

ACCURATE CHART LATTICING FOR LORAN-C

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Paper presented at the 17th Annual Canadian Hydrographic Conference, Patricia Bay, Sidney, B.C., 18-20 April 1978, and reproduced by kind permission of the Conference Organizers. Although Loran-C is not a global system, its coverage is extensive, its use in precise bathymetric surveying is not unusual and several countries chart its lattices. The variation found in land/sea interface distortion patterns and the experience of helicopter use for calibration are particularly called to the attention of hydrographers. (*Editor's Note*).

ABSTRACT

Unless the Loran-C lattice has much the same accuracy as any other feature shown, the chart is out of balance. There is not much point in charting hazards with great precision if the mariner must allow a large margin for positioning error in his navaid.

The Canadian Hydrographic Service's calibration program aims eventually to improve our knowledge of radio wave propagation so that we can rely on a calculated lattice with only a very few check points to verify the predictions. While we work towards this, we also map the lattice in the field so that we can put it on the chart accurately now.

We calibrated the Canadian West Coast Loran-C chain in the Spring of 1977, using Satnav offshore to give the ± 150 m accuracy needed for latticing small scale charts. We looked for and found the predicted coastal "phase recovery" using Trisponder and sextant fixing. And we made observations on shore by helicopter and calibration van to give propagation data for future predictions.

INTRODUCTION

In an old history of Newfoundland, one of those 19th century tomes four inches thick, there are maps dating from John Cabot's discoveries onwards. The early ones are quaint and imaginative — the sort of thing

you see reproduced on a Christmas card or Kleenex box. Then abruptly you come across one as correct in shape and detail as anything from the 20th century. It is James Cook's map, based on his surveys from 1765-68.

In a much smaller way, we aim to do for chart lattices what Cook did for Newfoundland. On past charts the smooth hyperbolas were mathematically correct, but in fact somewhat imaginary. The reason is that nature is not tidy. Because the radio wave is slowed down by land the Loran-C position line is not a plane hyperbola but an irregular curve. The kinks in it can be both large and abrupt.

For example, the Y position line of the West Coast Canadian Loran-C chain is shifted $2 \mu s$ (450 m) in 20 km at the entrance to the Strait of Juan de Fuca by the land effects illustrated in figure 4. (Note that 450 m — 1/4 n. mile — is the advertised maximum position error for Loran-C).

Another example: in St. Johns, Newfoundland, a fisheries officer recently related how a Portuguese fisherman anchored east of the Virgin Rock, on the Grand Banks, complained that the Decca readings put him to the west of the Rock. He was quite right, and if he had looked up "Radio Aids to Marine Navigation", he would have found a correction to Decca Purple that moves the fix two miles to the east. But how many mariners look up correction tables?

Computer controlled lattice drawing now allows us to warp the lattice to follow the real position line across the earth's surface. Using this technique, the Loran-C lattice should be correct to 1 mm on the chart like any other feature. After all, there is not much point in finding all hazards to navigation and positioning them with high precision, if the navigator cannot rely on the Loran-C lattice on the chart to keep him clear of them.

The problem is to know exactly where the position line runs. We would like to be able to predict this mathematically, but we don't yet know enough about radio wave propagation to do so. We therefore must go out and map the lattice using an independent positioning system, in much the same way as we map the depth contours. It is practicable to do this for offshore waters, where radio propagation is more predictable and the small scale of the chart hides small errors, so that we don't need very many samples. But inshore, the propagation anomalies are sharper and much more variable. Mapping them everywhere would be prohibitively expensive, and we are forced to leave the lattice off the chart until we learn more about propagation so that we can predict coastal anomalies from a small number of observations.

LORAN-C

Loran-C is a medium accuracy hyperbolic navaid, which combines long range with relatively low cycle ambiguity by making phase comparison type measurements on a pulsed groundwave. Its range is limited by the reliability of cycle selection to about 750 n miles over sea water, and to considerably less over land. The repeatability of a shipborne receiver is

0.2 μ s, which is 30 m on the hyperbolic baseline, and this sets a limit on the accuracy to which the chart lattice need be calibrated.

Loran-C has been adopted by the U.S. Coast Guard as their standard coastal/confluence zone aid to navigation, and by 1980-81 the entire U.S. Pacific and Atlantic Coasts and the Gulf of Mexico will be covered, plus the Canadian Pacific Coast and the Great Lakes. The Canadian Coast Guard is considering a further Loran-C expansion over the Canadian Atlantic Coast to part way up the Labrador Sea.

While the Coast Guard in each country is responsible for providing and operating radio nav aids, the U.S. National Ocean Survey and the Canadian Hydrographic Service are responsible for the lattices on the charts, without which the nav aid is useless. The calibration we describe here was done by the Canadian Hydrographic Service in the spring of 1977 to produce offshore charts of the areas covered by the West Coast Canadian Loran-C chain.

RANGE MEASUREMENT VERSUS TIME DIFFERENCE MEASUREMENT

The normal ship-borne Loran-C receiver measures the time difference between the arrival of the master and the slave signal. Over this short period of less than a tenth of a second a standard low cost oscillator gives adequate timing accuracy.

At the Loran-C transmitters, however, very precise atomic clocks (frequency standards) are used to control the slave coding delays and the preset interval at which the Loran-C transmissions are repeated. We have a similar atomic clock attached to our calibration receiver. If we synchronise our receiver to the arrival of one transmission, and to the pulse repetition rate, we can then predict when the next and all succeeding transmissions will arrive ; if a transmission arrives early, we have moved towards the transmitter, if it arrives late we have moved away.

Taking this a step further, if at the start we set up at a known point close to the transmitter, we can calculate the travel time (Δt) of the radio wave, from the known distance from the transmitter (Δd) and our best estimate of the propagation velocity (v) :

$$\Delta t = \frac{1}{v} \Delta d$$

We then synchronise the receiver to the instant of transmission instead of to the instant of reception. At this short distance, an error in the assumed velocity will make only a very small error in the synchronisation travel time.

Then we move out to another known point at a much greater distance from the transmitter, calculate the distance from the transmitter, predict the travel time of the radio wave to that distant point. Because a small difference in propagation velocity will have a proportionately larger effect at long range, this comparison should give us a very sensitive measurement of propagation velocity.

A range measuring receiver is strongly affected by a change in propagation velocity. But if you look at the difference in arrival time between transmissions from two stations, the effect is very much smaller, particu-

larly if the stations are nearly equidistant from the receiver. In addition, you cannot separate propagation velocity along the path to the master from velocity along the path to the slave. Hence a standard time difference receiver is not of much use in calibration if you are looking for information on propagation velocity. This is why we use rho-rho (in navigation these receivers fix the ship by measuring two ranges, hence the name "ρρ" or "rho-rho") receivers with atomic clocks. In addition, our Austron rho-rho receiver gives us the time differences provided by a standard receiver, plus auxiliary information on signal strength, signal to noise ratio, and pulse shape.

Measuring travel times requires some special precautions. Even an atomic clock is not perfect, and we are looking at very small time differences — 3 nanoseconds (10^{-9} s) per metre change in distance. There will inevitably be a very small difference in timekeeping between our atomic clock and the one running the chain. It will only be a few parts in 10^{13} , but this is enough to introduce an unknown clock rate of some 50 ns (15 m) per day. We have to determine this clock rate carefully, by returning to the same point at intervals and finding out the rate at which the range measured to the transmitter is changing. We can then eliminate clock rate error from our calibration results.

PROPAGATION VELOCITY AND THE PHASE LAG CURVE

A Loran-C receiver measures time, but the chart's dimensions are nautical miles, not microseconds. Only one position line, the baseline bisector along which master and slave are equidistant, is independent of the propagation velocity. All other hyperbolas move across the chart depending on what value of v is adopted in :

$$\Delta d = v\Delta t$$

Because we are interested in the corrections to a chart lattice, we tend to think in terms of shifts in metres, which we call phase lag, rather than of different velocities of propagation. The two are equivalent. For example, imagine you have two receivers set out so that they are both 500 km from the transmitter, but one has an all seawater path and the other a path over mountains. The wave travels so much faster over seawater that the pulse at the seawater receiver will be 1/3 way through the first cycle by the time the same pulse is just beginning at the land receiver.

The "overland phase lag" is 1/3 cycle, which is $3.3 \mu\text{s}$, or about 1 000 m at the Loran-C wavelength of 3 km. A hyperbolic lattice for the area around the land receiver, which was based on the seawater velocity, would be in error by 500-2 000 m, depending on the geometric spread of the hyperbolic position lines. In fact we would not use the seawater velocity for a land signal path, but choosing the correct alternative is not easy. Figure 1 shows how much the phase lag depends on ground conductivity, which is the main component of ground impedance. For example, at 100 km from the transmitter there is 350 m ($1 \mu\text{s}$) difference in phase lag between the middle curve for hilly land of conductivity 0.002 mho m^{-1} (such as New England) and the upper curve for mountains of equivalent

conductivity $0.0005 \text{ mho m}^{-1}$ (such as the B.C. Coast Range). Note how the phase lag offshore, represented by the right-hand half of the curves on figure 1, depends on the impedance of the land over the first part of the signal path.

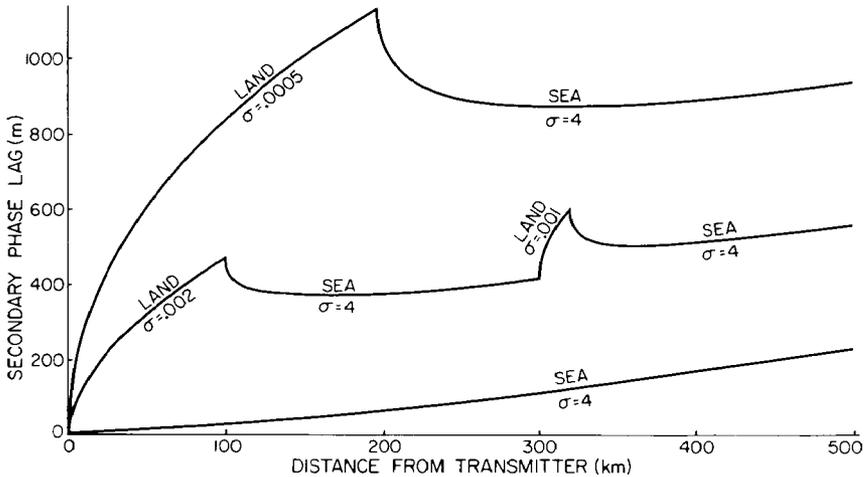


FIG. 1. — Secondary phase lag of radio wave travelling over mixed land-sea paths (From BRUNAVS, 1976).

The bottom curve shows an all seawater path. The middle curve shows 100 km of high impedance (rocky) land, then sea, with an island at 300 km. The top wave shows an initial 200 km of very high impedance mountainous land. Note the "phase recovery" on going from land to sea; it is large enough to have a serious effect on inshore chart lattices.

Phase lags are in metres, in comparison with a reference wave travelling through the atmosphere at $299\,691.2 \text{ km/s}$. Ground permittivity of 15 e.s.u. is used throughout, and the conductivity in mho/m is marked on the curve. The effect of the transmitter induction field is not shown.

FINDING GROUND IMPEDANCE: LAND CALIBRATION

We do not yet know enough to predict ground impedance accurately from the geology and roughness of the terrain. Incorrect estimates have produced lattice errors of $2 \mu\text{s}$, which created serious inaccuracies. At present the only effective approach is to go out in the field and measure phase lag, or propagation velocity, by making travel time observations close to the transmitter and a long way from it, as described above. From this we can deduce the impedance, and then use this information to calculate the travel time to other points (so long as we can reasonably assume the terrain on the path to them has the same impedance as the path we measured). After we have measured phase lag over a large number of lines, we hope to be able to predict impedance by matching similar paths on the geological map.

This Pacific Coast calibration was the first on which we have made land observations. We did so with some trepidation, as we knew that cliffs and mountainous terrain would cause phase anomalies, and so would power lines and buildings close to the observation points. For synchronisation, we chose geodetic points close to the master and Y slave transmitters that

avoided these problems, and for the distant point we used the wharf at Patricia Bay (south end of Vancouver Island). (See figure 2). We also planned observations at intermediate distances.

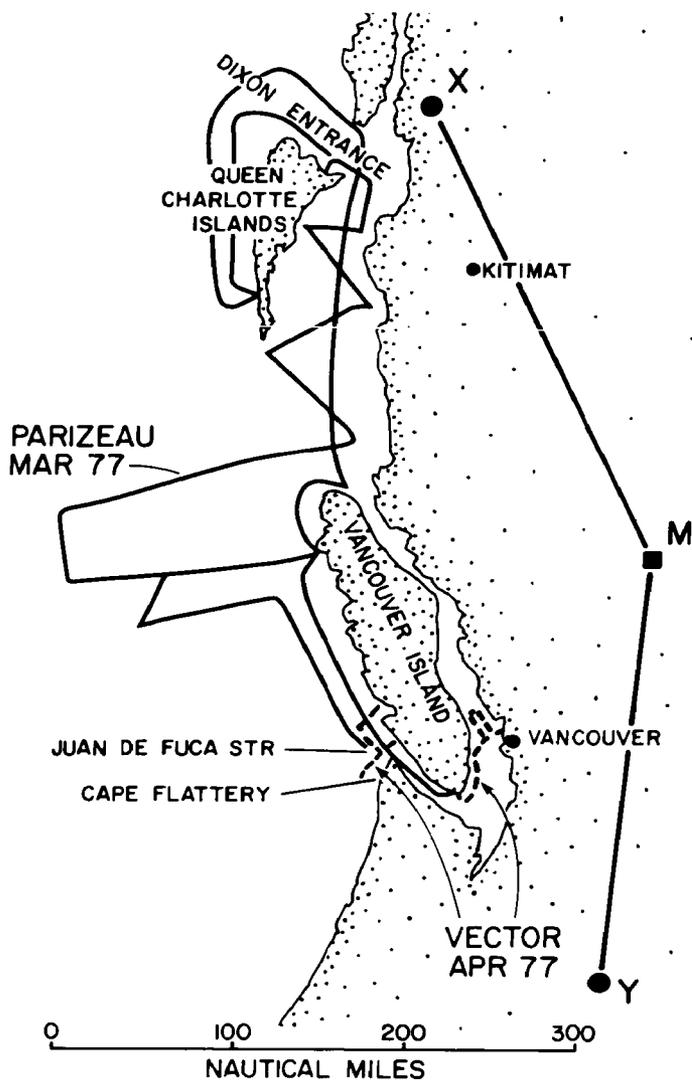


FIG. 2. — Tracks for offshore calibration (firm line) and inshore calibration (pecked line).

We decided to move by helicopter because of the long distance involved, and technician R. LOSCHIAVO fitted the rho-rho receiver and its no-break power supply in a Canadian Coast Guard Bell 212, where it worked very well. We landed close to the geodetic station, put out an 8 ft whip on a board over the plug, and connected it to the receiver with a shielded antenna lead. Having taken the reading we disconnected the antenna and moved to the next point. There was no need to track the Loran-C signal while flying, so long as we maintained synchronisation by means of the atomic clock.

After the usual initial problems all seemed to be going well with this operation when at one observation point we did an overdue experiment to verify the technique. We moved the antenna about 30 m towards the transmitter, expecting to see the equivalent 0.1 μs decrease in travel time. In fact the reading increased by 5 μs . Further tests made it clear that the reading varied depending on the orientation of the antenna lead relative to the transmitter, with shifts of up to half a cycle (5 μs) when within 50 km of the transmitter, and of 0.5 μs when 300 km from the transmitter. The antenna lead was perhaps acting as an auxiliary antenna, or as a re-radiator. We therefore moved the equipment into a van, where the antenna could be installed on the roof with a very short lead directly to the receiver. We found no measurable variation in reading as the van turned through different headings relative to the direction of the transmitter.

We repeated the land calibration with this set-up until over-heating in the van (which was not air-conditioned) stopped work for the summer. The results were reliable, but still not as accurate as we had hoped. Agreement between repeated travel-time measurements was in the worst case $\pm 1 \mu\text{s}$ over a travel time of about 1 000 μs , instead of the ± 0.1 we aimed for. The value for land conductivity deduced from these measurements varies, depending on the path, from 0.0006 to 0.0015 mho m^{-1} with an estimated reliability of ± 0.0005 mho m^{-1} .

We expect to improve on this reliability in future, using the experience of the first test. We suspect that the clock rate of the atomic frequency standards is one source of error ; we have to determine it very carefully, and in the calibration van it may vary due to vibration and temperature changes. We need to learn more about selecting good observation sites where the phase reading is not affected by buildings, trees, power lines, railway tracks, or nearby mountains. Finally, because the real life phase lag curve is not smooth, as shown in figure 1, but varies as the ground impedance changes slightly along the line, we must observe at a series of points along the curve and not just at both ends (as was planned for this test before time ran out).

OVER-WATER PHASE LAGS — OFFSHORE CALIBRATION

In the land calibration we attempted to tie down the left-hand over-land part of the phase lag curve (figure 1), with some success. Next, we looked at the flat right-hand over-water part of the curve — passing over the abrupt jump at the land/sea boundary, which we will come back to later. The impedance of sea water hardly varies and, so far as chart latticing is concerned, that part of the graph is really as smooth as it looks. However the level of the curve depends on the conductivity of the land in the first part of the signal path, and as noted we cannot yet predict that from geological information. Therefore we calibrated all along the B.C. coastline to measure this level, running a double line in order to verify any unexpected readings (see track of *Parizeau* on figure 2), and using Satnav as the position reference. As figure 3 shows, we could expect ± 150 m

accuracy from satellite fixes ; since the scale of the charts for which we were calibrating was 1 : 150 000, that would limit errors to ± 1 mm.

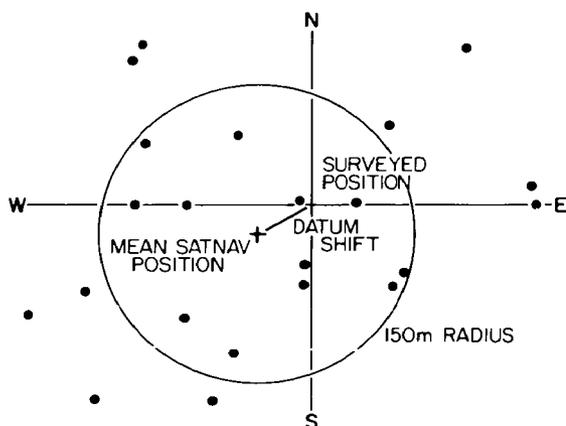


FIG. 3. — Errors in Satnav fix underway at sea.

A plot of the difference in position between simultaneous "Trisponder" precision microwave fixes having about 20 m accuracy, and Satnav fixes, observed on CSS *Hudson* in Lancaster Sound, September 1976.

More than 200 Satnav fixes along *Parizeau's* track provided corrections to chart lattices, and the two lines westward to the 200 mile limit, along a radial from the master transmitter, verified that the graph has the shape predicted by the mathematical model we are using. Observations in any one area agreed with each other to $0.5 \mu\text{s}$ (± 150 m in range), which agrees with the expected accuracy of Satnav.

The corrections were certainly significant. For example, we found an increase in correction to the X position line of $2.5 \mu\text{s}$ from west to east across chart 3652, which covers the approaches to the Strait of Juan de Fuca. If a mean correction were applied to this chart, the residual error of $\pm 1.2 \mu\text{s}$ at either side would move the position line 3 mm on the chart or 450 m on the ground. The chart alone would be contributing the maximum 1/4 n. mile error advertised for Loran-C. Fortunately, computer draughting makes it feasible to warp the lattice and put the line in the right place.

There is one error in any phase comparison system, Decca or Loran-C, that swamps all others. The receiver must make the correct decision on which whole cycle to track. A cycle selection error in Loran-C is $10 \mu\text{s}$, which could be catastrophic when close to hazards. Land path from the transmitters distorts the Loran-C pulse and makes such errors more likely, particularly at long range, and this chain has far more land path than any of the earlier Loran-C chains.

We had two standard hyperbolic receivers onboard, and at about half hour intervals the watchkeeper switched them both on for a cycle selection test. The results were alarming ; in the centre of the coverage area cycle selection tests were only 80-90 % correct, in Dixon Entrance with the southern Y slave at maximum range only 70 % correct, and in the Strait of

Juan de Fuca where the signal from the northern X slave has to struggle over the Coast Range neither receiver would acquire groundwave (MORTIMER, 1977). Adjustments at the transmitters, made after our tests, appear to have improved performance in the central part of the chain ; subsequent tests by the Coast Guard have shown 100 % correct cycle selection there, which is in line with extensive tests we have made on the Atlantic Coast. To remedy problems in the Strait of Juan de Fuca and in Dixon Entrance, a third slave will probably be built at the northern end of Vancouver Island. Meanwhile cautions on the chart warn mariners of cycle selection problems in these areas.

PHASE RECOVERY AT THE COASTLINE : INSHORE CALIBRATION

The most intriguing feature of the phase lag curve in figure 1 is the striking " phase recovery " at the coastline as the wave passes from land to sea.

Visualising the wave front in three dimensions, the lower part of the wave, slowed by the drag of the ground, lags further and further behind the upper part as the wave crosses the land. At the coastline it suddenly encounters the much lower impedance of the sea, and in a very short distance the bottom tries to catch up with the top, as though the whole wave front were an elastic balloon.

Phase recovery is predicted by the mathematical model we use, as a consequence of the " reciprocity principle ", which requires that the effective phase lag be the mean of the phase lag calculated from transmitter to receiver with that calculated from the receiver back to the transmitter (MILLINGTON, 1949 ; BIGELOW, 1963).

Phase recovery was verified during tests on Decca transmissions across the south coast of England by PRESSEY, ASHWELL and FOWLER (1956). They also found that the change within half a wavelength (1.5 km) of the coastline was larger and sharper than predicted ; and that there were variations from theory of about 1/100 cycle ($0.1 \mu\text{s}$ or 30 m for Loran-C) out to five wavelengths (15 km) from the shore.

All this is unfortunate, because most hazards to navigation lie within 15 km of the shore. Although mariners can use radar there, identification of the point returning the echo is often uncertain, particularly on a featureless or low-lying coastline, and it would be very valuable if the Loran-C lattice could be located reliably enough to put it on the chart.

The inshore calibration of last spring was intended to explore this problem rather than to solve it, which may take years. We aimed to check that phase recovery actually does occur with Loran-C ; to see how closely it follows the predictions ; and to find out what effect the extreme topography of a B.C. coastal fjord has on phase lag and performance in general.

The first test in the *Vector* consisted of running out from Vancouver Island across the entrance to the Strait of Juan de Fuca past Cape Flattery, shown as a pecked line on figure 2. We planned to use Trisponder posi-

tioning, but had trouble with one transponder and so used sextant fixing for part of the track.

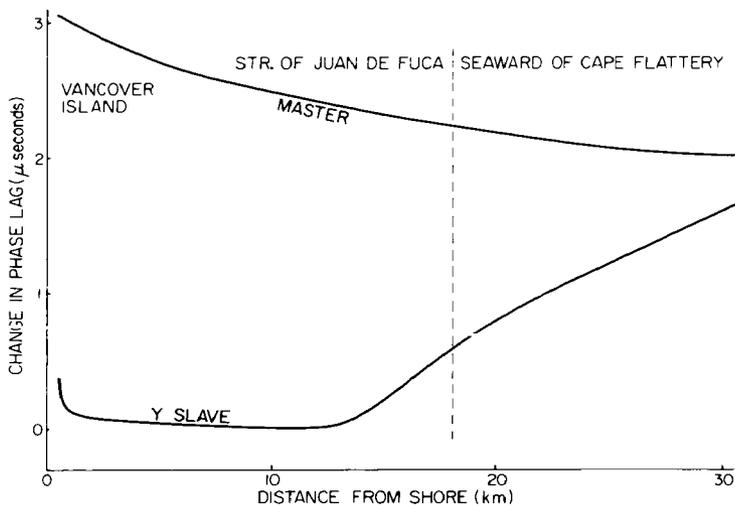


Fig. 4. -- Observed variations in phase lag close to shore.

The top curve shows a phase recovery of a microsecond in 30 km as the ship steams away from the land along a radial from the Master transmitter. The bottom curve shows an abrupt increase in phase lag as the ship passes from the strait, where the Y slave is seen over water, into the shadow of Cape Flattery.

The track followed is shown on figure 2.

The two lines running out to sea from Vancouver Island are radials from the master transmitter, and both showed clear evidence of phase recovery. The reduction in phase lag on leaving the shore was $1.0 \mu\text{s}$ for 30 km increase in distance from the transmitter, as shown on the top curve of figure 4. That is double the predicted phase recovery, perhaps due in part to the influence of the 1 000 m high mountains on Vancouver Island, which accentuate the land/sea boundary effect.

The path from the Y slave changes abruptly along the longer line running southwest from Vancouver Island. Initially, the signal arrives up the strait. Then as the ship comes under the "shadow" of Cape Flattery the path changes suddenly to land (figure 2). This shows up graphically on the lower curve of figure 4, with a $2 \mu\text{s}$ increase in phase lag over 20 km.

Finally, to test the performance of Loran-C among steep-to islands and in a mountain fjord, *Vector* spent four days in the Strait of Georgia, and up Howe Sound, just north of Vancouver. Howe Sound is only 3 km wide and the mountains go straight up to 1 500 m another 3 km back from the shore, and so it was not surprising the signal strengths were marginal and the receiver lost lock on making the turn at the head of the fjord. However, envelope shape diagnostics on the monitor receiver indicated that the signal was merely weak and not distorted.

Howe Sound is aligned almost exactly in the direction of the master transmitter, so that the signal was travelling along a channel one wave-length wide, with mountains on both sides at the head and all the way for

35 km down one side. One might have expected confused phase effects under these circumstances, but in fact the master signal showed a reasonably smooth phase recovery curve from 2 km short of the head of the fjord out into open water, with undulations of about $0.3 \mu\text{s}$ which may have been partly due to problems in sextant fixing on tangents of land (MORTIMER, 1978).

The signal from the Y slave crosses the fjord at about 60° , so that the ship was within the half wavelength ($1 \frac{1}{2}$ km) zone of very high phase change most of the time. This phase change turned out to be a sensitive indicator of how close to the shore the ship was, indicating that the change, though steep, is regular.

Howe Sound widens at the entrance, with a large island in the middle. Visualised in three dimensions, the phase lag in the vicinity of this island looks like a tree stump, with a large buttress root falling away in a south-westerly direction, representing the phase recovery in the master signal. We ran a series of lines across this "buttress", and did in fact find a ridge of phase lag in the lee of the island, decreasing in amplitude as we got further away. As before, the phase lag was greater than predicted, but the excess decayed in a regular fashion :

Table 1

<i>Excess of observed phase lag over predicted phase lag close in to mountainous B.C. coastline</i>	
Distance from coast-line (km)	Excess of observed phase change over predicted (μs)
< 1	1.5
2	0.5
7	0.4
10	0.2
25	0.0
32	0.0

The results from this inshore calibration agree quite closely with PRESSEY's conclusions quoted above. They indicate that some kind of scaling effect, probably associated with the ruggedness of the terrain, should be applied to predictions for the nearshore zone. An extension of P. BRUNAVS' work on approximating the effect of terrain by adjusting the ground impedance (BRUNAVS, 1976) may be successful in modelling this, as might the approach of JOHLER and DOHERTY in modelling the "Death Valley Anomaly" (DOHERTY, 1974).

The results indicate that the coastal effect depends mainly on the land on the direct line to the transmitter, and does not appear to be much affected by reflections from the land on either side. We need more evidence to be sure of this point, but it is a hopeful conclusion, as it would be impossible to predict phase lags if reflections had strong effect.

CONCLUSIONS

This calibration again demonstrated the effectiveness of Satnav in mapping the Loran-C lattice for small and medium scale charts, to 150 m accuracy.

We had moderate success in determining ground impedance from land observations ; we learned a lot, and expect improved results from future work.

We verified the general shape of the predicted phase lag curve, including phase recovery at the coastline.

We confirmed earlier observations that the phase change at the coastline is as much as $2 \mu\text{s}$ greater than predicted, but found that even under the extreme conditions of the B.C. fjord coastline, the excess phase change appears to be regular. This raises our hopes that extensions to the existing mathematical model will enable us to predict nearshore phase recovery, at least in less rugged terrain where the effect is not so pronounced.

ACKNOWLEDGEMENTS

The officers and ship's companies of the *Parizeau* and the *Vector* played a vital part in this work. Of the many other people who helped in the calibration, we particularly single out Mr. P. BRUNAVS, who pioneered precise phase lag measurement in the Canadian Hydrographic Service and has contributed to it ever since ; Lcdr. W. SCHORR, U.S. Coast Guard, who advised us on equipment and guided us when we had problems ; and Mr. R. LOSCHIAVO, the technician who made it possible to carry out the observations.

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