ON THE MYSTERY
OF MEAN SEA LEVEL SLOPES

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INTRODUCTION

In view of the current extension of geodesy into the ocean areas, the concept of the geoid at sea comes into focus. How is it related to that on land and how can we utilize oceanographic data to identify it? That sounds like a question in the wrong direction, since geodesists usually take mean sea level as a basic concept when talking about the geoid concept on land: the elevations on land are measured with respect to mean sea level; the Figure of the Earth is equated with mean sea level and its extension into the land, if we could get rid of the mountains. It seems such a simple graphic concept, considering that the oceans comprise about five sevenths of the Earth's surface, and thus constitute the major part of the Earth's Figure. We lived happily with this concept for more than 2000 years — at least since Pythagoras -- because there was no confrontation with oceanic data and no requirement for high precision. But now there are both. While it sounds like a dream come true — to get data about the marine part of the geoid, either by satellite altimetry or from oceanography or both — new problems appear.

THE PROBLEM

When precise leveling along the U.S. coasts was compared with mean water level at the tide gauges (fig. 1), it appeared that the water surface sloped down toward South with respect to the equipotential surface passing through the northern starting point — Neah Bay for the Pacific coast in figure 1, and Portland, Maine, for the Atlantic coast. Leveling has been

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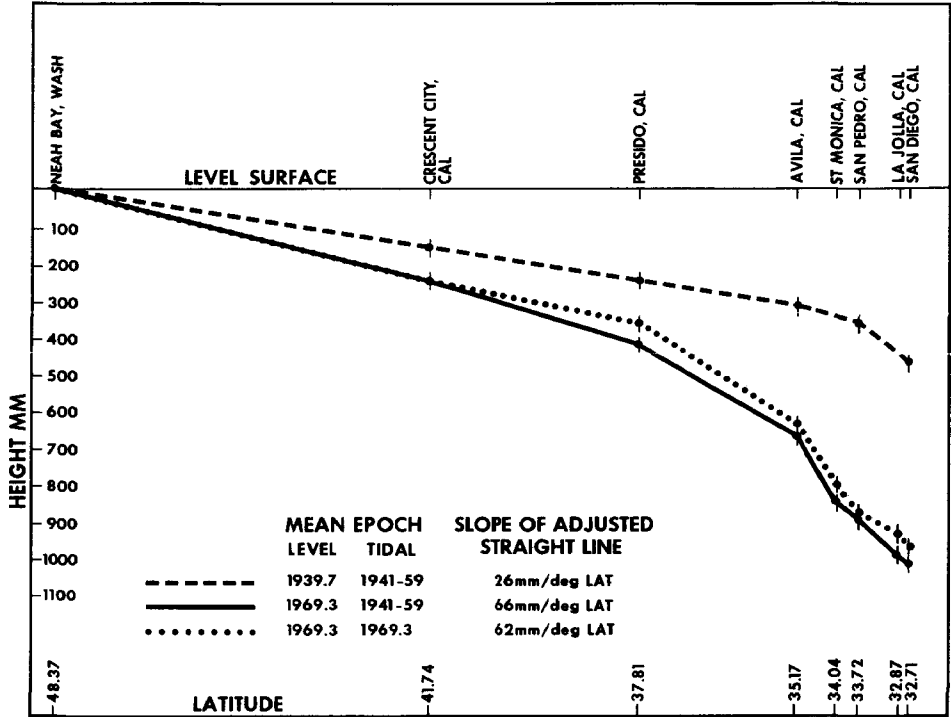repeated and improved, the numbers changed somewhat, *but the slope is still there* (E.I. BALAZS, 1973).

FIG. 1. — Heights of local mean sea levels — Pacific coast (BALAZS, 1973).

That in itself would be nothing to be excited about, since it is agreed that sea level is not necessarily level, although nearly so. Even the magnitude of the slope (less than a metre over the length of the coast) is acceptable, *except* for one disturbing fact: the oceanographers say that this is all wrong, because *their* data tell them that the slope must go the other way. Specifically, oceanographers and geodesists disagree on the north-south slopes, but agree in the east-west directions. Specialists on both sides of the fence are looking for errors or improvements in the observation — or reduction — procedures, but so far they have not found anything significant enough to resolve the dispute.

The present discussion does not deal with these efforts, important though they are, but is addressed to a different and additional aspect, which involves speculation about the *concepts* underlying such a comparison. Maybe, some unorthodox questions about concepts and tacit assumptions would help move the long-standing dispute off deadlock.

THE CONCEPT OF A SLOPE

Let us start with the concept of a slope. *Slope* is a relative concept, relative to a specific reference line. If slopes are to be compared directly,

they should have the same reference line, or parallel reference lines as second best. Otherwise, the slopes must be recomputed and/or redrawn with respect to the same reference, before a valid comparison can be made.

Geodetic leveling refers the slope to a level surface near the geoid. Oceanographic leveling (steric leveling) refers the slope to an isobaric surface several thousand metres below the ocean surface. Obviously, these two surfaces are not the same. The oceanographers say that the deep ocean references are practically level, and thus dynamically parallel to the geoid. But are they really? How level is "practically level"?

MYSTERIOUS EQUATORIAL BULGE

It appears that all oceanographic mean sea level topographies show an unmistakable systematic equatorial bulge of several metres, so that the dispute along the U.S. coasts, on the cm level, is only a very small part of a global puzzle. Figure 2 shows one example of a global sea level chart, by I. HELA and E. LISITZIN (1967), with a bulge of about 3.5 m. There is a similar chart by H. STOMMEL (1965) and several meridional profiles by W. STURGES (1974). The actual numbers differ, depending on the reference surfaces deep down in the ocean at 1000 - 2000 - 4000 db. The bulges are on the order of 1.5 to 3.5 m, respectively. One might wonder about such a systematic global feature even if one had never heard of the controversy along the coasts.
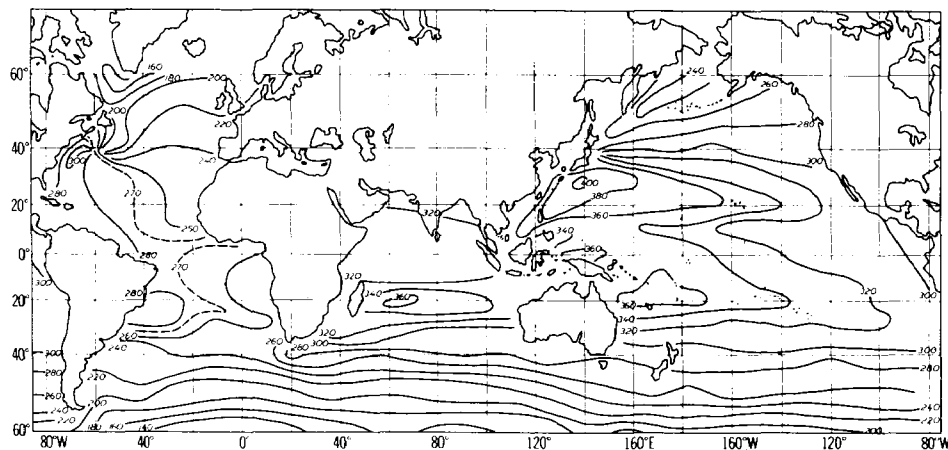

Fig. 2. — Distribution of different heights of mean sea level (dynamic centimetres) in the world ocean (referred to 4000 db) (HELA and LISITZIN, 1966).

For geodesists this systematic bulge is disquieting, because it seems to contradict the traditional definition of the geoid (according to G. BOMFORD) as "*that equipotential surface of the Earth's attraction and rotation which, on average, coincides with mean sea level in the open ocean. (Ambiguity due to mean sea level not being exactly an equipotential surface will not amount to one metre)*". The geoid then should fit as closely as possible through the data, leaving more or less random residuals on either side;

these residuals would represent the mean sea topography as deviations from the geoid. And *these* residuals we would want to compare with geodetic leveling.

Under this assumption, the deep ocean reference would *not* be parallel to the geoid dynamically, and we should find a way to redraw the oceanographic profiles with respect to the so defined geoid, for geodetic use and comparison. But how should we go about putting a surface or meridional curves through the data, that could represent an equipotential surface in the sense of BOMFORD's definition? Looking at the various meridional profiles of the sea topography, the eye can roughly distinguish between local disturbances and a systematic bell-shaped curve where the geoid might be expected to be. With this guidance I have made a tentative, exploratory attempt to formulate such a curve or surface, and to redraw the profiles accordingly (I. FISCHER, 1975).

A TENTATIVE EXPLANATION AND REMEDY

First, we remember that the dispute between geodesists and oceanographers refers to north-south profiles, and not to east-west directions. So we are looking for a latitude function. Next we remember that the hydrostatic equation is used in steric leveling, which — according to V. BJERKNES — is derived from the condition of equilibrium where the gradients of pressure and of potential are oppositely directed thus making isobaric and equipotential surfaces coincide. If it is maintained that the hydrostatic equation is meant to apply only to the vertical component of the pressure gradient, then there is another component which keeps the isobaric surface from being level. We will explore both alternatives.

Let us assume for the moment that the isobaric reference surface is level. Then we have the picture of two equipotential surfaces (one near the surface of the ocean and one several thousand metres down) where
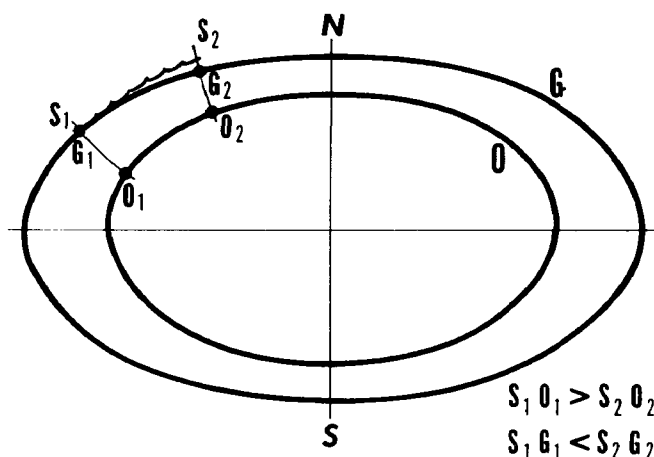

$$S_1 O_1 > S_2 O_2$$
$$S_1 G_1 < S_2 G_2$$

FIG. 3. — A meridional sea-surface profile S_1S_2 referred to the geoid G and to another equipotential surface O at several thousand metres of ocean depth.

the upper one shows an equatorial bulge with respect to the other. This brings to mind the textbook picture in figure 3 showing the orthometric non-parallelism between level surfaces. Could it be that a fractional orthometric admixture in the otherwise dynamic computations, a partial neglect of the variation of gravity with latitude, was the villain causing
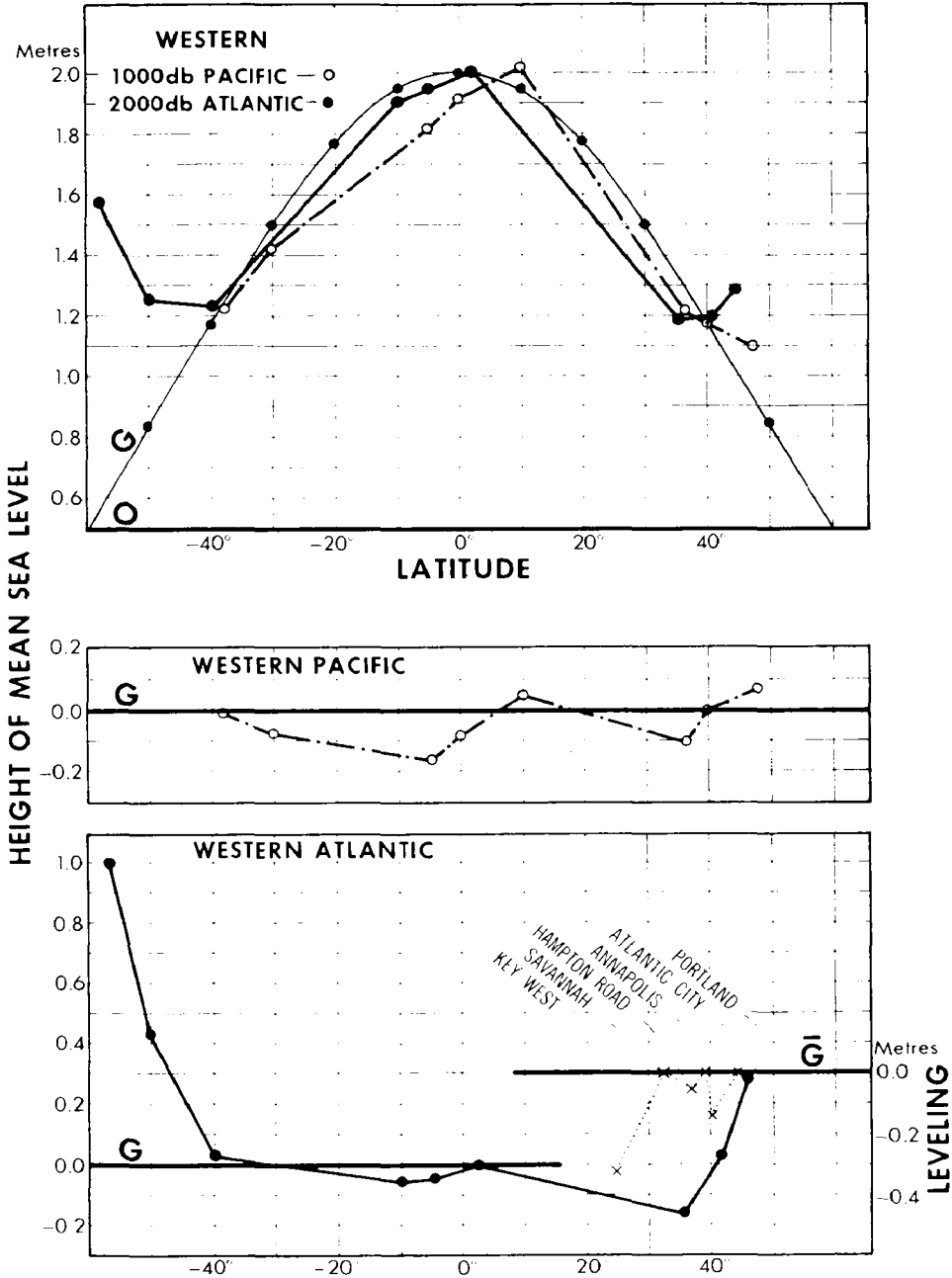

FIG. 4. — Western ocean profiles relative to a deep ocean isobaric surface O (W. STURGES, 1974, fig. 1) and to a near-geoid G. Levelling relative to Ḡ.
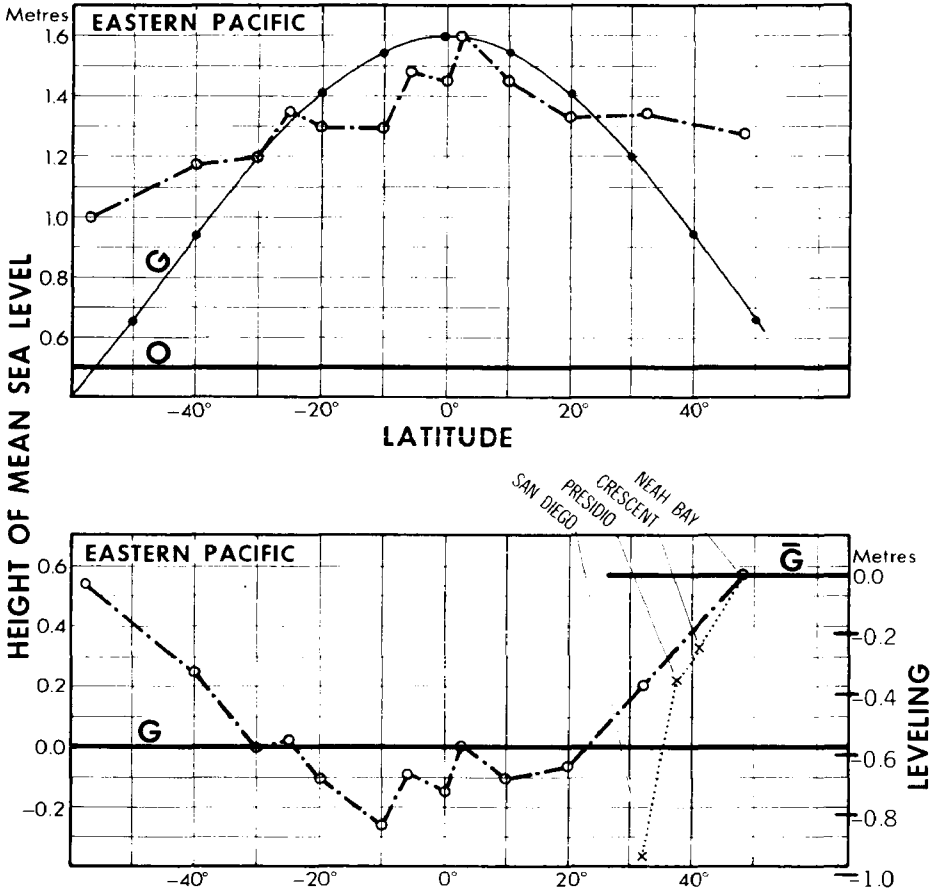
FIG. 5. — Eastern Pacific profile relative to 1000 db surface O (W. STURGES, 1974, fig. 1) and to a near-geoid G. Levelling relative to G̅.

the bulge in the oceanographic profiles? After all, any problem with level surfaces must have something to do with gravity variations. The oceanographic Tables in use are based on — or used in connection with — the round gravity value of 9.80 m/s² at the ocean surface. Maybe the 5 gal latitude variation was not always considered because of its insignificance for oceanographic regional phenomena, yet could accumulate to some significance over the whole spread of latitudes.

To get a feel for the magnitudes involved under this working hypothesis, we find from the gravity formula that a nominal separation of 100 m at the pole between level surfaces would increase by about 53 cm at the equator (which roughly includes all reasonable flattening values) and we can also get an approximate shape of the curve by computing the increase in 10° latitude increments. For a fractional effect, the equatorial increase would be 53 . p cm per 100 m separation, where p is an unknown scaling parameter between zero and one. For separations of roughly 1000 - 2000 - 4000 m, this effect amounts to approximately 5.3 p_1 m - 10.6 p_2 m - 21.2 p_i m, where the p_i could be used for a fit to the given data.

Let us try and apply this reasoning to W. STURGES' ocean profiles in the Western Pacific and Atlantic (fig. 4). Both profiles (in the upper part of the figure) show a peak of 2 m, referred to 1000 db and 2000 db respectively. For simplicity and because of the scarcity of the oceanic data points, the theoretical curve is here fitted to the peak only, giving
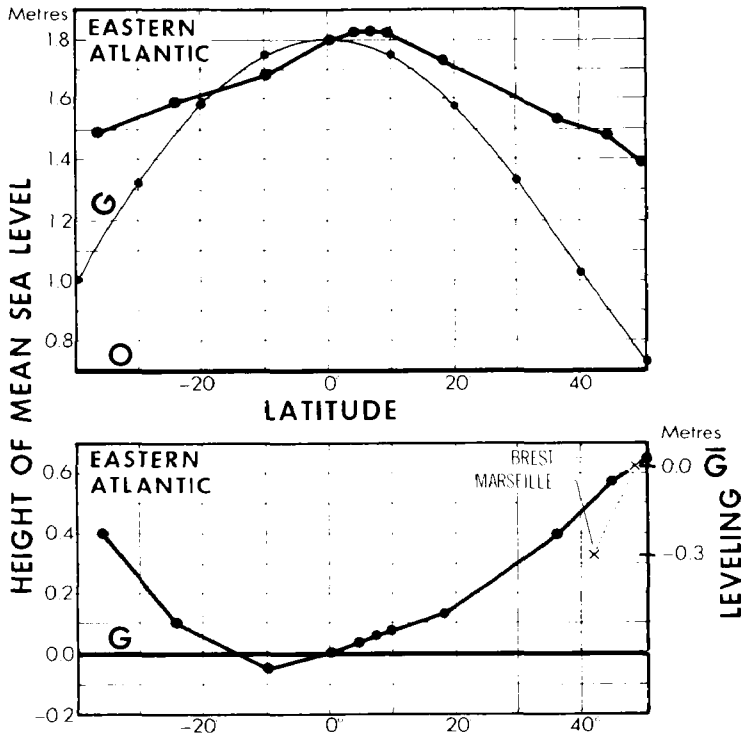
FIG. 6. — Eastern Atlantic profile relative to 2000 db surface O (W. STURGES, 1974, fig. 1) and to a near-geoid G. Levelling relative to G̅.
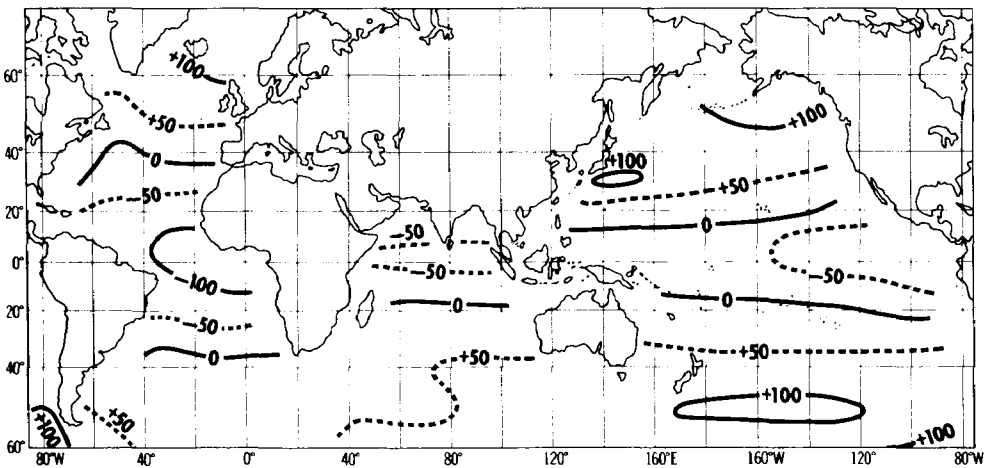
FIG. 7. — Mean sea level referred to the geoid (cm) derived tentatively from HELA and LISITZIN's data.

$p_1 = 2/5.3$ ($\sim 1/3$) and $p_2 = 2/10.6$ ($\sim 1/5$). The residuals with respect to this curve are plotted in the lower part of the figure, separately for each profile.

The geodetic leveling data on the Western Atlantic coast are plotted for a near-geoid level surface. We see that there are still discrepancies between the two types of profiles, but the overall direction of the slope goes *UP toward North* for both the geodetic *and* the oceanographic profiles.

The same was done for the other two STURGES profiles (figs 5 and 6), fitting a hypothetical curve to the profiles in the Eastern Pacific and Atlantic, plotting the deviations from that curve, and comparing them with the geodetic leveling data. Again, despite still persisting discrepancies, there is a rough agreement in the direction of the slope; it goes *UP toward North*.

On HELA and LISITZIN's global chart (fig. 2) which is referred to 4000 db, the equatorial peaks vary between about 2.8 m and 3.6 m, which would give a scaling factor p_4 of around 1/7 to 1/6, different for different meridians. Therefore, the hypothetical geoidal surface was determined in a global least squares fit to the data at 10° by 10° grid points such that the sum of the squares of its distances from the given sea topography at these grid points would be minimized. The residuals from that surface are plotted in figure 7. They also go *UP toward North*.

This exploratory exercise of redrawing the oceanographic profiles shows that a change of the deep ocean reference to one near the sea surface and conforming to BOMFORD's definition of the geoid *can* produce a rough agreement in the dispute about the direction of the slope. The magnitude of a possible fractional neglect of the latitude variation of gravity is not unreasonable; it is comparable to a fractional orthometric admixture, more for the shallower and less for the deeper references. The procedure used here is probably an oversimplication, but it served to demonstrate a possible avenue to a solution.

AN ALTERNATIVE EXPLANATION

Considering the other alternative, namely that the hydrostatic equation used in steric leveling is meant to apply only to the vertical component of the pressure gradient, we have an inclination of the isobaric surfaces with respect to level surfaces. Then, unless the oceanographic reference is changed to a level surface, sea topographies referred to these isobaric reference surfaces are *not* directly comparable with geodetic leveling profiles which *are* referred to a level surface. It is said, however, that the deep isobaric surfaces (especially the one at 2000 db) are so little different from level surfaces that they are practically level. But how little is little?

There is a series of charts by the oceanographer Albert DEFANT (1941), showing that these pressure surfaces are noticeably *non-level*. Figure 8 shows the 2000 db surface contoured at intervals of 5 dyn cm relative to the sea surface. DEFANT says: "The tabulated dynamical values for the standard pressures are referred to sea level as the starting point for counting; the topographies based on them are thus *relative* topographies
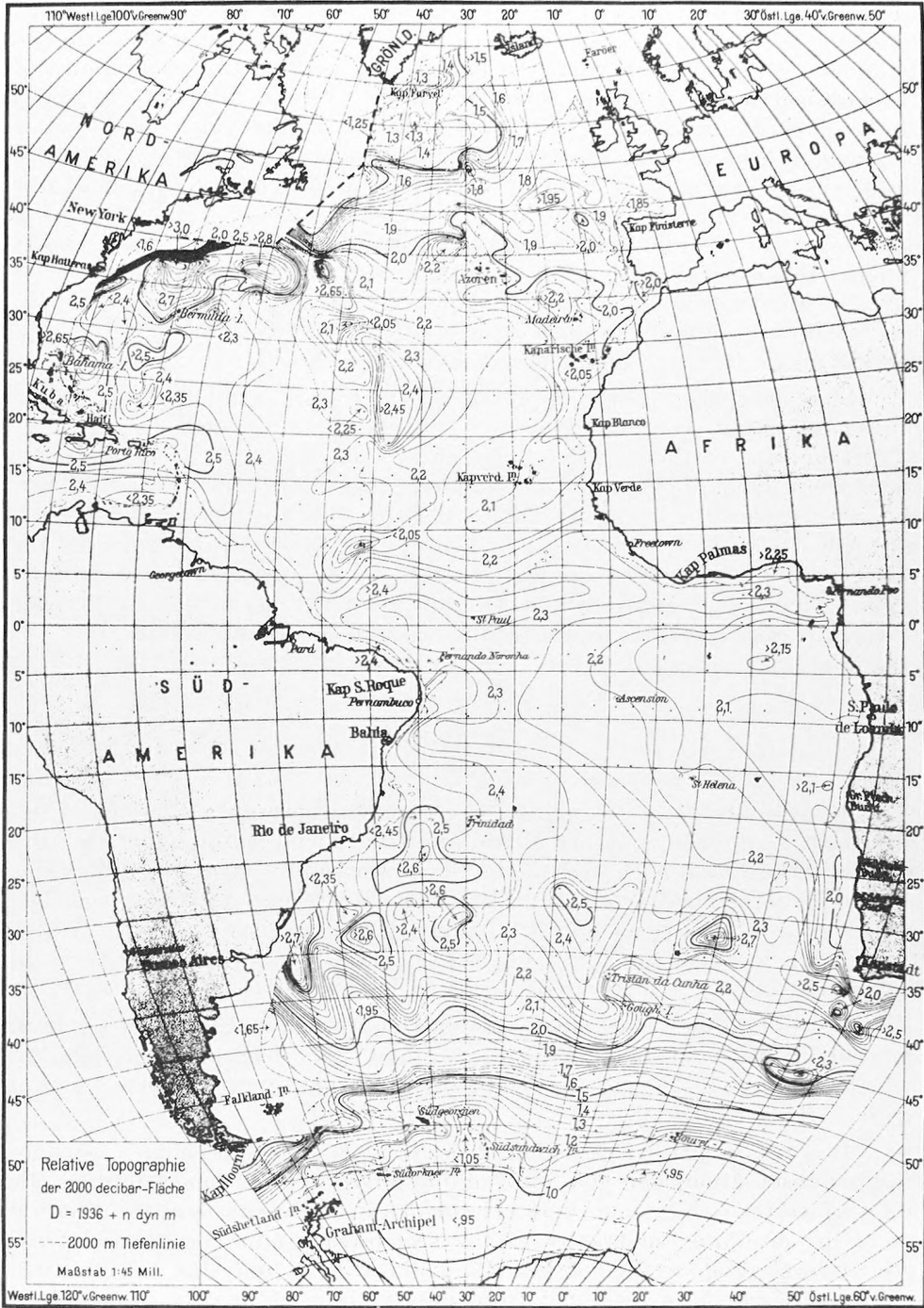
FIG. 8. — The relative topography of the 2000 db surface in the Atlantic Ocean
(A. DEFANT, 1941).

of the sea pressure with respect to the physical sea level considered as "level" (without the pressure of the atmosphere above the sea) ... that is unknown. One could interpret the charts also as absolute topographies of the physical sea level, *if* the isobaric surface given in the chart coincides at the given dynamic depth with a level surface of the Earth. But that is *certainly not* the case for the whole Atlantic ocean on any chart ... *Relative* topographies can only give qualitative clues in which direction one may expect changes in the position of the pressure surfaces relative to each other, but never quantitative ideas on their true slopes ... All charts (interpreted as referring to the physical sea level considered as level) agree in showing maximal dynamic depths in the subtropical areas of both hemispheres ... Going from these regions of maximal dynamic depth toward the poles, the dynamic depth decreases... ".

It appears here that the pressure surfaces do exhibit a systematic latitude variation, well distinguishable from local and regional disturbances. The magnitude of the negative bulge shown on the charts is of the same order we are looking for; and no wonder, since they were computed from the same Tables, the BJERKNES Tables of 1910. It is easy to see that if such a trough-like surface is used as a reference, and is then straightened out for purposes of a graphic, the sea level surface *would* appear with an artifical bulge. That bulge disappears when the apparently systematic difference between pressure surface and level surface is taken into account.

CONCLUSION

The two alternatives explored here seem to represent two sides of the same coin, as implied in figure 3: two reference surfaces, the upper showing an equatorial bulge with respect to the lower, and conversely, the lower showing a corresponding equatorial trough with respect to the upper. The Tables used in both cases do not distinguish the two cases; they only give the relative message that the layer between the two surfaces decreases in thickness from the equatorial to the polar regions.

The conflict between geodetic and oceanographic evaluation of sea level data along the coasts of the United States goes back several decades. H.U. SVERDRUP (1942) calls it "one of the most puzzling problems of recent years". He comments:

"It is not surprising that such discrepancies appear because ... oceanographic observations can give information only as to the topography of the sea surface *relative* to some selected surface in the ocean, and information as to the *absolute* topography of the sea surface must be derived from precise leveling along the coasts". He explains that "only the relative field of pressure can be determined from observations of density. Any added slope of the isobaric surfaces due to actual piling up of mass in certain instances can be derived from precise leveling along coasts, but in general it cannot be observed. It is of great importance to bear these facts in mind in order to avoid erroneous conclusions".

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A FIRST STEP IN 17TH-CENTURY CHART-MAKING

How to prepare the Parchment on which one wishes to draw a nautical chart

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You shall take a parchment skin, as beautiful and large as it is possible to find, and without any nodules. And if it is not also white and polished all over you shall first rub it with ceruse (white lead) and dry it with a white cloth: then you shall boil the parings of this same parchment in water until, on dipping your fingertip therein, you find it feels sticky. Next wipe over your wide-spread parchment with a cloth or sponge soaked in this liquid, and, when dry, rub it once again with ceruse. You will see your parchment become wondrously white, and without any rough protuberances to stop the pen. But remember to rub it well after putting on the ceruse for the second time: otherwise the writing would be obliterated and would fade away.

From: "Hydrographie — contenant La Théorie et La Practique de toutes les parties de la navigation" by Père Georges Fournier, first published in 1643.
Reprinted in 1973 by "Editions des 4 Seigneurs", Grenoble.
France.