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BERNOULLI EFFECTS ON PRESSURE-ACTIVATED WATER LEVEL GAUGES

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ABSTRACT

New developments in instrumentation engineering have led to the introduction of pressure-activated, submerged water level gauges such as the Aanderaa. These gauges may be placed in locations in which the velocity in the water column above the gauge and the variations in density cannot be neglected.

We show that significant errors can arise if the traditional hydrostatic assumption is made. The correct formulae are given for the conversion of pressure to water level, and the limits of applicability of these formulae are discussed.

Recommendations are given for the elimination of Bernoulli effects from water level records.

INTRODUCTION

Recent advances in the miniaturization and accuracy of data-loggers and pressure sensors have led to the introduction of submerged water level gauges. They are totally self-contained and may be placed in locations which are independent of the shoreline, allowing the collection of data from locations that have been, until now, virtually inaccessible. The most widely used gauge of this type, in Canada, is the Aanderaa. It consists of a metal cylinder, as shown in fig. 1, which is approximately 33 cm long by 13 cm in diameter, may be placed in up to 270 metres of water, and is totally self-contained for periods of up to one year. Canadian applications have been mostly for temporary gauging purposes in locations such as the high Arctic, on the continental shelves, on seamounts, and in estuaries and straits on both coasts. The data return has generally been good and the gauges have proved to be very easy to use.

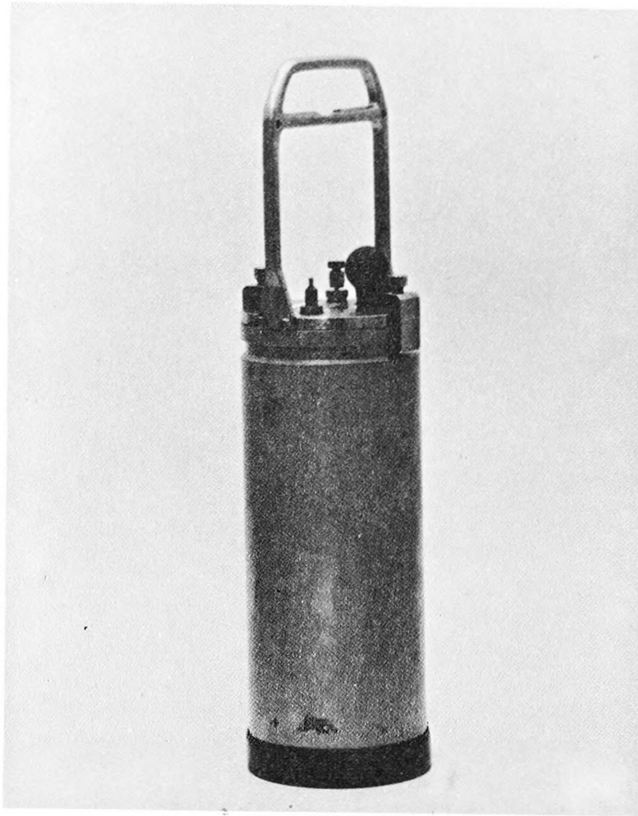


FIG. 1. — Aanderaa water level gauge.

The very fact that these gauges may be placed in virtually any location of interest, however, has brought forward a problem that has generally been ignored. It is fairly well-known that water velocity and density have an effect on the measurements obtained by most gauges. But, since they have been traditionally confined to the shore zone in locations where the water is not too deep and the velocities are fairly low, these Bernoulli effects have been ignored, even though they have been present in almost all water level records. In many cases, the new submerged water level gauges are now being placed in locations where neither the density nor the velocity effects may be ignored. It will be shown in this paper how these effects may arise and what their significance is. In some cases, the Bernoulli effects may be ignored, but in many other cases it is necessary to take them into account either by minimizing them before the water level record is collected or by correcting the water level record after it has been collected.

WATER LEVEL MEASUREMENT

Almost all types of water level gauges are misnamed since the gauges do not, in fact, measure water levels at all, but instead they measure water pressure. If the presence of the gauge in the water is assumed not to distort the pressure field, then it may be shown that the pressure measured by the gauge, p, is related to the height of the water column above the sensor, h, by the hydrostatic equation :

$$p = \bar{\rho}\, gh \tag{1}$$

in which $\bar{\rho}$ is the vertically averaged density of the water above the gauge and g is the local acceleration due to gravity, which may be assumed to be constant. In many instances $\bar{\rho}$ as well as h may vary significantly in time ; then in a small-time interval, Δt, equation (1) will become :

$$p + \Delta p = g(\bar{\rho} + \Delta\bar{\rho})(h + \Delta h)$$

and we can solve this equation for the difference in depth as a function of the difference in pressure as :

$$\left(1 + \frac{\Delta\bar{\rho}}{\bar{\rho}}\right) \Delta h = \frac{\Delta p}{g\bar{\rho}} - \frac{h\,\Delta\bar{\rho}}{\bar{\rho}} \tag{2}$$

The second term of this expression is due to the fluctuations in mean density of the water column, and in many cases should be taken into account even though the hydrostatic assumption is valid. The factor multiplying Δh will, in virtually all cases, be indistinguishable from 1.0, and so will be omitted from further discussion.

VELOCITY AND DENSITY EFFECTS

In many cases, the presence of a water level gauge in the fluid does significantly distort the velocity field, so that the pressure sensed by the gauge cannot be interpreted simply as the undisturbed hydrostatic pressure. From Bernoulli-type arguments it may be reasoned that the pressure measured by the gauge will be reduced below the hydrostatic pressure if the flow is accelerated by the presence of the gauge and will be increased above the hydrostatic pressure if the flow is decelerated by the presence of the gauge. The magnitude of the pressure change should be proportional to $\rho v^2/2$ where v is the speed of the flow undisturbed by the gauge, and ρ is the density of the fluid at the sensor.

If we assume that for the range of flow velocities encountered and for a fixed gauge geometry, with respect to the flow, the speed of the flow past the sensor is proportional to the ambient flow speed, v, then the pressure effect due to the distortion of the flow past the sensor is :

$$p' = -\frac{K\rho v^2}{2} \tag{3}$$

where ρ is the density of the water at the sensor and K is a constant which will be determined experimentally, but which will be dependent upon the exact geometry of the sensor and its attitude with respect to the flow.

By adding equation (3) to equation (1), the pressure recorded at the gauge is given by :

$$p = \bar{\rho} gh - \frac{K\rho v^2}{2}$$

and if $\bar{\rho}$, h, v and ρ are functions of time, then :

$$p(t) = g\bar{\rho}(t)h(t) - \frac{K\rho(t)v^2(t)}{2} \qquad (4)$$

and in difference form we have :

$$\Delta h = \frac{\Delta p}{g\bar{\rho}} - \frac{h\Delta\bar{\rho}}{\bar{\rho}} + \frac{K(\rho + \Delta\rho)(v + \Delta v)^2}{2\bar{\rho} g} \qquad (5)$$

The first term on the right-hand side of the difference equation represents the usual value employed for Δh, under the hydrostatic assumption. The second term gives the corrections necessitated by fluctuations in the mean density of the water column above the gauge while the third term gives the corrections necessitated by the ambient speed and density and the fluctuation in speed and density at the gauge. It will be shown that these correction terms may be significant in some instances.

DETERMINATION OF THE CONSTANT K

In equation (4), the constant K will depend upon a number of parameters. It essentially accounts for the distortion of the flow around the pressure sensor and will be highly dependent upon the geometry of the sensor design and the attitude of the sensor with respect to the flow. To determine the value of the constant for the Aanderaa water level gauge and also to check on the validity of the assumptions made in deriving equation (4), a standard Aanderaa water level gauge was towed at a variety of speeds and attitudes in the towing tank of the Canada Centre for Inland Waters.

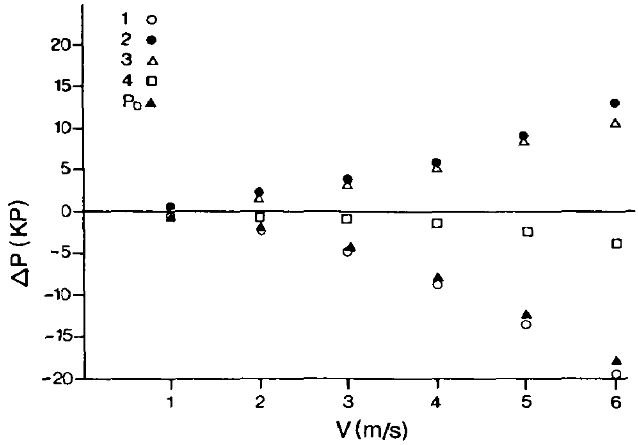
The configuration of the gauge was as shown in fig. 1 except that the acoustic transducer, which is the small projection on the top left of the cylinder, was removed. The handle was left attached, and the pressure port was the standard Swagelok fitting which extends about 22.5 mm above the cover plate and is the small projection on the top right of the cylinder. A rigid support structure attached the water level gauge to the tow carriage so that the flow pattern around the gauge was not affected by the support structure. Four sets of towing data were collected and these have been plotted in fig. 2. The four tests conducted were set out as follows :

1. The gauge was supported vertically in the normal field deployment attitude. Rotation of the gauge about its vertical axis made no difference to the values obtained. These values are plotted as open circles.

2. The gauge was supported horizontally with the longitudinal axis parallel to the flow direction. The pressure port pointed directly into the flow. These values are plotted as closed circles.

3. The gauge was supported with its longitudinal axis inclined at 45° from the vertical with the pressure port pointing into the flow. These values are plotted as open triangles.

4. The gauge was supported horizontally, as in test 2, but with the pressure port pointed directly away from the flow. These values are plotted as open squares.



V	1	2	3	4	P_D
1	-0·55	0·34	0·28	-0·17	-0·50
2	-2·14	1·52	1·24	-0·38	-2·0
3	-4·90	3·10	2·96	-0·76	-4·5
4	-8·83	5·17	5·38	-1·45	-8·0
5	-13·58	8·41	8·55	-2·55	-12·5
6	-19·72	10·41	12·96	-3·93	-18·0

FIG. 2. — Velocity effects on Aanderaa water level gauges.

During all of the tests, the pressure was recorded in kilopascals and then the static pressure, recorded while the gauge was at rest, was subtracted. Since the tow tank is filled with water, one decimetre of water will produce a pressure very nearly equal to one kilopascal. The values obtained are shown in fig. 2, and on the same graph are shown the values of the dynamic pressure (P_D) calculated from the formula :

$$P_D = -\rho \frac{v^2}{20\,000}$$

where : $\rho = 1.0$ g/cm³
v = towing velocity in cm/s

and the factor in the denominator converts the units to kilopascals (KP).

It is quite clear, then, from fig. 2 that the value of the constant K is a function of the angle between the longitudinal axis of the water level gauge and the direction of the flow. When the gauge is in its normal field deployment attitude, corresponding to test 1, the speed of the flow across the pressure sensor is somewhat higher than the speed of the surrounding water, causing a lowering of the pressure sensed by the gauge. In tests 2 and 3, a partial stagnation point develops in the pressure port, causing the gauge to read somewhat high. In test 4, the pressure port was in a highly turbulent zone and Bernoulli's equation should not be expected to hold at all in this region.

From these tests then, we can say that for the Aanderaa submersible water level gauge, in its normal field deployment position, the equation relating pressure to depth and velocity is given by equation (4) with the factor $K = + 1.09$ as derived from a linear regression between the observed and calculated values of the dynamic pressure.

DISCUSSION

Equation (5) relates the pressure recorded by the water level gauge to the mean density in the water column, the height of the water column, the velocity of the free-stream near the gauge and a constant K. Since neither the velocity of the flow near the sensor nor the mean density of the water column above the sensor is commonly measured, the calculation of water level fluctuations from recorded pressure fluctuations represents a problem. In many locations the second and third terms of equation (5) are significant and cannot be ignored. The obvious solution to part of this problem is to design a pressure port which has the proper geometry to make the constant K equal zero for the possible range of flow conditions. This has not yet been done, and may be a very difficult engineering task since the experiments with the Aanderaa gauge have shown that the numerical value of K depends upon the direction of the flow with respect to the longitudinal axis of the gauge. In the natural environment, especially if the sea bottom is not flat or if the gauge is placed on a compliant mooring, this direction will not remain constant and so, in general, the value of K may be a function of time.

It is generally assumed that in tidal waters the water level, the velocity and the density of the water column may be expressed as a sum of cosine terms whose frequencies are those of the standard tidal constituents. Each term in the expansion will have a unique amplitude and phase, depending upon the location. If each of the factors on the right-hand side of equation (4) has its expansion in terms of cosines of tidal frequencies substituted, then the resultant expression will be equal to the recorded pressure at the water level gauge. The algebraic manipulation would be quite tedious, but we can easily see that the first term on the right hand side of equation (4) would give rise to terms containing both the sums and differences of pairs of frequencies and also to terms containing first harmonics of all of the fundamental frequencies. The second term on the right-hand side of equation (4) will give rise to terms containing sums and differences of triplets of frequencies and also terms containing first and second harmonics of all of the fundamental frequencies.

These terms containing first and second harmonics and sums and differences of both pairs and triplets of the fundamental frequencies will arise simply because of the cross-product terms in equation (4). The recorded pressure record will therefore contain many more frequencies than will the actual water level. Since many of the tidal constituents and all of the shallow-water constituents are already combinations of other constituents, an analysis of the pressure record (or the pressure record divided by a constant density) may give misleading results. It may be necessary to correct the recorded pressure by means of equation (5) before any analysis is done.

As an example of the errors that could be introduced we can take a highly idealized case and put in some typical numbers. We assume that in an estuary we have placed an Aanderaa gauge in 50 metres of water (measured below mean sea level) which has a water level variation described by a single cosine term with amplitude 3 metres and frequency σ. Typical values from the Middle Estuary of the St. Lawrence River would have the mean density of the water column as 1.020 g/cm^3, the amplitude of the density variation as 0.004 g/cm^3 and the phase of the density variation as 15° with respect to the water level. The mean velocity is 10 cm/s, the amplitude of the tidal stream is 120 cm/s and the phase is 130° with respect to the water level. At the gauge, the mean density is 1.026 g/cm^3 and the variation is 0.001 g/cm^3, with a maximum at 15° after high water. Then, assuming the value of K is always equal to 1, and after doing a certain amount of algebra, we get an expression for the recorded pressure (P_r) at the gauge which would be :

$$\begin{aligned} P_r(t) = g(&5096.20 - 0.0004 \cos 3\sigma t + 10.32 \cos 2\sigma t \\ &+326.13 \cos \sigma t - 4.21 \sin \sigma t - 6.34 \sin 2\sigma t \\ &- 0.0019 \sin 3\sigma t) \end{aligned}$$

in kilopascals. If we make the usual calculation, we would divide this pressure by the mean density and g to obtain a recorded water level. The difference between the recorded water level and the true water level is then an estimate of the error that we have made by using the traditional method of relating pressure to water level. This error function will be (in centimetres),

$$E(t) = \frac{P_r(t)}{1.020\,g} - (5\,000 + 300 \cos(\sigma t))$$

so :

$$\begin{aligned} E(t) = &-3.73 + 10.12 \cos 2\sigma t + 19.74 \cos \sigma t \\ &- 4.13 \sin \sigma t - 6.22 \sin 2\sigma t. \end{aligned}$$

This function is time dependent, has a first harmonic of the original frequency, and has a mean value which is not zero. The maximum value is about 26 cm and the minimum value is about — 34 cm.

If then, we had made the hydrostatic assumption and just analyzed the pressure record, we would have been in error by finding one frequency that was not present in the physical situation and by having an absolute maximum error of more than 34 cm in the water level. If the simple example had been more realistic by having a number of tidal constituents present, then the errors could have been worse. It is important to notice that the exact form of the error function is very dependent upon the relative amplitudes and phases of the variable factors in equation (4).

CONCLUSIONS AND RECOMMENDATIONS

From the above discussion, the following conclusions may be drawn :

1) The proper equation relating depth changes to pressure changes is equation (5) given in the form :

$$\Delta h = \frac{\Delta p}{\bar{\rho} g} - \frac{h \Delta \bar{\rho}}{\bar{\rho}} + \frac{K(\rho + \Delta \rho)(v + \Delta v)^2}{2 \bar{\rho} g}$$

2) The value of the parameter K will depend upon the geometry of the water level gauge and its sensor and upon the attitude of the gauge in relation to the flow field.

3) The hydrostatic assumption should only be made when either $K = 0$ or $v(t) = 0$.

4) If the hydrostatic assumption is made incorrectly, the resultant computed water level may be significantly different from the true water level. The computed water level may have extraneous frequencies, and the amplitudes of the compound tidal frequencies could be distorted. These errors will be functions of time, and not constant.

From the conclusions, two major recommendations follow almost immediately. These are :

1) Proposed gauge locations should be carefully chosen to eliminate density and velocity effects. This means that gauge locations should have very little, or no, mean density variation with time and that the velocity of the water should be as small as possible.

2) Efforts should be taken to design pressure ports so that the K value is effectively zero.

If either one or the other of these recommendations is followed, Bernoulli effects may be ignored. If neither can be followed then the following procedure is suggested :

3) Water level gauges should be towed to determine the value of the K parameter and the effects of changes in the geometry of the flow.

4) Whenever a water level gauge is installed, a recording current meter should be placed on the same mooring.

5) The pressure record should be corrected for Bernoulli effects before any analysis is done.

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