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A NOTE ON EXTREME TIDAL LEVELS

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SUMMARY

A method of predicting extreme peaks of predominant semi-diurnal tides on the basis of 'Highest Astronomical Tide' (HAT) is investigated. CARTWRIGHT's (1974) views and comments hold in general, but some variations are observed between the extreme physical tides and HAT's. These variations are understandably due to nonlinearities in the response function of phase lags. The constituents of frictional origin tend to minimise the range of extreme physical tides. The order of these marginally reduced levels at various near-extremes can be easily reversed by nonlinearities of the response function. Although extreme peaks follow the moon's perigee these extremes occur when the longitude of the moon's node is near autumn equinox. It appears to be impossible to specify a simple rule for absolute determination of extreme tidal levels, but if a tolerance in accuracy of a few centimetres is allowed then the method as presented can work adequately.

THEORETICAL BACKGROUND OF THE METHOD

The object of this investigation is to help find a more economical way of computing extreme tidal levels than the customary one of predicting all tides over a period of 19 years and selecting the largest. It is obviously wasteful to compute neap tides and tides near the times of apogee, but it is not obvious how many other periods may be safely ignored or even which 19-year period should be covered. CARTWRIGHT (1974) gave a precise account of the peak tide-raising forces from purely astronomical reasoning. The extremes of physical tides can be easily predicted at places where the response of sea to the tide-generating forces is simply linear (that is, lacking sharp resonances or anti-resonances) as both are expected to be close on time scale. The differences grow in less simple linear regimes or as nonlinearities become more pronounced. Here, the possibility of establishing a rule which may help to predict extremes of physical tides in more general, predominantly semi-diurnal nonlinear regimes from peaks of tide-raising forces is examined. In the harmonic method the tidal elevations of a given place are predicted as:

$$\zeta = \sum_n a_n \cos (V_n - g_n) \tag{1}$$

summed over all significant harmonics,
where

a is the amplitude of a harmonic,
g is the phase lag relative to phase of astronomical term,

$$V = k_1\theta_1 + k_2\theta_2 + k_3\theta_3 + k_4\theta_4 + k_5\theta_5 + k_6\theta_6 \tag{2}$$

with

θ_1 = local mean lunar time reduced to angle;
θ_2 = the mean longitude of the moon;
θ_3 = the mean longitude of the sun;
θ_4 = the mean longitude of the moon's perigee;
θ_5 = the negative of the mean longitude of the ascending node of the moon;
θ_6 = the mean longitude of perihelion; and
k's are small integers known as argument numbers.

Equation (1) can be written as:

$$\zeta = a_m \cos (V_m - g_m) + \sum_r a_r \cos (V_r - g_r) \tag{3}$$

where subscript m denotes M_2 tide and the summation is over the remaining lines.
Substituting

$$\phi = V_m - V_r - (g_m - g_r) \tag{4}$$

$$q_s = \sum \frac{a}{a_m} \sin \phi \tag{5}$$

$$q_c = 1 + \sum \frac{a}{a_m} \cos \phi \tag{6}$$

equation (3) for predominant semi-diurnal tide, becomes

$$\zeta = a_m F \cos (V_m - g_m - \psi) \tag{7}$$

where

$$F = (q_s^2 + q_c^2)^{1/2} \tag{8}$$

$$\psi = tg^{-1} (q_s/q_c) \tag{9}$$

The conditions for peaks of semi-diurnal tide generating forces given by CARTWRIGHT (1974) can be interpreted in terms of orbital elements (θ_1, $\theta_2 \ldots \theta_6$) as

$$\left.\begin{array}{r} \theta_2, \theta_3, \theta_4 = 0, \pi \\ \theta_5 = \pi \end{array}\right\} \tag{10}$$

It is difficult for all these elements to satisfy conditions in equations (10) simultaneously; therefore some tolerance, say ε, is allowed for relaxation of these conditions. Times when these conditions are satisfied within the tolerance limit determine the peaks of semi-diurnal tide-raising forces, and extremes of the physical tides are expected to occur near these points with some perturbation due to phase lags of principal harmonics, harmonics of yearly cycle Sa, and the seasonally modulating harmonics such as MA_2 and T_2. The necessary procedure requires one to find times when conditions in equations (10) are satisfied and then to compute F for a few spring tides around this estimated time.

APPLICATION OF THE METHOD AND CONSIDERATION OF OTHER FACTORS AFFECTING THE EXTREME LEVELS

In shallow water areas, both the advective and frictional terms in hydro-dynamical equations of progressive tides can be significant and cannot be ignored. The tidal spectrum is seen to contain a large number of significant shallow water constituents. The development of these constituents is such that the phase lags of some of these tend to reduce the range of extreme tides while others help to increase the range. To illustrate this fact, consider the principal constituents of Southend tides

Table 1
Values of Solar and Lunar orbital elements on dates favourable to generate extreme tides

Year	Day	Time (h)	θ_1 (deg)	θ_2 (deg)	θ_3 (deg)	θ_4 (deg)	θ_5 (deg)	θ_6 (deg)
1905	21 March	0054	180.0	178.12	357.85	186.61	201.72	281.31
1922	21 Sept.	1207	180.0	181.319	179.59	178.92	180.25	281.61
1940	23 March	1150	0.0	177.95	0.818	171.14	158.79	281.91

(AMIN, 1976), given in table 1. At the time of an extreme tide one can expect that the following conditions should hold:

$$\left.\begin{aligned} V_{N2} - g_{N2} &= 0 \\ V_{M2} - g_{M2} &= 0 \\ V_{S2} - g_{S2} &= 0 \\ V_{K2} - g_{K2} &= 0 \end{aligned}\right\} \quad (A)$$

These conditions require that:

$$\left.\begin{aligned} V_{N2} &= g_{N2} = -30.72 \\ V_{M2} &= g_{M2} = -6.48 \\ V_{S2} &= g_{S2} = 49.41 \\ V_{K2} &= g_{K2} = 49.90 \end{aligned}\right\} \quad (B)$$

When these conditions are satisfied, the phases of shallow water constituents $2MK_2$, $2MS_2$, $2MN_2$, ... can be approximated as:

$$\left.\begin{aligned} V_{2MK_2} - g_{2MK_2} &= 2V_{M2} - V_{K2} - g_{2MK_2} \\ &= -168.32^\circ \\ V_{2MS_2} - g_{2MS_2} &= -167.82^\circ \\ V_{2MN_2} - g_{2MN_2} &= -160.94^\circ \end{aligned}\right\} \quad \text{(C)}$$

Similarly for Avonmouth:

$$\left.\begin{aligned} V_{2MK_2} - g_{2MK_2} &= -125.1 \\ V_{2MS_2} - g_{2MS_2} &= -133.2 \\ V_{2MN_2} - g_{2MN_2} &= -133.6 \end{aligned}\right\} \quad \text{(D)}$$

These shallow water constituents, clearly, oppose the astronomical tide-generating forces at the time when they are nearly in phase and therefore expected to generate extreme tides. However, there are some constituents such as M_4 and M_6 which are never exactly in phase with M_2 at the time of high waters but nevertheless always make some contribution, which may be positive or negative.

Furthermore, the phase lags of all principal constituents which are responsible for extreme tides can interact to shift the time of extreme tides, in a fashion which may not be easily predictable. In those cases where the response function of phase lags

$$g = f(\sigma) \quad (11)$$

is a linear function, the shift in times of high waters 't_s' can be easily approximated by the slope of the line as:

$$t_s = (g_i - g_j)/(\sigma_i - \sigma_j) \quad (12)$$

where (g_i, σ_i) and (g_j, σ_j) are two points of the function or, simply, phase lags and speeds of two principal constituents.

Observed tides in shallow water areas are not simple, and their response functions are, generally, nonlinear as shown in figure 1. There is no simple way of computing time shift 't_s', but one can approximate the optimal value. For optimisation, it is suggested that the response function may be represented by a straight line.

$$\hat{g} = \alpha + \beta\sigma \quad (13)$$

such that

$$\sum w_c(g_i - \hat{g}_i)^2 = \min \quad (14)$$

The weights w should be related to the amplitude of the associated constituents, i.e. may be chosen proportional to the amplitude of the constituents. The phase lags tend to displace the timings of observed high waters, and nonlinearities in the response functions of phase lags have an inverse effect on the predictibility of the time shift.

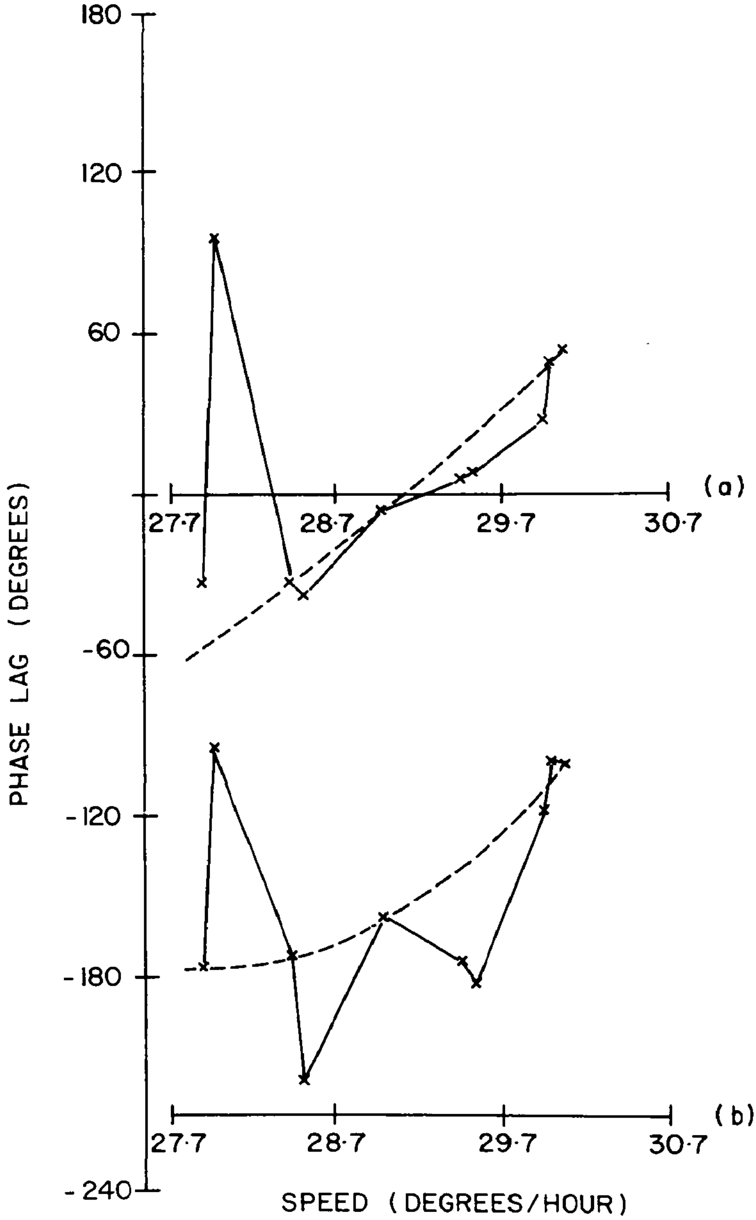

Fig. 1. — Response functions of phase lags of semidiurnal tides: (a) Southend, (b) Avonmouth.
x———x = as in observed tide with distortion due to the presence of shallow water constituents;
- - - - - - - = smooth functions.

Thus we see that these are two main factors which can obstruct the simple prediction of extreme tides:

(a) Some shallow water constituents tend to compress the range of extreme high tides and reduce the difference in range of various tides which are extreme or near extreme.

(b) Phase lags of principal constituents can shift the timing of extreme tides and a nonlinear response may conspire with unfavourable astronomical conditions to produce tides higher than those generated under astronomically favourable conditions.

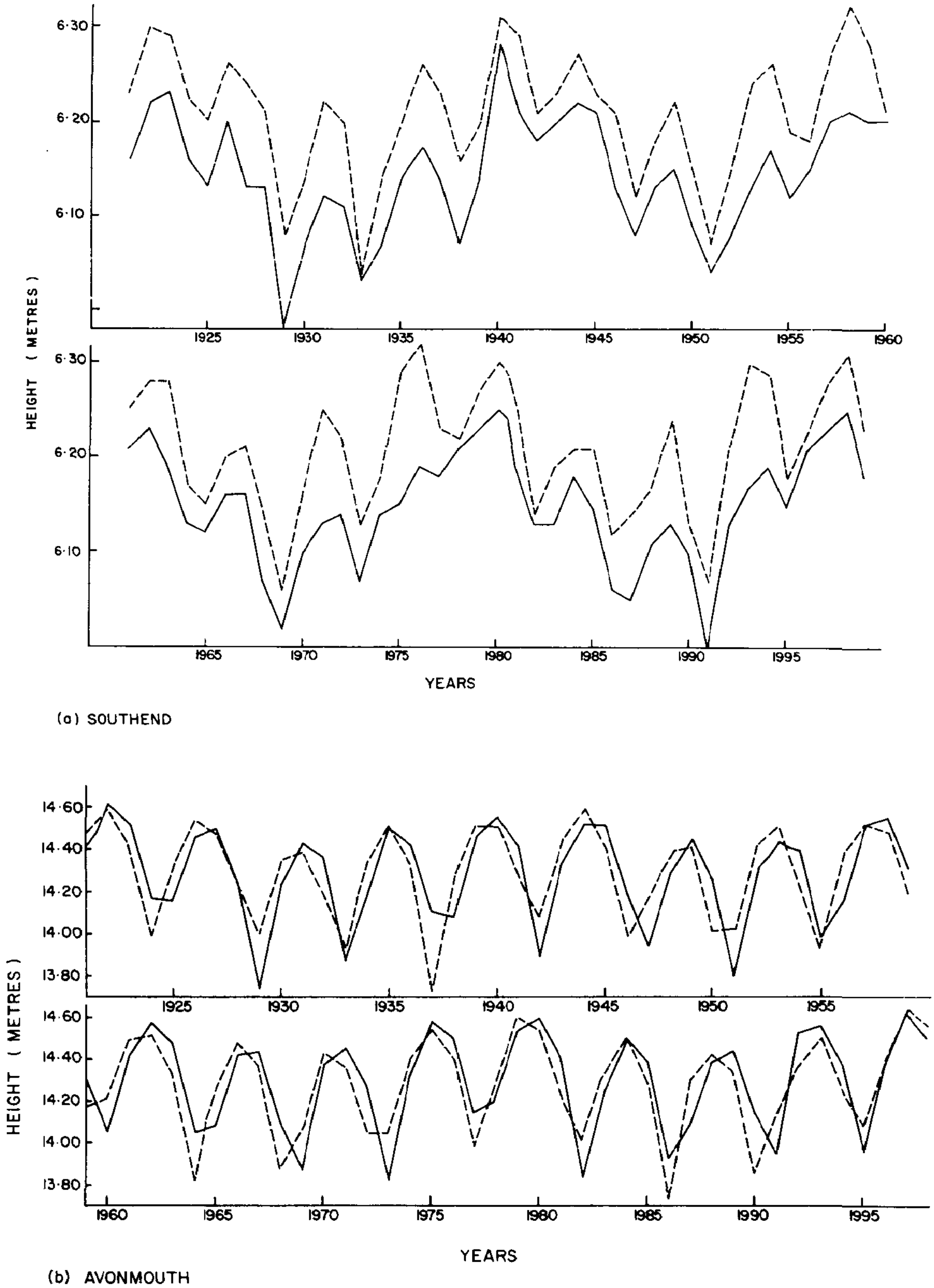

FIG. 2. — The plots of highest spring and autumn equinox tides in years 1921–1999.
———— = spring equinox.
- - - - - - - = autumn equinox.
The datum added is 3.024 m [2.9 m below Ordnance Datum (Newlyn)] for Southend, and 6.922 m [6.50 m below Ordnance Datum (Newlyn)] for Avonmouth.

DISCUSSION ON RESULTS

To find a feasible procedure, predictions were computed for Southend, using the harmonic constants derived in AMIN (1976). Their extreme values are plotted in figure 2. The Highest Astronomical Tide (HAT) expected on 21 September 1922, CARTWRIGHT (1974) was shifted to 23 September 1922 and the actual extreme high tide occurred a month later on 22 October 1922. The tides of 29 October 1905 and 3 October 1940 exceeded the highest tide of 1922 by 1 cm and 2 cm respectively. Astronomical conditions on the later dates were not favourable, as compared to those on 21 September 1922, (see table 2) to generate high tides, but these conditions conspiring with phase lags produced the highest tide on 3 October 1940. This suggests that nonlinearities in the response of

Table 2

List of principal semi-diurnal constituents and other major constituents which play an important role in determining the peak tides. Phases of $2MS_2$ *and* $2MN_2$ *are assumed to be the same as* μ_2 *and* L_2 *respectively. Constant terms in argument number are neglected.*

Argument number	Constituent	Southend		Avonmouth	
		Amplitude (m)	Phase lag (deg)	Amplitude (m)	Phase lag (deg)
0 0 1 0 0 0	Sa	0.055	215.42	0.098	195.32
1-1 0 0 0 0	O_1	0.131	187.74	0.084	8.81
1 1 0 0 0 0	K_1	0.111	11.22	0.062	136.58
2-2 0 0 0 0	$2MK_2$	0.049	105.46	0.135	269.41
2-2 0 2 0 0	$2N_2$	0.041	305.54	0.095	177.48
2-2 2 0 0 0	μ_2	0.053	310.20	0.098	183.40
	$2MS_2$	0.173	103.40	0.445	276.78
2-1 0 1 0 0	N_2	0.349	329.28	0.730	187.83
2-1 2-1 0 0	ν_2	0.108	320.43	0.197	152.18
2 0-1 0 0 0	MA_2	0.038	306.13	0.056	138.62
2 0 0 0 0 0	M_2	2.044	353.52	4.221	201.89
2 1 0-1 0 0	L_2	0.054	20.10	0.116	225.56
	$2MN_2$	0.093	359.70	0.242	169.60
2 2-2 0 0 0	S_2	0.590	49.41	1.477	260.63
2 2 0 0 0 0	K_2	0.172	49.90	0.434	254.34
4 0 0 0 0 0	M_4	0.097	8.67	0.337	346.69
4 2-2 0 0 0	MS_4	0.035	72.37	0.298	26.09

the sea may displace the extreme tides on a time scale measured by single or multiple periods of orbital elements θ_2, θ_3, θ_4 and θ_5. It will not be easy to measure such a displacement because of different rates of increase of orbital elements. The sequences of extreme high tides, shown in figure 2, are in close agreement with CARTWRIGHT (1974) in that the longitude of perigee of the moon plays a more important role than the node of the moon in determination of the cycle of extreme levels. However, years of extreme tides are not exactly as expected in the general form as suggested by CARTWRIGHT (1974). High peaks at Southend are predicted for years 1922, 1940, 1958, 1976, 1998 and 2015, and at Avonmouth for years 1922, 1944, 1962, 1980, 1997 and 2015. Avonmouth tides are shifted by half a perigee cycle since 1940 from the years of HAT. A similar shifting takes place at Southend from year 1993 to year 1998. This appears to result from the interaction of phase lags of individual constituents of the physical tide and the 186 years cycle (approximately 21 cycles of θ_4 or 10 cycles of θ_5). In the case of Southend, extreme tides generally took place in the same years as HAT, but tides expected in years 1975 and 1997 occurred (and should occur) in years 1976 and 1998 respectively. Annual extremes at Southend are always in autumn whereas at Avonmouth they occur near both equinoxes, without any obvious pattern. This may be a consequence of the fact that the response function of phase lags is almost linear for Southend and therefore the weight of constituents Sa and MA_2, which come in phase with M_2 in autumn, may have compensated or exceeded any small difference due to tide-raising forces. A comparatively high nonlinearity in the response of Avonmouth upsets this system as the phase lags of principal constituents are considerably more effective than Sa and MA_2. It is also possible that the diurnal components affect the two parts differently at the times of large semi-diurnal tides. However, it could be concluded from results in figure 2 that the most suitable conditions occur when:

$$\left.\begin{aligned} \theta_5 &\simeq \pi \\ \theta_3 &\simeq 0 \text{ or } \pi \\ \theta_3 &\simeq 0 \text{ or } \pi \end{aligned}\right\} \qquad \text{(E)}$$

If these conditions are suitable, then the mean lunar time 'θ_1' and θ_2 can be adjusted to zero or π value whichever is suitable, at the cost of some divergence of θ_3 and θ_4 from their optimal values. Although the observed extreme value may not be exactly in the same month or year as that predicted, the predicted value is expected to be within 5 cm of the nearest extreme value.

In conclusion, it appears to be impossible to specify a simple rule for the absolute determination of extreme (high or low) tidal levels, but if a tolerance in accuracy of a few centimetres is allowed, then predictions around the equinoxes of a few certain years would appear adequate for predominantly semi-diurnal regimes.

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