THE ADMIRALTY METHOD OF TIDAL PREDICTION, N. P. 159

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A new edition of the Admiralty Method of Tidal Prediction (N.P. 159) was issued in January 1976 [1], and it is felt that this is a suitable opportunity to describe the method in some detail, with the reasons for the alterations that have now been made.

The method is intended to supply a prediction of hourly heights for all those ports for which Harmonic Constants are published in Admiralty Tide Tables (A.T.T.) [2]. When used with the data given for Secondary Ports, this provides the best available prediction of both hourly heights and High and Low Waters. When used for Standard Ports, a very convenient method of providing hourly height predictions is available, but account must be taken of the high and low water predictions published in A.T.T. These latter are based on a very large number of Harmonic Constants and the use of very large electronic computers, and are thus of a much higher standard than any prediction provided by the Admiralty Method. By plotting the Standard Port predictions from Part 1 of A.T.T. on the Form B (N.P. 159) and then drawing a curve to pass through these points while following the general shape of the curve originally obtained on Form B, a prediction of a very high standard can be obtained.

For some Secondary Ports in A.T.T. there is no suitable Standard Port available. In these circumstances, the letter "p" is given instead of time differences, and the only way a prediction can be obtained is by the use of N.P. 159.

The Method has been very carefully designed with the needs of the mariner always borne in mind. Thus, simplicity has been kept to the forefront as far as possible, without reducing the accuracy of the predictions to such a level that they are no longer of any practical value. The equipment required has also been kept to the minimum, so that the only tools needed in addition to the book of forms and A.T.T. are a pencil, pair of dividers, and parallel rule.

Other equipment such as a pocket calculator or slide rule can be used to advantage if available; however, a small table of logarithms is included inside the covers of N.P. 159.

The Method involves the combination of the Tidal Angles (A) and Factors (F) for the day with the four main Harmonic Constituents M₂, S₂, K_1 and O_1 for the place concerned. The vectorial sum of the two semidiurnal (S.D.) constituents is obtained by plotting on Form A. Hourly heights for the S.D. tide are extracted from Form A using the dividers and plotted on Form B referred to the line of Mean Level which, after being corrected for known seasonal variations, has been drawn across the form. An hourly speed of 29° per hour has been assumed for the total S.D. tide. The values of A and F are calculated for 0000 Zone Time on the day in question and corrected to 1200 Zone Time by the addition of the angles α . Thus the phases are correct at noon but become progressively in error due to the assumption of a single speed for all the S.D. constituents. The error involved in using this simplified method of calculation is nil at 1200 and increases progressively as the time increases or decreases, becoming a maximum in tidal height of about 14 % of M₂ at 0000 and 2400. However, due to the fact that this error varies with the relative phases of the constituents concerned, this maximum is not often attained. The central time for these calculations was chosen to be noon as it was considered that most navigation takes place during daylight hours and that therefore a prediction with its greatest accuracy at mid-day would be most useful. If the angles α are ignored in the calculation on Form A, the accuracy then becomes greatest at 0000 on the day in question. An estimate of the errors involved in a particular prediction from this cause can best be found by predicting two consecutive days and obtaining the difference between the two predictions at midnight. If the mean of these two predictions is accepted as the best available for midnight, and the differences found are used to correct the two curves progressively from midnight towards noon, the predicted curves so obtained will evidently give a better result than that obtained from one day's calculations.

The Diurnal prediction is obtained in a similar way using the vectorial sum of K_1 and O_1 and an assumed speed of $14\frac{1}{2}^{\circ}$ per hour. Again some progressive error must arise from noon towards midnight in each direction. In this case it is much more difficult to assess the magnitude of the errors that may arise from this cause due to the considerable variations in the relationships between K_1 and O_1 from area to area.

However experience has shown that the considerable additional computation necessary if the actual speeds of the constituents are all to be used cannot be justified for this approximate method of prediction particularly if the meteorologically caused perturbations in the actual tidal levels are considered.

TIDAL ANGLES AND FACTORS

In both the semi-diurnal and diurnal predictions, Tidal Angles (A) and Factors (F) are used to provide the astronomical data for the day in question, and daily values of these are published in Admiralty Tide Tables as Table VII (Vols 2 and 3 only). Some consideration of the source of

this data, and of the differences between A & F and the normal Astronomical Arguments (also published in these volumes as Table VIII) is necessary.

Firstly, the data is published in the general form $A = 360^{\circ} - (E + u)$, where E is the normal astronomical argument and u is the nodal correction. This ensures that the calculation on Form A is a simple addition instead of combining addition and subtraction. Secondly, the data is amended to include the effects of minor constituents whose speeds approximate to those of the major constituents used and for which it is possible to assume that the phase lags g are the same as those of the major constituents. The minor constituents are therefore grouped as follows :

Semi-Diurnal	$\left\{ egin{array}{c} \mathbf{M}_2 \ \mathbf{group} \ \mathbf{S}_2 \ \mathbf{group} \end{array} ight.$	$egin{array}{llllllllllllllllllllllllllllllllllll$
Diurnal	K ₁ group O ₁ group	$\begin{array}{l} P_{1}, \ J_{1}, \ \pi_{1}, \ \phi_{1}, \ \theta_{1}, \ \psi_{1}, \ SO_{1} \\ Q_{1}, \ 2Q_{1}, \ \rho_{1}, \ \sigma_{1}, \ M_{1} \end{array}$

The amplitudes of these constituents are assumed to have the same relationships as those given by the equilibrium theory and commonly used in the inference of constants.

The method by which these corrections are applied is considered in detail in the Admiralty Tidal Handbook No. 3, paragraphs 4, 5 and 9 [3]. The values of the Nodal Factors f, Astronomical Arguments E and Nodal Corrections u are obtained from the normal formulae used in tidal prediction.

The improvement made possible by the introduction of these additional constituents has to be balanced against the inaccuracies due to the assumptions that the 'g's of each group are all the same and that the speeds of each pair of groups are 29° and $14\frac{1}{2}°$ per hour respectively. In the semidiurnal group the improvement is on balance of the order of 20% of M₂, while in the diurnal group the improvement is similar or slightly less, and somewhat more variable. However, practical experience has shown that, as no additional labour was required on the part of the mariner, the improvements can be justified and that they make a highly significant contribution to this method of prediction.

The Tidal Angles and Factors described above were used in the previous Method of prediction formerly known as Form H.D. 289 [4], but in the new method the plotting of the data and its transfer to the tidal height plot Form B has been greatly simplified with no loss of accuracy — in fact there has been some improvement here as well.

SHALLOW WATER CORRECTIONS

The major addition to the previous method of prediction is the introduction of a simple and remarkably effective system for the application of Shallow Water Corrections. These corrections are taken to consist mainly of the higher harmonics of the tidal constituents, and in this method only the Quarter and Sixth Diurnals are taken into account. The system is based on the principle, given in the Admiralty Manual of Tides [5], that "at any place the amplitude of the Quarterdiurnal tide varies approximately as the square of the Semidiurnal tide, and the amplitude of the Sixthdiurnal tide varies approximately as the cube of the Semidiurnal tide.

Based on this principle it is possible to calculate values for the relationships for all those ports where the Shallow Water Corrections are of importance. These have been called F_4 , f_4 , F_6 and f_6 , where the suffixes refer to the species of the tide; as usual, the capital letters are the ratios to be used, and the lower case letters refer to the phase differences involved. Data for these are now published in A.T.T. vols. 2 and 3 for all ports where the necessary constants are known and where the corrections are sufficiently important to be of practical navigational significance. Similar data, and the four main harmonic constants, will be included in A.T.T. vol. 1 for 1980 and subsequent years.

The mathematical principle involved is given in Appendix I. The method of plotting the correction is given on Form C and is very similar to that used for the other constituents. The squaring, cubing etc. of the Semidiurnal tide is carried out at the top of the form, using the logarithm tables inside the covers if necessary, and then reading off the hourly corrections on the plotting form below. On this form the speeds of the constituents are assumed to be 58° per hour for the Quarterdiurnal and 87° per hour for the Sixthdiurnal. Again these assumptions introduce minor errors but experience has shown that the improvements obtained by this method for the introduction of Shallow Water Corrections fully justify the system.

In order to simplify the plotting and extracting of data for these corrections the principle of symmetry has been used so that the right half of the form is used for the Quarterdiurnals and the left half of the form for the Sixthdiurnals. At the same time it has been found possible to reduce the number of radial lines used to a minimum by accepting minor errors in the positioning of these lines. The details of these errors are given in Table II. It will be seen that these nowhere exceed 6° of phase and that this error only occurs in two hourly values of the Sixth-diurnal, all the remaining errors being + or -3° except for those values at 1200 where there is no error. It would, of course, have been possible to include on Form C all the radii necessary to take off the exact values for each hourly correction, but it was considered that the very great increase in the complexity of the form for only a marginal gain in accuracy was not justified. Errors of 3° or so in the phase of these usually small corrections have very little effect on the shape of the final curve plotted on Form B, and are certainly well within the overall accuracy of the system.

COMPUTER APPLICATIONS

The results obtained by this method of prediction can be improved by the use of a small computer such as the Hewlett Packard 9810A Desk Top Programmable Computer. In this case it is possible to use the more accurate individual speeds of the constituents and other similar improvements of detail while retaining the general principles. This results in a marked improvement in the run-over differences between one day and the next. However these improvements are only of the order of one or two decimetres so that the simple, hand-drawn method using N.P. 159 remains an extremely valuable system for tidal prediction.

The main advantage is a reduction in the time taken. Using a small computer as suggested above, a prediction for 24 hours can be prepared in about 4 minutes while the hand-drawn method, when used by a practised mariner, takes about 20 minutes.

<u>. </u>			TABLE I			_
Group	Constituents	Speed Magnitude		E	u	$\int f$
		Deg./h	Ratio	at 0000 hrs	u	
	M ₂	28.98	1.000	-2s + 2h	M ₂	M ₂
	N ₂	28.44	0.194	-3s+2h+p	M_2^-	M ₂
	v ₂	28.51	0.038	-3s + 4h - p	M_2	M ₂
M ₂	L ₂	29.53	0.028	-s + 2h - p + 180	L_2	L ₂
	2 N ₂	27.90	0.026	-4s+2h+2p	M ₂	M ₂
	μ_2	27.97	0.024	-4s+4h	M ₂	M ₂
	λ_2	29.46	0.007	-s + p + 180	M ₂	M ₂
	S ₂	30.00	1.000	000	000	1.000
S_2	K ₂	30.08	0.272	2h	K ₂	K ₂
52	T ₂	29.96	0.059	-h + 282	000	1.000
	R ₂	30.04	0.008	h + 258	000	1.000
	K ₁	15.04	1.000	h + 90	K ₁	K ₁
	P ₁	14.96	0.331	-h + 270	000	1.000
	J ₁	15.59	0.079	s+h-p+90	J	J ₁
	M ₁	14.49	0.071	-s + h + 90	M ₁	M ₁
K ₁	π_1	14.92	0.019	-2h + 192	000	1.000
	ϕ_1	15.12	0.014	3h + 90	000	1.000
	θ_1	15.51	0.008	s - h + p + 90	J	J
	ψ_1	15.08	0.008	2h + 168	000	1.000
	SO ₁	16.06	0.006	2s - h + 90	$-O_1$	О ₁
	O ₁	13.94	1.000	-2s + h + 270	O_1	O_1
	Q ₁	13.40	0.194	-3s+h+p+270	000	1.000
0 ₁	ρ_1	13.47	0.038	-3s + 3h - p + 270	000	1.000
	$2\overline{Q}_1$	12.85	0.026	-4s+h+2p+270	000	1.000
	σ_1	12.93	0.012	-4s + 3h + 270	000	1.000

TABLE I

 $u. M_2 = 2.14 \sin N$ $K_2 = -17.74 \sin N + 0.68 \sin 2N - 0.04 \sin 3N$ $K_1 = 8.86 \sin N + 0.68 \sin 2N - 0.07 \sin 3N$ $J_1 = 12.94 \sin N + 1.34 \sin 2N - 0.19 \sin 3N$ $O_1 = 10.80 \sin N - 1.34 \sin 2N + 0.19 \sin 3N$ f. $M_2 = 1.0004 - 0.0373 \cos N + 0.0002 \cos 2N$ K₂ $1.0241 + 0.2863 \cos N + 0.0083 \cos 2N - 0.0015 \cos 3N$ K₁ $1.0060 + 0.1150 \cos N - 0.0088 \cos 2N + 0.0006 \cos 3N$ J₁ $1.0129 + 0.1676 \cos N - 0.0170 \cos 2N + 0.0016 \cos 3N$ $O_1 = 1.0089 + 0.1871 \cos N - 0.0147 \cos 2N + 0.0014 \cos 3N$ $L_2 f \cos u = 1.000 + 0.2505 \cos 2p - 0.1102 \cos (2p - N) - 0.0156 \cos (2p - 2N)$ $-0.0370 \cos N$ $\int \sin u 0.2505 \sin 2p = 0.1102 \sin (2p - N) = 0.0156 \sin (2p - 2N)$ - 0.0370 sin N $M_1 f \cos u = 2 \cos p + 0.4 \cos (p - N)$ $f \sin u = \sin p + 0.2 \sin (p - N)$ s = 277.025 + 129.3848 (Y - 1900) + 13.1764 (D + L)h = 280.190 + 0.23872 (Y - 1900) + 0.98565 (D + L) p = 334.385 + 40.6625 (Y - 1900) + 0.11140 (D + L)N = 259.157 - 19.3282 (Y - 1900) - 0.05295 (D + L)where : Y = YearD = No. of days elapsed since 0000 on lst. January in year Y L = integral part of 0.25 (Y - 1901)

7 T	Diurnal	Semi diurnal Radii	Ģ)uarterdiurn	al	Sixthdiurnal			
Zone Time	Radii		Phase Radii		Error	Phase	Radii	Error	
0600	273°	186°	012°	009° P	3°	198°	195° V	- 3°	
0700	287 ^{1/2}	215	070	067 Q	3	285	282 W	3	
0800	302	244	128	125 R	3	012	015 V	+ 3	
0900	316 ^{1/2}	273	186	189 – P	+ 3	099	102 - W	+ 3	
1000	331	302	244	247 – Q	+ 3	186	180 A	6	
1100	345 ^{1/2}	331	302	305 R	+ 3	273	270 X	3	
1200	000	000	000	000 A	0	000	000 A	0	
1300	$ \begin{array}{r} 014^{1/2} \\ 029 \\ 043^{1/2} \end{array} $	029	058	055 S	- 3	087	090 - X	+ 3	
1400		058	116	113 T	- 3	174	180 - A	+ 6	
1500		087	174	171 U	- 3	261	258 Y	3	
1600	058	116	232	235 — S	+3 + 3 + 3 + 3	348	345 Z	3	
1700	072 ^{1/2}	145	290	293 — T		075	078 Y	+ 3	
1800	087	174	348	351 — U		162	165 Z	+ 3	

TABLE II

APPENDIX I

The method employed for the determination of the quarter-diurnal tidal vector at 1200 results from the following empirical relationships between the constants H and g of the semi-diurnal and quarter-diurnal constituents of the tide. They were derived from a study of the results of a large number of year's analyses for ports all over the world, as published in the 1.H.B. lists of Harmonic Constants. To a degree of accuracy adequate for the present purpose, if $H_x H_y g_x g_y$ denote the values of H and g of any two semi-diurnal constituents of speeds x and y while $H_{xx} H_{xy} g_{xx} g_{xy}$ denote the H and g of quarter-diurnal constituents of speeds 2x and (x + y), then :

The ratio F_4 and the angle f_4 are constants for all the quarter-diurnal constituents at any port. They vary from one port to another. For example :

$$H_{M_4} = F_4 \cdot H_{M_2}^2 \qquad g_{M_4} = 2 \cdot g_{M_2} + f_4 \\ H_{MS_4} = 2 \cdot F_4 \cdot H_{M_2} \cdot H_{S_2} \qquad g_{MS_4} = g_{M_2} + g_{S_2} + f_4 \\ H_{S_4} = F_4 \cdot H_{S_2}^2 \qquad g_{S_4} = 2 \cdot g_{S_2} + f_4 \\ H_{MN_4} = 2 \cdot F_4 \cdot H_{M_2} \cdot H_{N_2} \qquad g_{MN_4} = g_{M_2} + g_{N_2} + f_4$$

The nodal factor f of any Q.D. constituent is the product of the nodal factors of the two S.D. constituents from which it is derived. i.e. :

$$f_{xx} = f_x^2$$
 and $f_{xy} = f_x \cdot f_y$ (2)

The (E + u) of any Q.D. constituent at any time is the sum of the (E + u)'s of the two S.D. constituents from which it is derived at the same time; hence if E_i denotes the (E + u) of the constituent of speed x at 1200 on the day of the prediction, then :

$$E_{xx} = 2 \cdot E_x \quad \text{and} \quad E_{xy} = E_x + E_y \tag{3}$$

Combining (1) (2) and (3) gives :

(a)
$$f_{xx} \cdot H_{xx} \cdot e^{i(E_{xx} - g_{xx})} = F_4 \cdot f_x^2 \cdot H_x^2 \cdot e^{i[2(E_x - g_x) - f_4]}$$

(b) $f_{xy} \cdot H_{xy} \cdot e^{i(E_xy - g_xy)} = 2 \cdot F_4 \cdot f_x \cdot H_x \cdot f_y \cdot H_y \cdot e^{i[E_x - g_x + E_y - g_y - f_4]}$
(4)

There is one of these vector equations for each Q.D. constituent, i.e. for each S.D. constituent of type (4a) and for each pair of S.D. constituents of type (4b).

Adding the lot together gives :

$$\sum_{xx} \{f_{xx} + H_{xx} + e^{i(E_{xx} - g_{xx})}\} + \sum_{xy} \{f_{xy} + H_{xy} + e^{i(E_{xy} + g_{xy})}\}$$

$$= F_{4} + e^{-if_{4}} \left[\sum_{xx} \{f_{x}^{-2}, H_{x}^{-2}, e^{i(2(E_{x} - g_{x}))}\} + 2\sum_{xy} \{f_{x} + H_{x} + e^{i(E_{x} - g_{x})} \times f_{y} + H_{y}, e^{i(E_{y} - g_{y})}\} \right]$$

$$= F_{4} + e^{-if_{4}} \left[\sum_{x} \{f_{x} + H_{x} + e^{i(E_{x} - g_{x})}\} \right]^{2}$$
(5)

The L.H.S. of this equation is the Q.D. tidal vector $= H_4 e^{ih}$.

$$\sum_{x} (f_x + \mathbf{H}_x + e^{i(\mathbf{E}_x - g_x)})$$

is the S.D. tidal vector $- H_2 e^{ih_{2.}}$

Hence the equation (5) states that :

$$H_4 e^{ih_4} = F_4 \cdot H_2^2 \cdot e^{i(2h_2 - f_4)}$$

i.e. the Q.D. tidal vector at 1200 on the day of prediction is of magnitude F_4 , H_2^2 in direction $(2h_2 - f_4)$ where H_2 in direction h_2 is the S.D. tidal vector at 1200 on the day.

The ratio F_4 and angle f_4 are constants for the port which have to be determined from the harmonic constants of the port, or otherwise by analysis from observations.

In a similar manner, comparison of the harmonic constants H and g of the Sixth-diurnal (6-D) and semi-diurnal (S.D.) constituents gives the following relationships between them at any port. If $H_x H_y H_z$ and $g_x g_y g_z$ denote the values of H and g of any three different S.D. constituents of speeds x y and z respectively, then the possible 6-D constituents have speeds of 3x, (2x + y) and (x + y + z), e.g. M_6 , $2MS_6$, MSN_6), denoted by suffixes xxx, xxy and xyz respectively.

Sufficiently accurately for the present purpose :

$$H_{xxx} = F_{6} \cdot H_{x}^{-3} \qquad g_{xxx} = 3 \cdot g_{x}^{-1} + f_{6}$$

$$H_{xxy} = 3 \cdot F_{6} \cdot H_{x}^{-2} \cdot H_{y} \qquad g_{xxy} = 2 \cdot g_{x}^{-1} + g_{y}^{-1} + f_{6}$$

$$H_{xyz} = 6 \cdot F_{6} \cdot H_{x} \cdot H_{y} \cdot H_{z} \qquad g_{xyz} = g_{x}^{-1} + g_{y}^{-1} + g_{z}^{-1} + f_{6}$$

the ratio F_6 and angle f_6 being constants for the port.

In a similar manner to that set out for the Q.D., it can be shown that it follows from these relationships that :

$$H_6 e^{ih_6} = F_6 \cdot H_2^{-3} \cdot e^{i(3h_2 - f_6)}$$

i.e. the 6-D tidal vector at 1200 on the day of prediction is of magnitude $F_6 \cdot H_2^{-3}$ in direction $(3h_2 - f_6)$. F_6 and f_6 are constants for the port which must be determined from its harmonic constants.

The angles h_2 h_4 and h_6 in the preceding discussion are the phases of the S.D., Q.D. and 6-D tide vectors at 1200 on the day of prediction.

The angle plotted on the circle of NP 159, due to the use of Tidal Angles (A.T.T., Table VII) in its computation, is $h'_2 = 360 - h_2$. This is allowed for in the subsequent graphical process for obtaining the hourly heights of the S.D. tide. We want to use a similar process to obtain the hourly Q.D. and 6-D tides, so we require to plot $h'_4 = 360 - h_4$ and $h'_6 = 360 - h_6$ at 1200.

$$h'_4 = 360 - h_4 = 360 - (2h_2 - f_4) = 2h'_2 + f_4$$

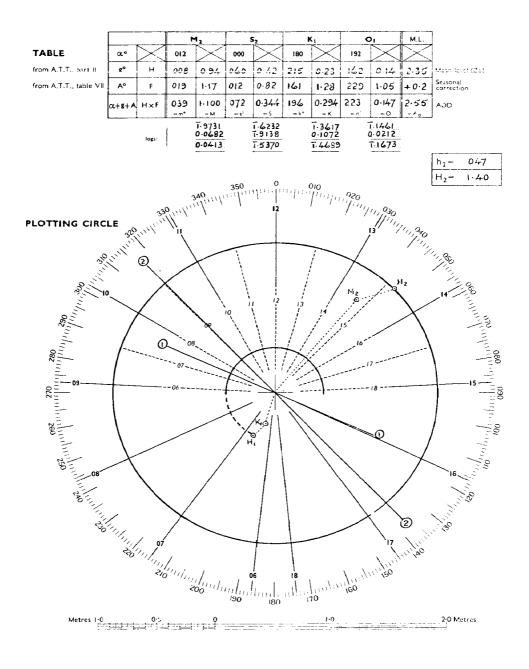
Similarly : $h'_6 = 3h'_2 + f_6$

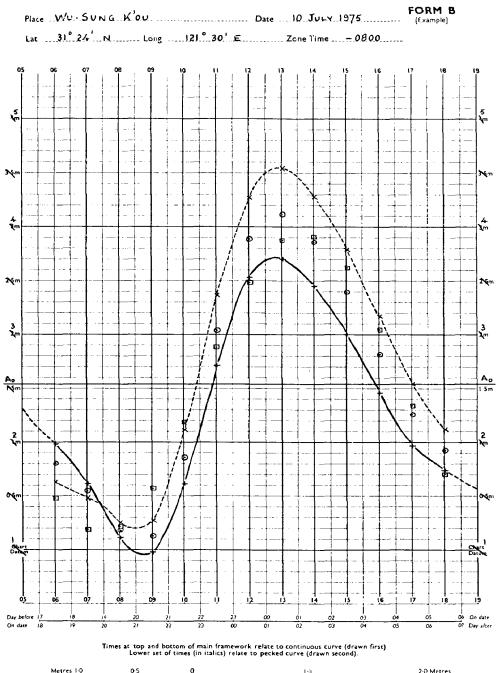
i.e. if the 1200 S.D. tide vector on NP 159 is H_2 in direction h_2 , the Q.D. and 6-D tide vectors at 1200 are $F_4 \cdot H_2^2$ in direction $2h_2 + f_4$ and $F_6 \cdot H_2^3$ in direction $3h_2 + f_6$ respectively.

APPENDIX II

 Place
 WU-SUNG K'OU (ATT No 7284)
 Date
 10 JULY 1975
 FORM A [Example]

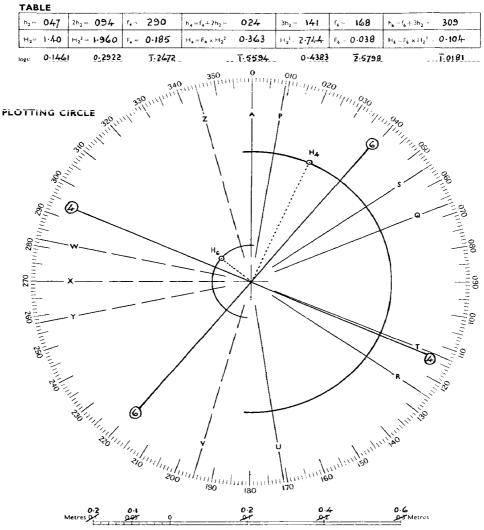
 Lat
 31° 24'
 N
 1.00g
 121° 30'
 Zone Time
 - 0800







Plac	WU-SUNG	K'OU	Date 10	JULY 1975	FORM C [Example]
Lat	31° 24' N	Long 121° 30' E		ne Time - 0800	



0.2	0.1		0.2	0.4	0.6
Metres	وجوع	ò	44	<u>م</u>	Metres

TABLE													
Time	0600	0700	0800	0900	1000	1100	1200	1300	1400	1500	1600	1700	1800
QD	P +350	°.266	R .068	- ^P -350	- 9-266	-R + 068	^ + ·336	⁵ • 3 14	T+010	U -304	-s -314	- 1.010	-0.304
6-D	v -042	w.088	-~.042	-w.088	- 1062	× t oso	^ . 5 62	-×-080	-^.062	Y .062	z .082	-Y -062	-z .082
Sum	+.308	+.354	026	438	328	+ .148	+ .398	+.234	052	242	232	- 072	+.222

ACKNOWLEDGEMENT

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GREAT SEA WAVES

Since time immemorial, seafaring men have been telling the world in their inarticulate way that storm waves attain heights which seem incredible to the rest of mankind. In the absence of satisfactory proof in specific cases, it has been easy to doubt the accuracy of the observations.

Possibly the controversy began in 1837 when Dumont D'Urville estimated and reported a wave 100 feet high off the Cape of Good Hope. It is significant that an authority of to-day should consider this statement "of such dubious character that probably only seafarers would agree with it".

"Great Sea Waves", R.P. WHITMARSH, U.S. Naval Institute Proceedings, Vol. 60, No. 8, August 1934.