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SOME COMPARISONS OF RESPONSE AND HARMONIC TIDE PREDICTIONS

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ABSTRACT

Empirical tests compared response and harmonic tide predictions for Atlantic City and Pensacola (semidiurnal and diurnal tidal regimes respectively). Three years of hourly heights were analyzed by both methods in the frequency range of one to six cycles per day. The results were used to predict another three-year period, the predictions were subtracted from the observations, and energy calculations were made for the frequency bands in each of the six tidal species. Once more, response methods were somewhat better, but the differences are small compared to the total (unpredictable) continuum. The study disclosed: (1) the need to include third-order nonlinear interactions of diurnal tides in response predictions for some stations, (2) the need for National Ocean Survey to examine carefully its rejection limit in analyzed amplitudes of 0.03 foot and a practice of inferring T_2 regardless of its amplitude, and (3) the need to examine an annual modulation of M_2 presumably due to some local seasonal effect.

INTRODUCTION

The 'response method' of tidal analysis was designed by MUNK and CARTWRIGHT (1966) primarily as a research tool and to aid physical understanding of tidal processes. It also provides very good tidal predictions, not only in the nearly linear regimes studied by Munk and Cartwright (1966) but also for regimes which require fairly strong nonlinear terms, as demonstrated by Cartwright (1968) and Cartwright and Rossiter (1972). In fact, in all references just cited, and in others (e.g. Unesco, 1975), the residual variances from 'response' predictions have invariably been shown to be less than those from modern 'harmonic' predictions. The Unesco report included a caution, "It is like the difference between a Kodak (harmonic method) and a Hasselblad (response method): with little input, the former gives the better pictures, but when properly used, the Hasselblad can improve the result." The response method has never been used in the operational production of tide tables, partly because of unfamiliarity with the technique, and partly because the harmonic method gives good enough predictions for most purposes. The present experiments were designed to re-assess the relative accuracies of harmonic and response predictions in the context of the standard operational procedures of the U.S. National Ocean Survey. Because of Cartwright's greater experience in nonlinear response procedures, his offer to participate in the tests was gratefully accepted by the other two authors.

The two stations chosen for the experiment were Atlantic City, New Jersey, on the United States east coast and Pensacola, Florida, on the Gulf of Mexico. The Atlantic City tide is primarily semidiurnal, $(K_1 + O_1)/M_2$ being about 0.3. Because of the broad continental shelf, extreme sea level fluctuations are almost as large as the tidal range (figure 1 from Zetler and Lennon, 1967). This is manifested by a high continuum in the low-frequencies; the residual energy after a tidal analysis exceeds 10 % of the observed total energy (in the frequency range of 0

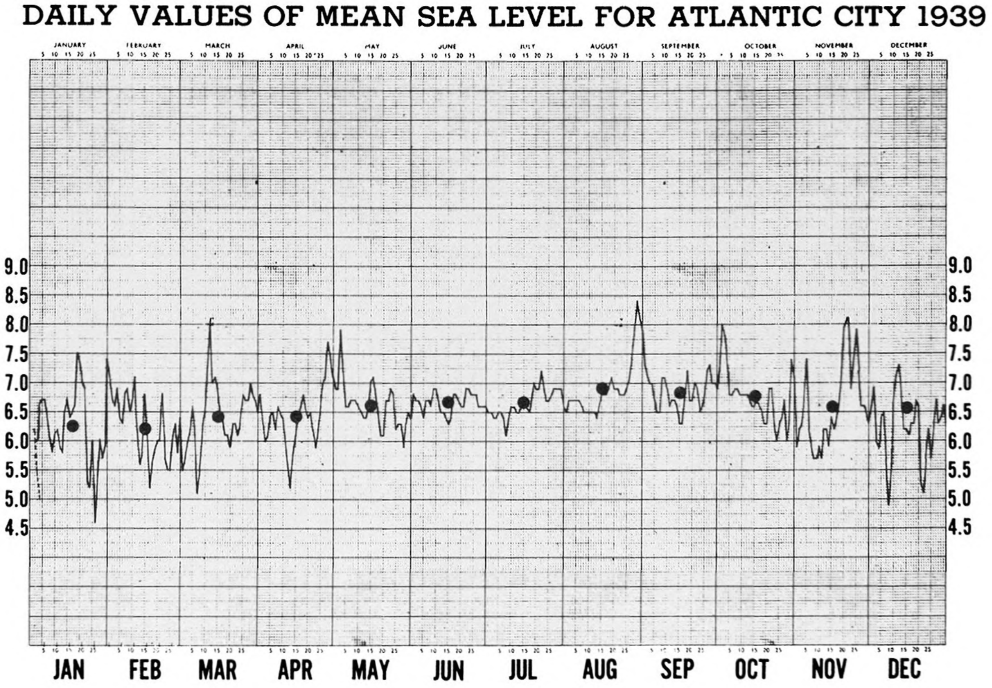

Fig. 1.

to 12 cycles per day). In a similar study of San Francisco tides, the residual energy was about 3 % of the observed energy.

Pensacola has a dominantly diurnal tide, $(K_1 + O_1)/M_2$ being about 15. The mean diurnal range is small, about 0.4 m, and a tidal analysis leaves a residual energy about 50 % of the observed energy.

The plan called for optimum analyses of three years of hourly heights by both methods, response and harmonic predictions for a different three years, subtraction of predictions from observations, and determinations of residual energy in six tidal bands, from one to six cycles per day. Inasmuch as harmonic methods obtain Sa and Ssa from averaged monthly means for as many years as are available, species 0 was omitted from the calculations. This decision also ruled out the use of the monthly and fortnightly tides, Mm, Mf and Msf, but experience has shown that these are not defined satisfactorily by routine harmonic analysis because of their low signal/noise ratio. The National Ocean Survey ordinarily solves for M_8, but this is known to be small at the two stations so it was omitted also, making 6cpd the highest frequency resolved.

ANALYSIS PROCEDURES

An editing procedure for eliminating data errors called for a quintic polynomial as a test for smoothness. However, when a roughness limit of 0.3 foot (CARTWRIGHT, 1968) was applied to Atlantic City data, roughly 2 % were flagged. It was evident that this is due to the large sea level fluctuations; a larger roughness limit, 0.6 foot, was necessary. Using the latter, the only values flagged were during a 1954 hurricane; no changes were made to these data points.

Although synodic periods are less necessary for data analysis of long series by either the response or the least-square harmonic method (all constituents within each species resolved in one matrix) than in classical harmonic procedures, it was agreed that it would be prudent to allow for this aspect. The choice of an optimum period was based on proximity to integers for the number of synodic periods of S_2 and M_2, M_2 and N_2 and K_1 and O_1. On this basis, 1107 days (3 × 369 d) was chosen in preference to 1065 days (3 × 355 d) or to 1093 days (2 × 369 + 355 d). The response analysis would cover the whole period in one analysis, thus permitting the separation of gravitational and radiational tides. Since National Ocean Survey (NOS) procedures ordinarily use 369 days, it was agreed to average harmonic constants from three 369-day analyses.

NOS analysis of 369-day series ordinarily includes 37 constituents, but with Sa, Ssa, Mm, Mf, Msf and M_8 omitted, this would leave 31. Ordinarily, any constituent with an analyzed amplitude of less than 0.03 foot is not used; if an inference can be made from one or more nearby major constituents, using relationships in the equilibrium tide or similar reference tide (not possible for shallow-water constituents) and the inferred amplitude is at least 0.01 foot, then inferred values for the constituent are used in future predictions. T_2 is always inferred from S_2.

Table 1

Available Reference Series for Response Analysis

SPECIES 1-6

SPECIES 1				
	G_2^1 (2)	G_3^1 (1)	R_1^1	$(I)^{1-0}$
	G_2^1 (1)	G_3^1 (0)	R_2^1	$(I)^{1+0}$
	G_2^1 (0)	G_3^1 (−1)		$(I)^{2-1}$
	G_2^1 (−1)			$(I)^{1+1-1}$
	G_2^1 (−2)			$(I)^{1+2-2}$
SPECIES 2				
	G_2^2 (2)	G_3^2 (1)	R_2^2	$(I)^{2-0}$
	G_2^2 (1)	G_3^2 (0)	R_4^2	$(I)^{2+0}$
	G_2^2 (0)	G_3^2 (−1)		$(I)^{1+1}$
	G_2^2 (−1)			$(I)^{2+1-1}$
	G_2^2 (−2)			$(I)^{2+1-2}$
SPECIES 3				
	G_3^3 (1)		R_4^3	$(I)^{2+1}$
	G_3^3 (0)			$(I)^{2+2-1}$
	G_3^3 (−1)			$(I)^{1+1-1}$
SPECIES 4				
			R_4^4	$(I)^{2+2}$ (0,0), (0, ½), (½, ½) (0, – ½), (– ½, – ½)
				$(I)^{2+2-0}$
				$(I)^{2+2+0}$
				$(I)^{2+1+1}$
				$(I)^{2+2+2-2}$
SPECIES 5				
				$(I)^{2+2+1}$
				$(I)^{2+2+2-1}$
SPECIES 6				
				$(I)^{2+2+2}$(0,0,0), (0,0, ½), (0, ½, ½),(½, ½, ½)
				$(I)^{2+2+2-0}$
				$(I)^{2+2+2+0}$

Other ground rules for NOS analysis procedures were tested in the course of our experiments; as a result, some procedures previously routinely accepted are now being questioned.

Table 1, extracted from Cartwright and Rossiter (1972), is a listing of various possible reference series in a response analysis for species 1 to 6. The G_n^m series (m is order and n is degree) represent complex time series of the total gravitational potential, the R series are comparable expressions of the Sun's radiational potential, and the $(I)^{i \pm j}$ series are the product of primary tide predictions, species i and j, conjugate for — sign. Symbols $(I)^{j \pm o}$ represent annual modulations to first order predictions of species j; we did not use them in this work because no corresponding constituents are used in the NOS harmonic procedure. Numbers in brackets are time lags in 2-day units.

Table 2 lists reference series used in the Atlantic City analysis. Time lags are listed here in hours (rather than 2-day units) and the lags are centered on tidal ages for each species, rather than zero, as suggested by Zetler and Munk (1975). However, when Cartwright also experimented

Table 2

Reference Series used in Atlantic City Analysis

(Numbers denote lags in hours)

SPECIES 1			
G_2^1 (72)	G_3^1 (24)	R_1^1 (24)	$(I)^{2-1}$
G_2^1 (24)			$(I)^{1+2-2}$
G_2^1 (−24)			
SPECIES 2			
G_2^2 (120)	G_3^2 (24)	R_2^2 (24)	$(I)^{2+2-2}$
G_2^2 (72)			
G_2^2 (24)			
G_2^2 (−24)			
G_2^2 (−72)			
SPECIES 3			
G_3^3 (24)			$(I)^{2+1}$
SPECIES 4–6			
			$(I)^{2+2}$
			$(I)^{2+2+1}$
			$(I)^{2+2+2}$

with lags centered on zero, he found no significant differences in residual variances. The principal advantage in considering the tidal age appears to apply to analyses using fewer than the optimum number of weights. For further reference, note that $(I)^{1+1-1}$ was not used.

In computing residual variances in the six species frequency bands, successively wider limits were used for frequencies higher than species 2 because of the greater spread implicit in nonlinear interactions of the principal constituents. In calculating the residuals for 355-day series, this involved the following:

	Cycles per day	±	cycles per month	Harmonics
Species 1	1	±	4½	285-401
Species 2	2	±	4½	628-744
Species 3	3	±	5½	958-1100
Species 4	4	±	6½	1288-1456
Species 5	5	±	7½	1618-1812
Species 6	6	±	8½	1948-2168

ATLANTIC CITY RESULTS

Table 3 shows residual energy in six frequency bands from IOS and IGPP response predictions and two sets of residuals from NOS harmonic predictions. NOS #1 values represent routine procedures, in particular substituting inferred amplitudes and phases for any constituent whose analyzed amplitude is found to be less than 0.03 foot; inferred constituents

Table 3

Atlantic City results in cm² (3 × 355 days)

		Residuals			
		Response		Harmonic	
	DATA	IOS	IGPP	NOS # 1	NOS # 2
SPECIES 1	79.47	3.90	3.86	3.93	3.85
SPECIES 2	1989.95	4.58	4.73	6.07	5.91
SPECIES 3	1.50	1.25	1.24	1.46	1.31
SPECIES 4	1.28	0.68	0.69	0.91	0.71
SPECIES 5	0.81	0.76	0.76	0.82	0.85
SPECIES 6	0.92	0.68	0.70	0.82	0.84
Species total...... 2073.92					
Overall.......... 2368.19					
Non-tidal........ 294.27					

are used in prediction only if the inferred amplitude is at least 0.01 foot. The procedure also includes inferring T_2 regardless of analyzed amplitude; since T_2 and S_2 (used as the base for the inference) are both gravitational and radiational, an inference based on the gravitational potential only leaves something to be desired. In NOS #2 tests, analyzed values only are used; of the 31 constituents resolved, only S_6 and ρ_1 are rejected for small amplitudes and widely-varying phases. The improvement in results (smaller residual energy) indicates a reconsideration of NOS routine procedures is advisable.

The residuals from IOS and IGPP response predictions are slightly different; no attempt was made to determine the cause of the small differences and either set may be taken as representative of response analysis and prediction. The principal advantage of response over harmonic methods is found in the species 2 band. Although the difference appears to be significant, it is less than 0.1 % of the energy in the species 2 band. In the light of the species 2 difference, it was puzzling that the residuals in species 1 are about equal. Our subsequent analyses of Pensacola tides may have furnished an explanation.

PENSACOLA RESULTS

Table 4 shows results obtained with Pensacola data. Response analysis #1 (at IGPP) used a reduced set of reference series:

Species 1: G_2^1 (—48,0,48), G_3^1 (O), R_1^1 (O)
Species 2: G_2^2 (—48,0,48), G_3^2 (O), $(I)^{1+1}$ (O)
Species 3: G_3^3 (O), $(I)^{1+1+1}$ (O).

Table 4
Pensacola Results in cm² (3 × 355 days)

	Data	Residuals			
		Response # 1	Response #2	Harmonic # 1	Harmonic # 2
SPECIES 1	152.24	3.21*	1.43	3.64	3.02
SPECIES 2	2.68	0.28	0.29	0.59	0.42
SPECIES 3	0.13	0.10	0.11	0.15	0.15
SPECIES 4	0.12	0.12	0.12	0.15	0.12
SPECIES 5	0.08	0.08	0.09	0.12	0.12
SPECIES 6	0.09	0.09	0.10	0.14	0.13
Species total......	155.34				
Overall..........	307.35				
Non-tidal........	152.01				

* This value was obtained using either 3 or 5 weights for G_2^1 reference series.

The decision to omit species 4, 5 and 6 was based on the very small amplitudes in species 2, hence no significant overtides involving M_2 or S_2 can be expected. The residual energy in these species therefore equals the data energy; other analyses did no better. Tests were also made with five weights for G_2^1 and G_2^2 respectively. There was no difference in the mean residuals in species 1; species 2 admittances, using 5 weights, fluctuated so wildly that this portion of the tests ended here.

Response analysis #2 (at IOS) added the reference series $(I)^{1+1-1}$ (O) after noting large residual lines at the frequencies $2OK_1$ $(2O_1 — K_1)$ and $2KO_1$ $(2K_1 — O_1)$. The results sharply decreased the species 1 residual energy. Table 5, showing apparent admittances from NOS harmonic constants, indicates clearly the need for a species 1 nonlinear reference series. A linear response analysis, required to be smooth over a narrow frequency band, could not reflect adequately the sharp changes at the extremes of the band. The presence of triple interactions like $(I)^{1+1-1}$ indicates friction in the tidal system.

Table 5

Pensacola tidal admittances (1 cpd)

Harmonic Constants from NOS Analysis — 3 years — 1952 to 1954

Constituent	Speed (°/h)	Ampl. (ft)	Coef.	Ampl./ Coef.	Phase (°)
$2Q_1^*$ (almost $2OK_1$)	12.854	0.025	0.0097	2.58	268.7
Q_1	13.399	0.090	0.0730	1.23	311.4
ρ_1	13.472	0.017	0.0142	1.20	311.2
O_1	13.943	0.398	0.3771	1.06	323.2
M_1	14.497	0.013	0.0209	0.62	352.6
P_1	14.959	0.120	0.1755	0.68	336.8
S_1 (radiational only)	15.000	0.013	–	–	91.5
K_1	15.041	0.403	0.5305	0.76	333.0
J_1	15.585	0.014	0.0297	0.47	370.4
OO_1^{**} (same as $2KO_1$)	16.139	0.034	0.0163	2.09	325.3

* Inferred $2Q_1$ would be 0.010 ft, 313.4°

** Inferred OO_1 would be 0.017 ft, 342.8°

In retrospect, it is possible that a frictional term could have slightly improved the species 1 predictions for Atlantic City as well. Table 6 is a comparable table for Atlantic City. Although not as pronounced, the admittances show non-smooth variability at the extremities of the bandwidth and therefore suggest a potential improvement if $(I)^{1+1-1}$ was added to the species 1 reference series. However, with a predominantly semi-diurnal regime one would expect interactions involving the species 2 tide to be the more important, and we do not consider this case warrants further complication.

Table 6

Atlantic City tidal admittances (1 cpd)

Harmonic Constants from NOS Analysis — 3 years — 1952 to 1954

Constituent	Speed (°/h)	Ampl. (ft)	Coef.	Admittances	
				Ampl./ Coef.	Phase (°)
$2Q_1^*$ (almost $2OK_1$)	12.854	0.007	0.0097	0.72	141.0
Q_1	13.399	0.041	0.0730	0.56	95.0
ρ_1	13.472	0.009	0.0142	0.63	38.5
O_1	13.943	0.245	0.3771	0.65	90.9
M_1	14.497	0.010	0.0209	0.48	129.9
P_1	14.959	0.109	0.1755	0.62	101.1
S_1 (radiational only)	15.000	0.035	–	–	37.7
K_1	15.041	0.363	0.5305	0.68	107.0
J_1	15.585	0.016	0.0297	0.54	92.3
OO_1^{**} (same as $2KO_1$)	16.139	0.015	0.0163	0.92	117.9

* Inferred $2Q_1$ would be 0.006 ft, 74.6°

** Inferred OO_1 would be 0.011 ft, 123.2°

Harmonic #1 is the traditional NOS procedure, using a rejection limit of 0.03 for analyzed values. Harmonic #2 averages all 31 constituents for 3 years and uses the mean values for predictions. Once again, even more obviously this time, the evidence suggests that the traditional rejection limit decreases the accuracy of future predictions. Although harmonic analysis does not implicitly require a smooth admittance within a species bandwidth, table 5 indicates other problems in a harmonic prediction. At the high frequency end of the 1 cpd range, OO_1 has exactly the same frequency as $2KO_1$ (the nonlinear $2K_1 — O_1$). The analyzed OO_1 is really the vector sum of the two and therefore node corrections designed for OO_1 only will be somewhat inaccurate. At the low frequency end of the species spectrum, the analyzed $2Q_1$ must be contaminated by $2OK_1$ (the nonlinear $2O_1 — K_1$) which is 1 cycle per 4½ years away in frequency. It seems reasonable that harmonic predictions can be improved slightly by substituting inferred harmonic constants for the analyzed $2Q_1$ values, thus removing the effect of $2OK_1$ sidebands; when this was tried, the residual variance for 1 cpd was greater than that shown in table 4 for Harmonic #2. Another test, estimating harmonic constants for $2OK_1$ from computed sidebands at the $2Q_1$ frequency for three consecutive years, also failed to improve the table 4 residuals.

CONCLUSIONS

Response tidal analysis and prediction have once more been found to produce more accurate results than classical harmonic procedures. However, the differences between the results are small compared to the total unpredictable variance which is concentrated primarily in frequencies less than 1 cpd.

Harmonic predictions were improved by departing from usual National Ocean Survey procedures; the latter include rejecting the harmonic constants for those constituents whose analyzed amplitude is less than 0.03 foot and a practice of inferring T_2 from S_2 regardless of the analyzed T_2 amplitude.

A need was found for including third-order nonlinear interactions of diurnal tides in response predictions for some stations.

MISCELLANEOUS RESULTS OF INTEREST

The residual by both methods showed an annual modulation of M_2 of amplitude about 1 cm, as usual predominantly at the lower sideband frequency (2 0-1). Such terms have previously been identified, especially at ports in the North Sea, by CORKAN (1934), CARTWRIGHT (1968), and PUGH and VASSIE (1976). They can be accommodated in a response prediction by $(I)^{2-0}$ as mentioned in reference to table 1.

Both analysis procedures identified significant lines with amplitudes of about 0.5 cm for S_3 and S_5 at Atlantic City, a rather unusual situation; S_4 and S_6 are much smaller. S_1/K_1 at Atlantic City is 0.096 whereas at Pensacola the ratio is 0.032, thus indicating a relatively larger-than-normal S_1. It has been suggested that the S_3 and S_5 amplitudes may be due to a thermal response of the tide gauge to sunlight on its housing. Environmental conditions appear to be a more plausible reason. The tide gauge at Atlantic City is located at the far end of a pier extending into the ocean. On occasion, the thermograph adjacent to the tide gauge has shown a sudden rise in temperature of about 10 °F within an hour. An investigation showed there is a very shallow inlet just north of Atlantic City and that the sudden change in temperature was related to the tidal current transporting the heated lagoon water past the tide gauge and thermograph; thus a diurnal solar frequency interacting with a species 2 tidal current regime may explain the anomalous S_3 and S_5 amplitudes. It is open to question, however, whether anyone would wish to include such small and highly localized effects in a tidal prediction.

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Information has been received that the National Ocean Survey has accepted the values for T_2 from analyses of series of one year. They are now considering adoption of the remaining recommendations. (*Editor's note*).