinations which fall on or near those of constituents of direct gravitational origin : these are shown in table 1. The constituents of shallow water origin such as NO_3, MK_3, M_6, etc. whose frequencies are well removed from those of the fundamental constituents present no particular problems in general except when two distinct combinations yield very close frequencies such as M_5 and $2MNO_5$, M_3 and NK_3, etc.

We distinguish three different types of situations in table 1 :

(a) brackets enclose pairs of constituents of identical frequencies whose nodal modulations are quite similar. Under such circumstances it is virtually impossible in practice to distinguish between the elements of the pair from a succession of analyses and it is best to assign the amplitude and phase analyzed at that frequency to the constituent of direct gravitational origin. This will cause little error in the prediction since both constituents have similar phases and nodal modulations.

(b) a star pinpoints a shallow water constituent whose frequency differs slightly from that of a constituent of direct gravitational origin. This difference cannot be detected from a one-year analysis, but a succession of analyses may reveal marked fluctuations in the amplitude and phase analyzed at that frequency whenever the shallow water constituent has an appreciable magnitude.

(c) no marks of any kind indicates a pair of identical frequencies but of different nodal modulations. If the shallow water constituent contributes significantly this will be indicated by a drift in the succession of analyzed values or by an intolerably large variability. It is possible at times to separate the individual components of such pairs.

TABLE 1

Shallow water constituents whose frequency falls near or coincides with the frequency of constituents of direct gravitational origin (*)

Diurnal Band			
Direct	Shallow	Direct	Shallow Water
Name and Doodson Number		Name and Doodson Number	
(Q_1	NK_1)	(K_1	MO_1)
1 −2 0 1 0 0	1 −2 0 1 0 0	1 1 0 0 0 0	1 1 0 0 0 0
(O_1	MK_1)	SP_1	K_1^{sw}
1 −1 0 0 0 0	1 −1 0 0 0 0	1 1 0 0 0 0	1 1 0 0 0 0
τ_1	MP_1	(J_1	MQ_1)
1 −1 2 0 0 0	1 −1 2 0 0 0	1 2 0 −1 0 0	1 2 0 −1 0 0
NO_1	NO_1	(SO_1	SO_1)
1 0 0 1 0 0	1 0 0 1 0 0	1 3 −2 0 0 0	1 3 −2 0 0 0
P_1	SK_1)		
1 1 −2 0 0 0	1 1 −2 0 0 0		
Semidiurnal Band			
QQ_2	*OQ_2	M_2	KO_2
2 −3 0 1 0 0	2 −3 0 3 0 0	2 0 0 0 0 0	2 0 0 0 0 0
(ϵ_2	MNS_2)	L_2	$2MN_2$
2 −3 2 1 0 0	2 −3 2 1 0 0	2 1 0 −1 0 0	2 1 0 −1 0 0
O_2	*$2N_2$	S_2	KP_2
2 −2 2 0 0 0	2 −2 0 2 0 0	2 2 −2 0 0 0	2 2 −2 0 0 0
(μ_2	$2MS_2$)	(K_2	K_2)
2 −2 2 0 0 0	2 −2 2 0 0 0	2 2 0 0 0 0	2 2 0 0 0 0
N_2	KQ_2	ζ_2	*MSN_2
2 −1 0 1 0 0	2 −1 0 1 0 0	2 3 −2 1 0 0	2 3 −2 1 0 0
Terdiurnal Band			
		M_3	*NK_3
		3 0 0 0 0 0	3 0 0 1 0 0

(*) We overlook the low frequency band which contains Sa, Ssa, MSm, Mm, MSf, Mf, etc. since the ratio of signal to noise is small there and makes any attempt at a careful analysis in that band futile.

A TECHNIQUE FOR THE IDENTIFICATION OF CONSTITUENTS

This technique is based on the postulate that the amplitude and phase of the tidal constituents are constant at a given station. Without this assumption we would not bother preparing tide tables and even less so making analyses. The stability in amplitude and phase may be marred by :

(a) the interference of close constituents;
(b) the blurring effect of the noise.

The interference of close constituents is virtually eliminated in a one-year analysis (one "year" may mean 355, 365, 369 days or whatever time interval appears suitable; we use 355 days) and all the fundamental constituents are fully resolved. On the other hand the presence of signals of non-tidal origin causes a range of fluctuation in the amplitude and phase given by (GODIN, 1970) :

$$\Delta A \simeq \frac{\pm\, 4s}{(2N+1)^{1/2}} \tag{2}$$

$$A\Delta\alpha \simeq \frac{\pm\, 4s}{(2N+1)^{1/2}} \tag{3}$$

where ΔA and $\Delta\alpha$ are the ranges in amplitude and phase (measured in radians), s^2 is the variance of the noise and $2N+1$ is the number of hourly observations. A is the mean amplitude and $A\Delta\alpha$ has units of length similar to ΔA : the fluctuations in amplitude and phase are directly comparable with such formulas. Formulas (2) and (3) are often violated in short analyses because of the presence of unresolved constituents, but they should hold for a one year analysis. We therefore formulate the rule for the proper identification of a constituent (or of a pair) : "A constituent is properly identified if, in a succession of yearly analyses, its amplitude and phase vary between the limits given by (2) and (3)".

The quantity s may be evaluated by various techniques (LECOLAZET, 1956; VENEDIKOV, 1966; DE MEYER, 1974). Personally we estimate it from the ranges observed in the analyzed constituents; these are known from experience to vary between rather constant limits for each band (GODIN, 1973) at a given station. Once these are established, any constituent that shows any systematic drift from year to year or which exhibits fluctuations which exceed the norm, is scrutinized and its identification is questioned. The following possibilities have to be considered :

(a) a single constituent contributes at this frequency (or near this frequency) but its identity is mistaken (leading to the use of the wrong astronomical argument and nodal modulation);

(b) two constituents of different frequencies but of comparable amplitudes contribute in the vicinity of the frequency chosen for analysis (their interference causes the observed variability);

(c) two constituents of identical frequencies contribute at the chosen frequency.

A SINGLE CONSTITUENT IMPROPERLY IDENTIFIED

It is quite possible — in the higher frequency bands which are filled by the contribution of the shallow water constituents — that two possible combinations of the fundamental constituent may explain the signal observed at a given frequency : for instance M_3 or NK_3 could explain a signal at 43°.5/hour, M_5 or $2MNO_5$ at 72°.5/hour, etc. These are of close but distinct frequencies; however a single one-year analysis cannot differentiate between M_3 or NK_3, M_5 or $2MNO_5$. It is only by doing a succession of yearly analyses that one can decide for one or the other. Indeed by assuming the wrong constituent, one chooses the wrong Doodson number and therefore will calculate a Greenwich phase lag g which will drift from year to year; a switch to the proper Doodson number will automatically remove this drift in phase and bring $\Delta\alpha$ to the limits given by (3). We illustrate this statement by studying a succession of analyzed values at Trieste and Quebec in table 2. The regular drift in phase of NK_3 at Trieste and M_5 at Quebec is quite evident in table 2. The reason why we had first picked NK_3 as the contributor to that frequency is that our experience in Canadian waters had led us to assign the observed signal at that frequency to NK_3 and our programs had been written accordingly. However the evidence at Trieste indicates quite clearly that M_3, which is of direct gravitational origin, is the cause of the signal. When it comes to M_3, we had assumed that since it originated from a third order interaction ($M_2+M_2+M_1$) it would tend to predominate over a term originating from a higher order interaction ($M_2+M_2+N_2-O_1$); however we had overlooked the fact that M_1 is a very faint line and does not constitute a fundamental constituent.

TABLE 2

Examples of the proper identification of constituents

Location	Frequency °/hour	Assumed Constituent	Doodson Number	Mean Amplitude	Measured Greenwich Phase (degrees)								$A\Delta\alpha$
					Yearly Sample No.								
					1	2	3	4	5	6	7	8	
Trieste	43.5	NK_3	300100	0.8 cm	70.2	112.8	160.9	199.6	222.9	268.1	300.6	343.4	–
		M_3	300000	0.8 cm	108.5	107.7	114.4	114.2	103.2	112.7	110.9	115.2	0.2 cm
Quebec	72.5	M_5	500000	0.025 ft	334.7	329.2	308.4	173.8	164.6	156.5	149.0	15.0	–
		$2MNO_5$	500100	0.025 ft	110.8	143.0	163.2	80.8	97.8	135.7	158.2	66.6	0.04 ft

A PAIR OF CONSTITUENTS

We may notice an abnormal behaviour in a sequence of analyzed amplitudes and phases at a given frequency; such behaviour may be due to the presence of a pair of constituents contributing at or near that frequency. If this is so, the raw analyzed amplitude A and phase α can be expressed by :

$$A_{\sin}^{\cos}(\alpha) = f_1 A_1 {}_{\sin}^{\cos}(V_1 + u_1 - g_1) + f_2 A_2 {}_{\sin}^{\cos}(V_2 + u_2 - g_2) \tag{4}$$

where (A_1, g_1) and (A_2, g_2) are the amplitudes and phase lags of the individual constituents making up the response while V_1 and V_2 are their astronomical arguments at the central time of the analysis. (f_1, u_1) and (f_2, u_2) are their individual nodal modulations. If we define :

$$x \equiv A_1 \cos g_1 \qquad y \equiv A_1 \sin g_1 \qquad \xi \equiv A_2 \cos g_2 \qquad \eta \equiv A_2 \sin g_2$$

and :

$$V + u \equiv V^1$$

equation (4) may be written as :

$$\begin{aligned} A \cos \alpha &= f_1 \cos V_1' \,.\, x + f_1 \sin V_1' \,.\, y + f_2 \cos V_2' \,.\, \xi + f_2 \sin V_2' \,.\, \eta \\ A \sin \alpha &= f_1 \sin V_1' \,.\, x - f_1 \cos V_1' \,.\, y + f_2 \sin V_2' \,.\, \xi - f_2 \cos V_2' \,.\, \eta \end{aligned} \tag{5}$$

A one-year analysis yields one set of (A, α); then we have two equations in the four unknowns $(x, y; \xi. \eta)$.

Two years of observations would resolve the pair in theory. If the constituents have slightly different frequencies, their astronomical arguments differ considerably from year to year; in the case of constituents of identical frequencies, their astronomical arguments are also equal and coefficients of the unknowns in (5) may differ by only a few percents from year to year. In the first case, one may consider that a solution is possible; one may check on this by extrapolating the resolution into the future or the past to see if it compares with observed amplitudes and phases. If it does, the resolution is satisfactory. In the second case, one should aim to use the results of all the analyses available to require a solution by least squares. The imposition of the latter to (5) yields a redundant set of equations which can be reduced to a single set by a simple linear combination of addition and subtraction. We write these in matrix form as :

$$\begin{pmatrix} A_1 & P \\ P^T & A_2 \end{pmatrix} \begin{pmatrix} X \\ \Xi \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \tag{6}$$

where T indicates the transpose :

$$A_i \equiv \sum_j \begin{pmatrix} f_{ij}^2 & 0 \\ 0 & f_{ij}^2 \end{pmatrix} \qquad P \equiv \sum_j f_{1j} f_{2j} \begin{pmatrix} \cos(V_{2j}' - V_{1j}') & \sin(V_{2j}' - V_{1j}') \\ \sin(V_{1j}' - V_{2j}') & \cos(V_{1j}' - V_{2j}') \end{pmatrix}$$

$$X \equiv \begin{pmatrix} x \\ y \end{pmatrix} \quad \Xi \equiv \begin{pmatrix} \xi \\ \eta \end{pmatrix} \qquad Q_i \equiv \Sigma_j A_j f_{ij} \begin{pmatrix} \cos(V_{ij}' - \alpha) \\ \sin(V_{ij}' - \alpha_j) \end{pmatrix}$$

The summation over j denotes the summation over the yearly samples labelled by j. Equation (6) under this form requires the knowledge of the astronomical argument at the central time of the analysis and the raw amplitude and phase (A, α) before the application of the nodal corrections.

It may happen at times that an abnormal behaviour in a sequence of annual values of a constituent is noticed well after the analysis is performed and when the relevant information about V has been lost. If the two constituents of the pair have exactly the same frequency it is possible to rewrite (6) exclusively in terms of the observed Greenwich phase lags and the mean values of the nodal corrections over the years. We note simply that for constituents of equal frequencies :

$$\begin{aligned} V_1 &= V_2 \\ \alpha &= V_1 - g \\ V_1' - \alpha &= u_1 + g_1 \\ V_2' - \alpha &= u_2 + g_2 \\ V_1' - V_2' &= u_1 - u_2 \end{aligned} \tag{7}$$

which leads to a set of equations of the form :

$$\begin{pmatrix} A_1 & p \\ p^T & A_2 \end{pmatrix} \begin{pmatrix} X \\ \Xi \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \tag{8}$$

where :

$$p \equiv \sum_j f_{1j} f_{2j} \begin{pmatrix} \cos(u_{2j} - u_{1j}) & \sin(u_{2j} - u_{1j}) \\ \sin(u_{1j} - u_{2j}) & \cos(u_{1j} - u_{2j}) \end{pmatrix}$$

$$q_i \equiv \sum_j f_{ij} A_j \begin{pmatrix} \cos(u_{ij} + g_j \\ \sin(u_{ij} + g_j) \end{pmatrix}$$

(A_j, g_j) are the analyzed amplitudes and Greenwich phase lags (before applying the nodal corrections) while the sequence of f's and u's may be read off from a table of nodal corrections.

EXAMPLES

(a) A pair with different frequencies

In this case one must have recourse to (5) or (6) and know the astronomical arguments at the central times of each analysis. For example, at frequency 27°.9/hour, a pair that may contribute is $2N_2$ at 27°.895/hour and O_2 at 27°.886/hour. Table 3 shows the sequence of analyzed amplitudes and phases at 27°.9/hour at Quebec between the years 1962 and 1969 as well as the values extrapolated for the years 1962 to 1967 after having solved (5) using the analyzed values for 1968 and 1969. The error in the fit is relatively large at times but we must keep in mind that the constituent is rather small and that the fluctuations fall within −0.045 and +0.031 feet which is acceptable for the semidiurnal band at Quebec. Figure 1 shows that the trend in the successive annual values is followed quite well by a direct fit to the years 1968 and 1969 and the resolution of the system of equations (5).

TABLE 3

The resolution of the pair $2N_2$ and O_2 at Quebec

Year	Observed		Values extrapolated from 1968 and 1969		Error	
	Amplitude A feet	Phase g degrees	Amplitude A feet	Phase g degrees	Amplitude A feet	Phase g degrees
1962	0.067	210.1	0.062	180.0	−0.045	−30.1
1963	0.089	68.0	0.120	85.7	+0.031	+17.7
1964	0.218	116.0	0.208	124.2	−0.010	+8.2
1965	0.212	158.2	0.200	169.4	−0.012	+11.2
1966	0.137	197.5	0.101	225.4	−0.036	+27.9
1967	0.049	9.7	0.073	21.7	+0.024	+12.0
1968	0.197	89.9	—	—	—	—
1969	0.251	138.9	—	—	—	—

Resolved Pair : $2N_2$ = 0.10 ft. 139°9
O_2 = 0.11 ft. 220°5

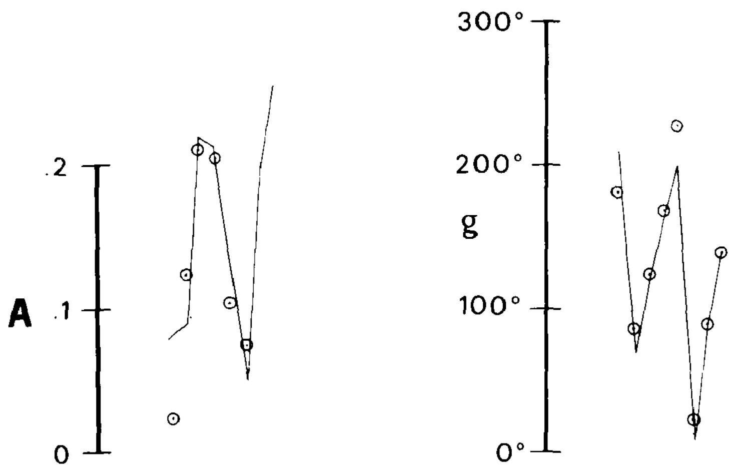

FIG. 1. — The resolution of $2N_2$ and O_2 at Quebec (from GODIN, 1973). The sample amplitudes and phases analyzed at the frequency of $2N_2$ are plotted with the year of observation along the abscissa; the fine line joins the points observed. The circled dots represent the values extrapolated from 1962 to 1967 from the observations of 1968 and 1969 using equation (5) and assuming the presence of both $2N_2$ and O_2 at that frequency.

(b) **A pair with identical frequencies**

We had noticed a steady drift in the analyzed amplitudes of M_2 at Quebec; this could not be interpreted as a secular change since it was not

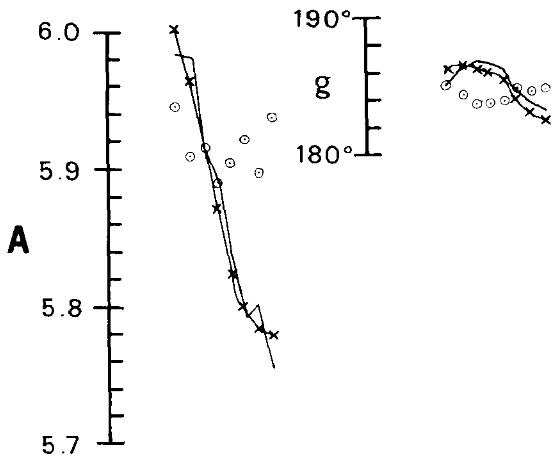

FIG. 2. — The resolution of M_2 and KO_2 at Quebec (from GODIN, 1973). The fine line joining the points represents the observed values of the amplitude and phase analyzed at the frequency of M_2. The points denoted by x's are the fitted values assuming the presence of M_2 and KO_2 and using the set of equations (8). The circled dots are the demodulated values : these should be constant over the years.

corroborated by neighbouring stations. Since the shallow water distortion is very marked there and since KO_2 is quite likely to be felt at the M_2 frequency we used the analyzed amplitudes and phases at that frequency in conjunction with the system of equations (8) in order to attempt to separate the individual contributions of M_2 and KO_2. Table 4 shows the results of our calculations.

TABLE 4

The resolution of the pair M_2 *and* KO_2 *which have identical frequencies at Quebec*

Year	Observed		Least Square Fit		Demodulated	
	Amplitude A feet	Phase g degrees	Amplitude A feet	Phase g degrees	Amplitude A feet	Phase g degrees
1962	5.98	185.5	6.01	186.2	5.95	185.0
1963	5.97	186.4	5.97	186.3	5.91	184.2
1964	5.92	186.7	5.92	186.2	5.92	183.8
1965	5.90	186.6	5.87	185.9	5.89	183.6
1966	5.84	186.0	5.83	185.5	5.91	183.8
1967	5.80	184.5	5.80	184.8	5.92	184.7
1968	5.80	183.8	5.79	184.1	5.90	184.7
1969	5.76	183.0	5.78	183.5	5.94	184.7
				Range	0.06 ft	0.04 ft.

Resolved Pair : M_2 = 5.68 ft. 186°.3
KO_2 = 0.31 ft. 144°.9

We call "demodulated values" the vector sums of the two resolved constituents to which the deviation between the fitted value and the observed value has been added vectorially; this stands for the values which would have been analyzed at that frequency on that year, had the nodal modulation been absent. It represents the closest estimate we can make of the constituent pair, and the variability it exhibits should conform to criteria (2) and (3). The size of KO_2 may come as a surprise but, for a reason which is not understood, the diurnals contribute quite strongly to the formation of shallow water constituents at Quebec.

CONCLUSIONS

Constituents cannot be properly identified from a single yearly analysis; one needs a sequence of such analyses to attempt such an identification. Usually the constituent whose identity is dubious is small and an error of this type will be scarcely felt in the predictions. However, that such an identification is possible is a tribute to the power of contemporary methods of analysis. On the other hand, major constituents such as M_2 and N_2 can be strongly modulated by satellites of an appreciable magnitude and then the proper identification and separation of these constituents becomes highly relevant. This had been noticed in the past by DOODSON (1924) whose attention was drawn to peculiar modulations in the M_2 values at Saint John, New Brunswick, but he apparently failed to assign these to the phenomenon we have just mentioned.

Finally the considerations of this paper show that the results of any analysis can often be misleading unless subjected to a very close scrutiny. It follows too that the preparation of a "standard list" of constituents ignores the reality of an analysis and the need to question the presence and identity of all the constituents sensed.

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"TUBBY'S" REVENGE

In the early 1890s the Admiralty decided to make a large scale survey of Mudros harbour (North Aegean Sea), a bit of foresight which paid off some twenty years later when it became the base for the Dardanelles expedition. So a ship was sent, commanded by an officer called Corry, and among his officers was a young one called Lockyer, whose figure had already earned him the nickname of "Tubby".

At the western side of the entrance are four hills (or pimples, almost) inscribed on the chart from West to East as Yam, Yrroc, Eb and Denmad.

Somehow Lockyer had fallen foul of his C.O. and, as a punishment, had been ordered to put in the 100 foot contour on those hills, an arduous job in the height of summer, as Lockyer found.

He was determined to get even, and reported that the local names of these hills were as above. Mudros had formerly been in Turkish hands for very many years and so these names, although obviously not Greek, did, to a non-linguist, have a Turkish flavour; they were therefore accepted and placed on the fair chart. This duly went to the Hydrographer, who passed it. The chart plate was then engraved and the chart issued.

Nobody in those days realised that, reversed, the names read "May Corry be Damned". So for eighty years Lockyer has had his revenge and will, I imagine, continue to do so as I cannot see any Hydrographer amending the chart * — at least I hope not.

No doubt all who have served in the Hydrographic Department know this story but I am sure there are many people who do not.

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(*) Admiralty Chart 1661 : Port Mudros, Lemnos Archipelago.