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THEORETICAL OFFSHORE TIDE RANGE DERIVED FROM A SIMPLE DEFANT TIDAL MODEL
COMPARED WITH OBSERVED OFFSHORE TIDES

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ABSTRACT

A simple Defant model, based on the M_2 constituent, is presently used by the National Ocean Survey to estimate the offshore range of the tide, with observations at coastal tide stations of the actual tide supplying the necessary boundary condition for the model. The calculated values provide preliminary tide correctors for soundings obtained in offshore hydrographic surveys. Using offshore tide data from Deep Sea Tide Gage (DSTG) deployments and Offshore Telemetering Tide System (OTTS) buoys, the quantitative effects of the continental slope and shelf on the incoming semi-diurnal tide are discussed, and anomalies in the predicted tide curve due to Hurricane Belle are shown. Comparison of the observed tide range with the theoretical tide range reveals the need to modify the initial Defant model, which is accomplished by decomposing the range into its major harmonic constituents, resulting in an improved calculated offshore range.

INTRODUCTION

The National Ocean Survey (NOS), NOAA, has recently collected offshore tide data in support of major hydrographic surveys in the New York Bight and in the waters of the Mid-Atlantic States. The data obtained have proven valuable in the reduction of hydrographic soundings to chart datum and are used here to verify a simple model which provides preliminary tidal zoning for survey ships performing offshore hydrography (MARTIN and EARLE 1976). In general, because hydrographic soundings are taken throughout the tidal cycle and not just when the tide is at a particular chart datum, corrections (called tide reducers) must be applied to individual soundings for the height of tide at the time of the sounding. The soundings are corrected, or reduced, to chart datum (Mean Low Water on the U.S. east coast, Gulf Coast Low Water Datum on the Gulf of Mexico coast, and Mean Lower Low Water on the U.S. west coast). Tide reducers are usually one of the most significant corrections applied to hydrographic soundings; inaccurate reducers can lead to problems in junctioning separate hydrographic sheets and in resolving individual cross-line discrepancies within the sheets (UMBACH 1976).

Soundings from inshore hydrographic surveys are usually reduced by applying tide data collected from a nearby station or by interpolating tide data obtained from two or three of the nearest stations that were in operation during the survey. However, for offshore surveys it has been necessary to extrapolate tidal characteristics from an onshore station out onto the continental shelf. In many past instances, the range of the tide at a coastal tide station was simply assumed to be the same offshore, and the phase of the tide was estimated using shallow water wave theory as applied in the *Manual of Tide Observations* (USC&GS, 1965).

General wave theory states that an incoming tide wave will have its basic sinusoidal form modified once it advances over a continental shelf. The wave increases in amplitude due to bathymetric effects. With this knowledge, NOS has adopted a simple Defant model (or method) to compute the range of the tide offshore (DEFANT, 1929). The phase of the tide offshore is estimated by applying shallow water wave theory.

Given the range of tide at appropriate coastal stations and the bathymetry of the continental shelf along transects running from the stations out normal to the shelf, the Defant model provides the theoretical range of tide at any location along the transects. These theoretical ranges and phase of tide are then used to 'zone' the area of hydrography for a particular survey. The number of zones required depends on the rate of change in the area's tidal characteristics. Thus, a sounding taken in a specific area will have a specific time and range corrector applied to the predicted tide (preliminary zoning used during the survey) or observed tide (final zoning after the field work) at a coastal station.

The preliminary tide zoning for NOAA's MESA (Marine EcoSystems Analysis) and DELMARVANC (Delaware, Maryland, Virginia, North Carolina) projects are shown in figure 1. This zoning is based on transects run out from Sandy Hook, New Jersey, for MESA and out from Bethany Beach, Delaware, and Virginia Beach, Virginia, for DELMARVANC. Figure 2 shows the relative locations of the offshore tide gages from which data were collected during both surveys.

Deep sea tide data were collected for 6 months in the MESA Project (August 1975 to February 1976) and the DELMARVANC Project (June 1976 to November 1976) using gages deployed in deep water off the continental shelf and slope. Tide data from the shelf gages shown on the MESA transect (MESA 9, 10, 11, and 22) are presented in the MESA New York Bight Atlas Monograph 4 (SWANSON, 1976). The OTTS gages (OTTS 1 and 2) collected useful continuous data for 15 days in August 1976.

The deep sea tide gage (Model B-DSTG, Gulf General Atomic) (fig. 3) is a self-contained pressure recorder capable of measuring small changes in hydrostatic pressure in depths of water up to 5,000 meters. The pressure transducer is a bourdon tube connected to a small tube that opens into an oil reservoir that is separated from the outside sea water by a membrane.

The changes in hydrostatic pressure brought about by variations in sea level cause the bourdon tube to rotate. The rotation of the bourdon tube is translated into an electrical signal by means of a stationary light source, a mirror mounted on the tube, and an array of photoelectric cells.

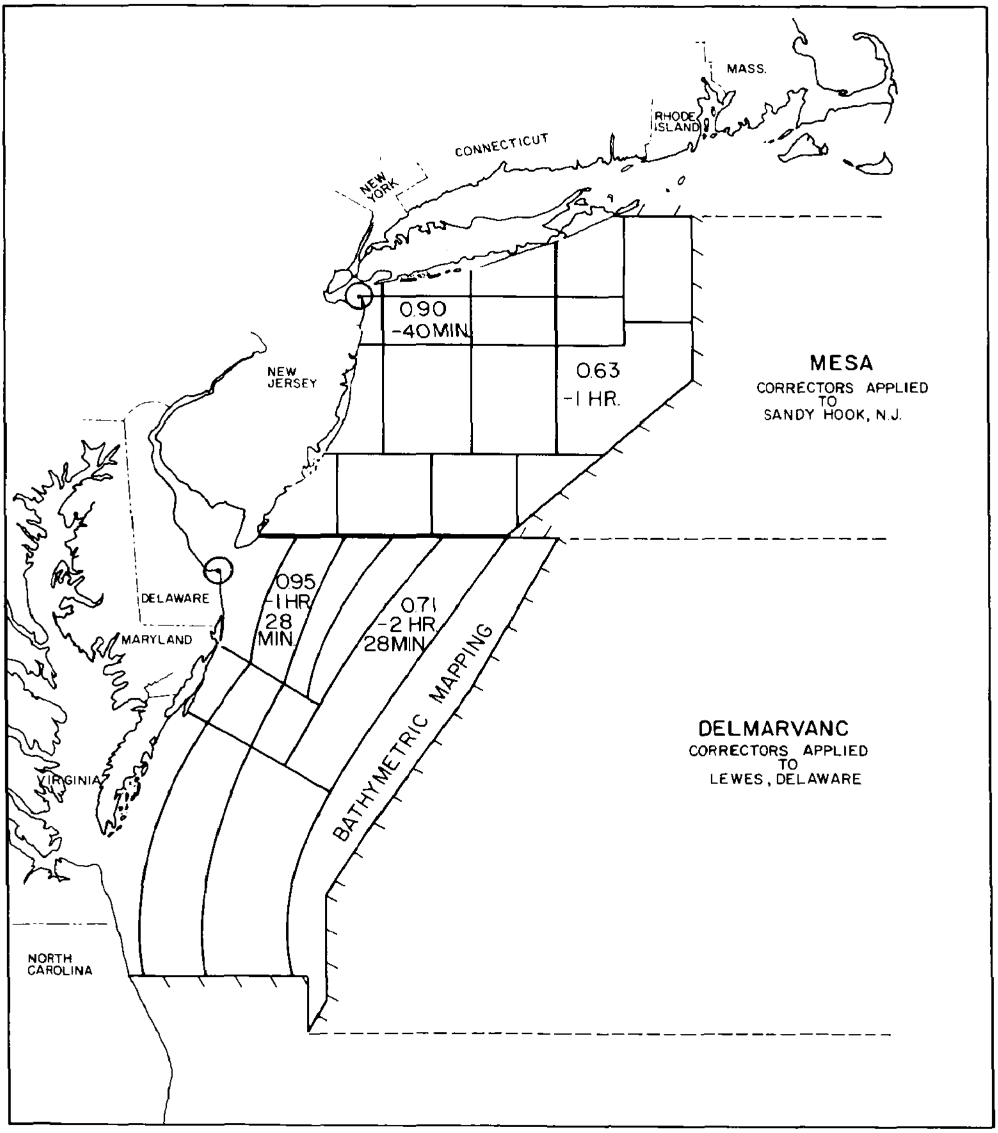

FIG. 1. — Preliminary tidal zoning where the upper number is the height ratio and the lower number the time correction.

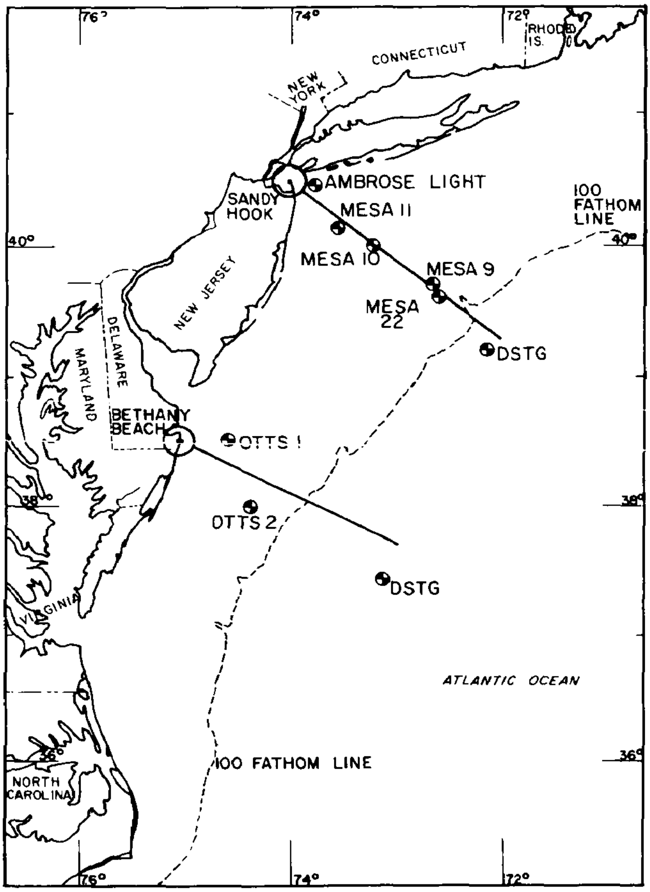

FIG. 2. — Locations of the offshore tide gages and transects.

This signal is then recorded as a function of time on a Rustrack recorder. The gage can record data up to half a year at a sampling rate of 30 minutes.

Data obtained from the MESA Deep Sea Tide Gage (DSTG) agree well with the data obtained from a Lamont-Doherty Geological Observatory gage located just northeast of the MESA DSTG. The amplitudes and phases of the M_2 constituent of the tide from the MESA and Lamont gages are 40.5 cm/191.2° and 43.9 cm/207.7°, respectively (MARTIN and EARLE, 1976). All recent east coast DSTG deployments by NOS show a consistent range of tide of approximately 90 cm in a north-south band just off the continental shelf from the New York Bight south to the South Carolina-Georgia border (MARTIN and EARLE, 1976). The ranges and phases of tide found with these gages are also consistent with the estimates of REDFIELD (1958).

The differences between the theoretical (line labeled M_2) and observed range (circles) of tide along the transects shown in figure 2 can be seen in figs. 4 and 5. These differences increase as the depth of water increases in each case, and the theoretical range of tide is consistently less than the

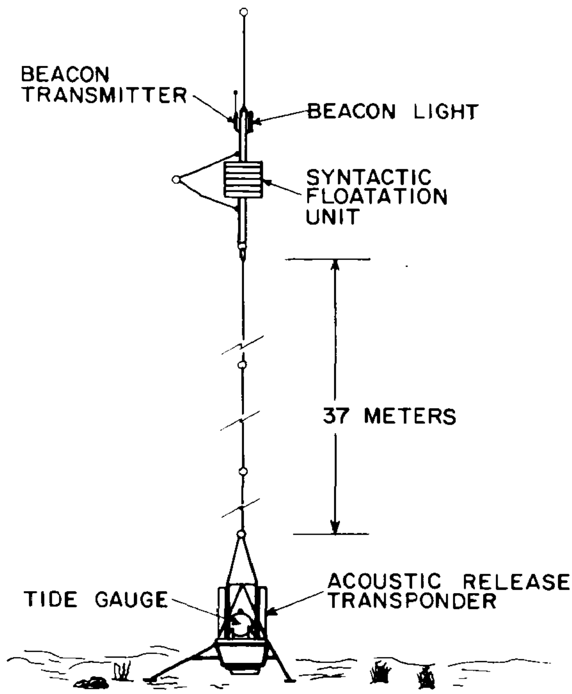

FIG. 3. — Deep sea tide gage mooring.

observed range. Analysis of the DSTG and OTTS data has led to significant changes in the preliminary zoning originally provided to the hydrographic surveys as shown in figure 1. Although the theoretical and observed range differences at the locations of the DSTG's are on the order of 10 cm and 20 cm, progress has been made in refining the model. Figures 4 and 5 clearly show the effect of the continental shelf on the theoretical and observed tide. Figures 6 and 7 show the difference in the theoretical and observed wave travel times for each transect. The time differences are just under 1 hour for DELMARVANC and just over 1 hour for MESA at the locations of the DSTG's. The phase diagram for the MESA data (fig. 6) exhibits standing wave characteristics, while the corresponding diagram for the DELMARVANC data (fig. 7) exhibits progressive wave properties.

An unexpected situation observed during the DELMARVANC project was the collection of simultaneous tide data across the continental shelf during the passage of Hurricane Belle in August 1976. The approximate path of the hurricane in relation to the tide gage locations is shown in figure 8. Approximate hourly positions are indicated by the dots superimposed on the path. Figures 9 and 10 are the plots of the height of the tide versus time for a 2-day period around the passage of the hurricane for the OTTS 1 and DSTG, respectively. The predicted hourly height curves reflecting the preliminary zoning are also plotted for each location. The predicted curves for the offshore locations were formulated by applying the appropriate tide correctors shown in figure 1 to the predicted tide

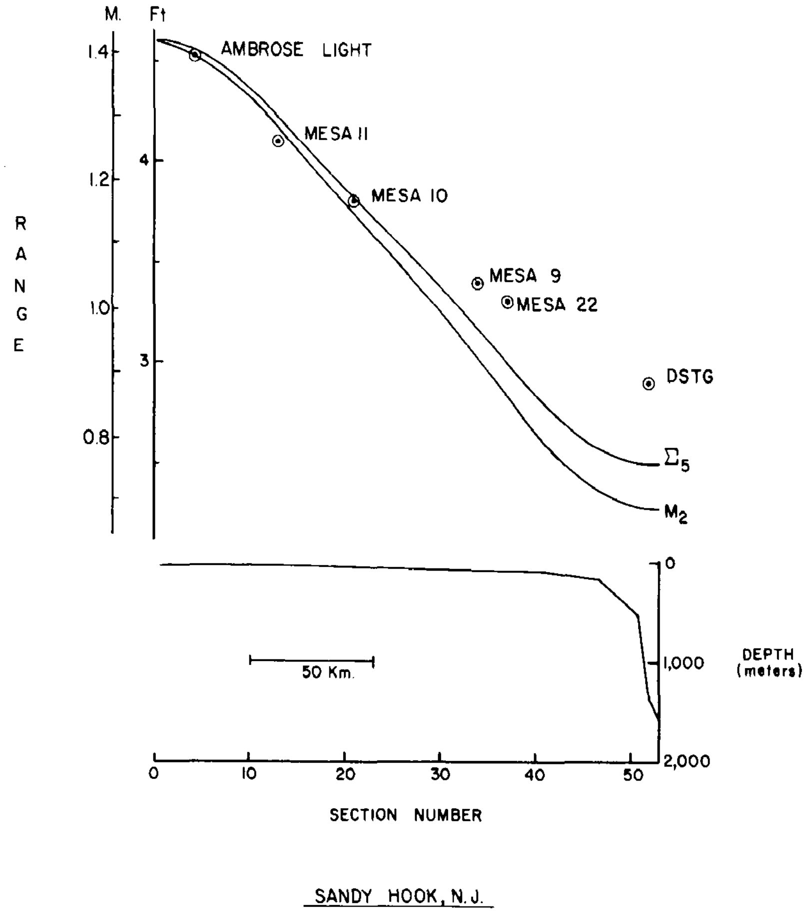

FIG. 4. — Differences between observed (circles) and theoretical (M_2 and Σ_5) range of tide offshore.

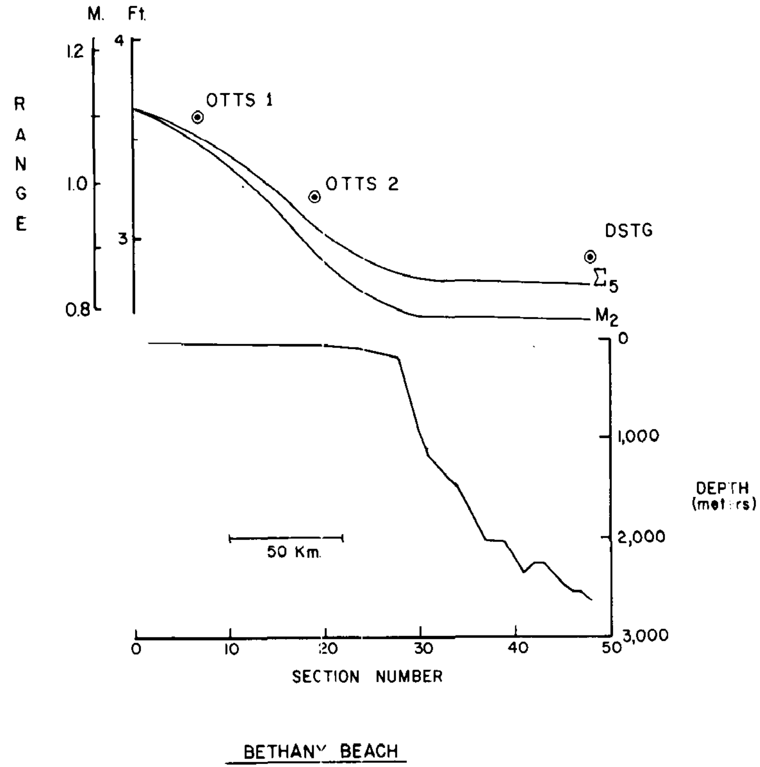

FIG. 5. — Differences between observed (circles) and theoretical (M_2 and Σ_5) range of tide offshore.

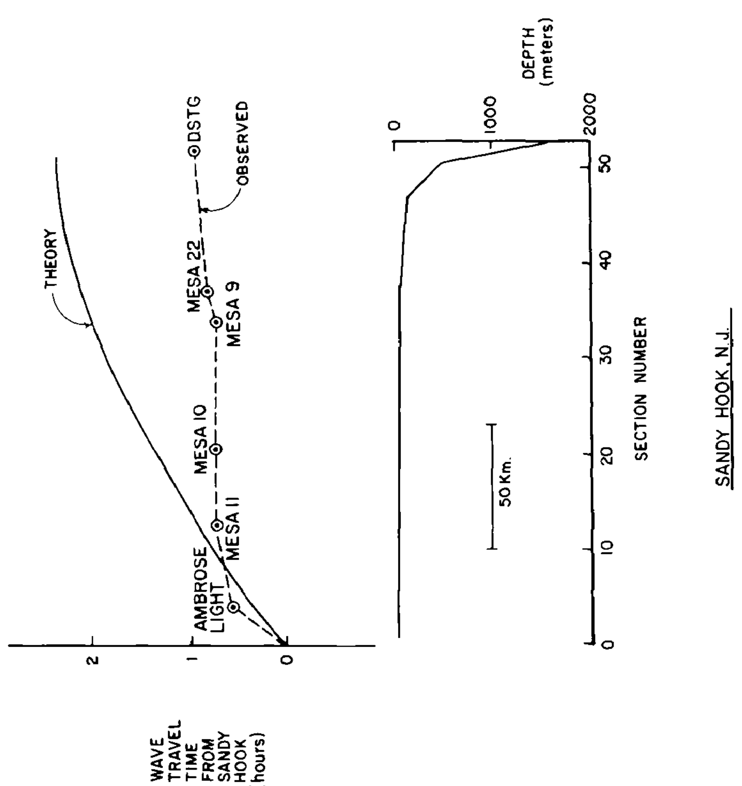

Fig. 6. — Phase diagram for observed and theoretical wave travel times.

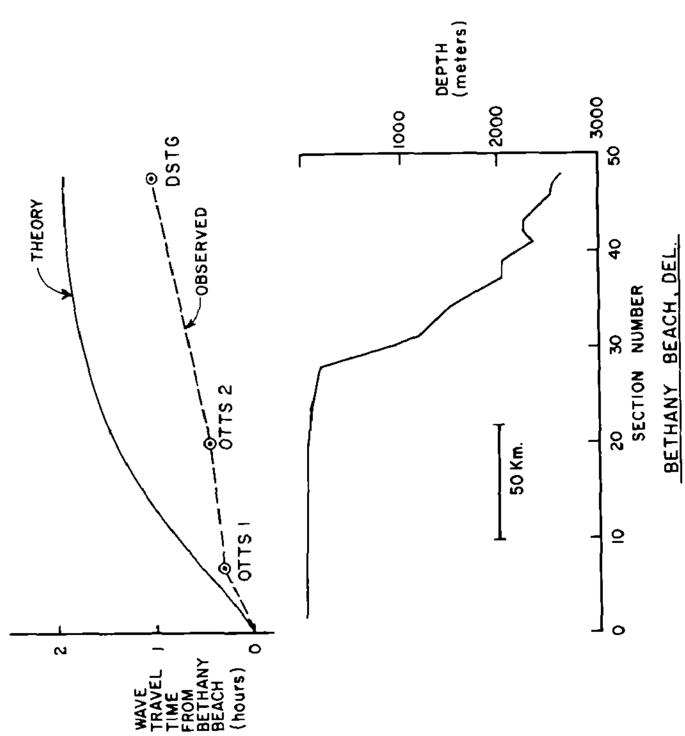

Fig. 7. — Phase diagram for observed and theoretical wave travel times.

at Lewes, Delaware. Wind speed and direction at 3-hour intervals at Atlantic City, New Jersey, are shown below each plot.

The effects of Hurricane Belle on the second high tide of August 9 can be seen when the observed and predicted curves are compared. The piling-up effect due to the hurricane winds on the high tide appears to decrease as the depth of the water increases. The OTTS 1 data show a large deviation from the predicted heights during the hurricane, while the DSTG plot shows no perceptible deviation that could be attributed to the hurricane.

These hourly height plots bring out the importance of knowing the correct phase of tide offshore as well as the range of the tide when considering hydrographic applications. The Defant method seems to satisfy the accuracy requirements for range correctors, especially in deeper waters. However, use of the shallow water wave theory to estimate time correctors can introduce significant errors. The OTTS 1 and DSTG plots (figs. 9 and 10) show that the phase difference in the theoretical curve and observed curve at any particular time may lead to a height corrector which is as much as 1 foot in error.

DEFANT MODEL

The following is a brief description of the Defant model and of a modification that increases the accuracy of the range estimates offshore.

The coordinate system originates on the coast where the sea level intersects the shore (fig. 11). The gravity vector is in the negative y-direction, and the x-direction is orthogonal to the bottom contours. The controlling physics are continuity and a balance between the horizontal pressure gradients and the horizontal acceleration of the fluid. With the assumption of no advection, no rotation, and inviscidness, the controlling equations are:

1. The equation of motion:

$$\frac{\partial^2 \xi}{\partial t^2} = - g \frac{\partial \eta}{x}$$

where g = gravity
x = distance from the origin
t = time
η = vertical displacement
ξ = horizontal displacement.

2. The continuity equation:

$$\eta = \frac{1}{b(x)} \frac{\partial}{\partial x} (S(x)\, \xi)$$

where S(x) = the cross sectional area of a plane perpendicular to the x-axis

b(x) = the width of plane measured perpendicular to the x-axis.

This set of equations can be solved by imposing a frequency of oscillation for the basin which is either some intrinsic frequency for a closed basin or some forcing frequency of an open basin. The boundary conditions are then enforced, and the set of equations are numerically piecewise integrated. This approach is from Defant's method (1929) as shown by SVERDRUP, JOHNSON, and FLEMING (1942).

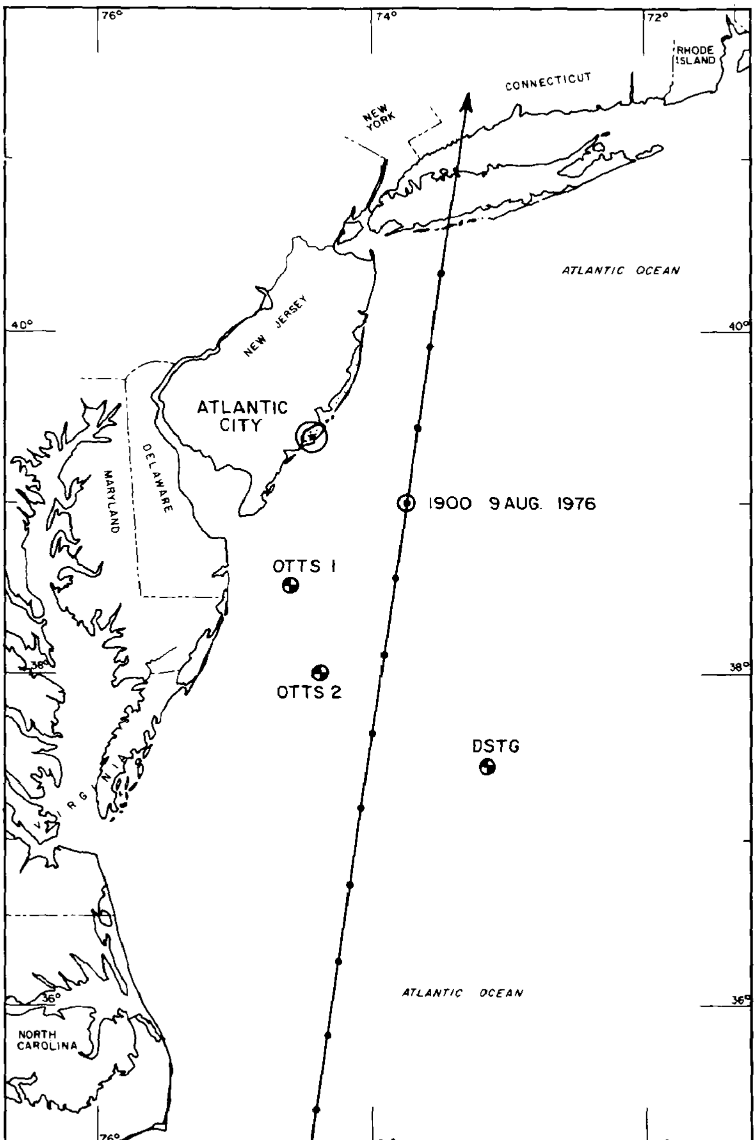

FIG. 8. — Path of Hurricane Belle showing hourly positions.

For the open coast, certain modifications must be made in the formulation of this problem. The assumptions are that the tide wave propagates inshore such that the wave crests are parallel to the shore and to the contours of equal depths. This model has been used by NOS in the past by assuming that the frequency of the wave corresponds to the M_2 con-

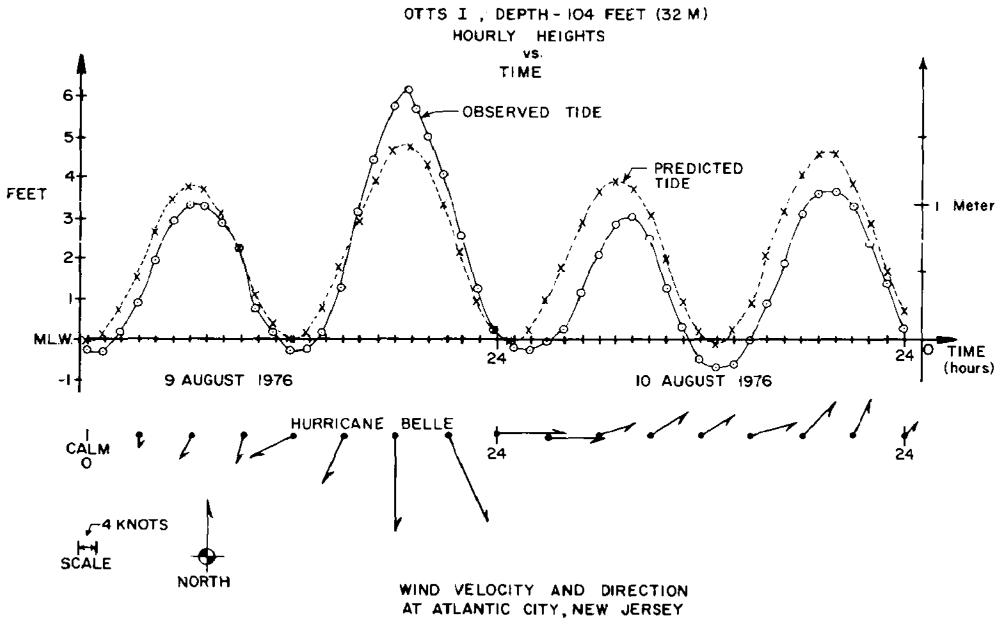

FIG. 9. — Predicted and observed tides at OTTS 1 during Hurricane Belle.

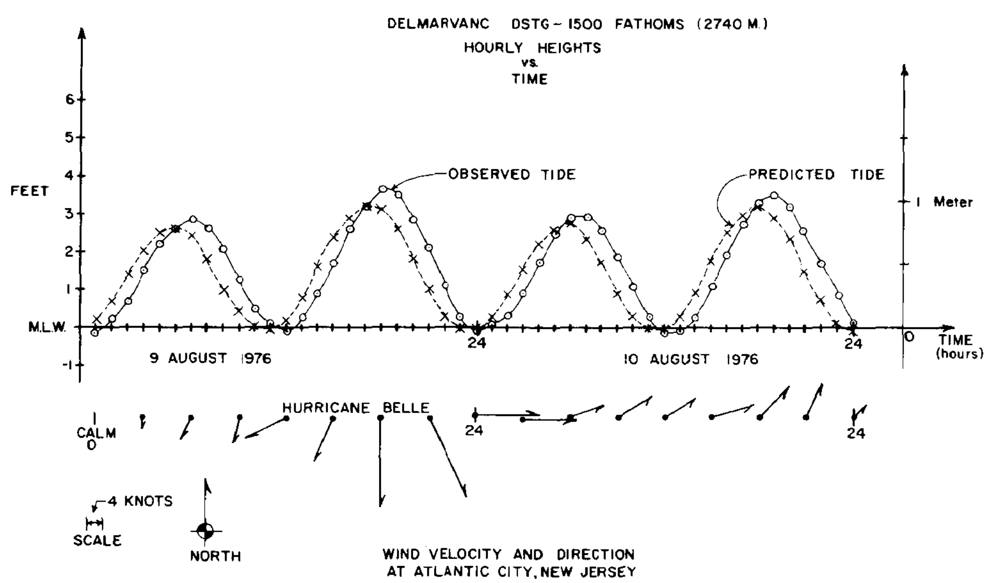

FIG. 10. — Predicted and observed tides at the DSTG during Hurricane Belle.

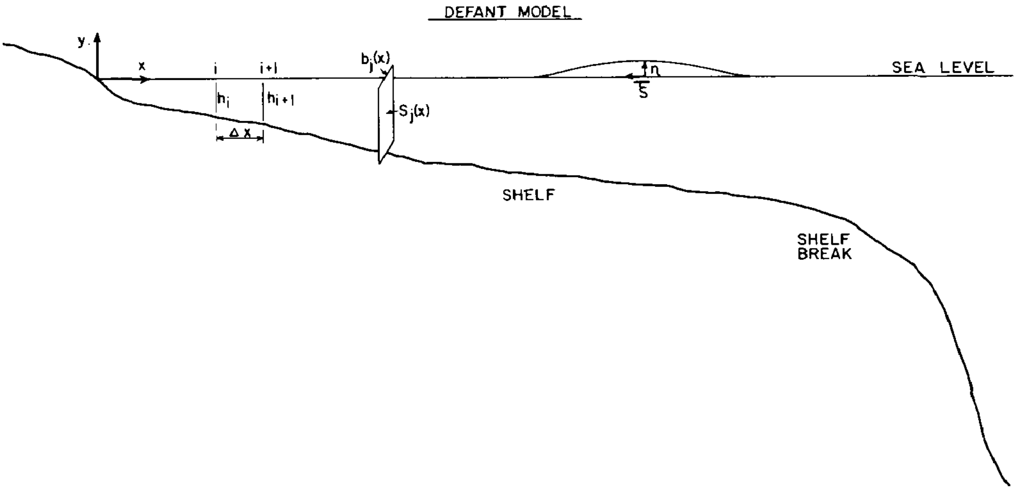

FIG. 11. — Schematic for formulation of the Defant model.

stituent period, that the transect integrated over was orthogonal to the bottom contours, and that the point of origin is a coastal tide station. Examples of the transects for the MESA and DELMARVANC projects are shown in figure 2 where Sandy Hook, New Jersey, and Bethany Beach, Delaware, were used as the points of origin. The integration steps applied were 4.00 km and 4.17 km, respectively.

The depths used for the integration were derived from NOS nautical charts, and the input frequency used was the M_2 frequency (or a period of 12.42 hours). The results are shown by the curve labeled M_2 in figures 4 and 5. The curves show a monotonically decreasing range of tide from the shoreline to the continental break. A constant range is present in the deep water off the continental shelf. This is indeed as it should be according to wave theory. As the wave propagates into the shallower water, it increases in amplitude. It should be noted that the computed range falls below the observed range in each of the cases. This underestimation is due to the model constraints. An input parameter that has a large effect on the results of the model is that of the wave's period. The shorter the period, the faster the wave's range will decrease moving offshore. It was this fact that led to the modification of the basic method by considering each tidal constituent separately. This procedure involves using the harmonic constituents of the tidal data obtained at a controlling coastal tide station such as Sandy Hook or Bethany Beach. The five largest constituents are M_2, S_2, N_2, K_1, and O_1. It was assumed that all the significant tidal information was contained in these five components. Essentially the model is run five separate times and the results accumulated. The range of tide at the controlling station was multiplied by the amplitude of a particular constituent and then divided by the sum of the amplitudes of the five constituents. The frequency used corresponded to the amplitude of the specific tidal component. The final result is determined by adding the five separate partial ranges at each integration step. In this manner, the major forcing frequencies of the Moon and Sun are taken into account, not just the M_2 component as previously used. The result is that the range of tide does not attenuate as quickly offshore, because the other diurnal tidal constituents that are taken into account, specifically K_1 and O_1, have very large periods and decrease slowly offshore. The resulting estimates of the range offshore are therefore closer to the observed range. The estimated ranges labeled Σ_5 in figures 4 and 5 show an approximately 50 percent increase in accuracy.

The model is thus modified to give a more accurate result; however, the remaining discrepancies between theory and observation are noteworthy. These differences are due to the fact that in the model friction was ignored and the tide wave was assumed to propagate along the transect. The modifications show a better fit to the observed data at the Bethany Beach site than at Sandy Hook. The assumption that the wave travels along the transect is satisfied better at Bethany Beach than at Sandy Hook in the New York Bight. The New York Bight has a unique tidal response, in that the wave is affected by Long Island and is funneled into a 'corner' where the model assumptions are not satisfied.

Efforts to test the usefulness of the Defant model in areas other than the Atlantic coast are continuing in the Gulf of Mexico and on the U.S. west coast. The Gulf is a complicated area because of the dominance of the diurnal tide, with some areas switching from diurnal to mixed tidal characteristics within a month. The cophase lines for the major solar constituents in the Gulf are not parallel to the shoreline. Also, on the Pacific coast, the cotidal and corange lines are not consistently parallel to the coast, thus violating the model constraints. The Atlantic seaboard appears to be one of the better suited sites for applications of the model presented in this paper, although further studies may make it more practical for these other areas as well.

CONCLUSION

The necessity to observe the actual tide during hydrographic surveys of an area is obvious; tide predictions are based on harmonic constituents which, in turn, are based on previous observations and do not take into account localized meteorological effects. The argument for the real-time observation of tides in offshore areas can be further advanced after noting the difference in the effects that an offshore hurricane can have on inshore and offshore tides. Hydrography, of course, is not conducted during hurricanes, but a storm surge can affect water levels for periods longer than the localized residence time of a storm. This may lead to situations where tide correctors applied to an offshore station do not reflect true offshore conditions.

The cost benefits of offshore tide gage development and deployment must be weighed along with the accuracy of the data needed for an overall survey objective. If tide gages are not deployed, offshore surveys must rely on the accuracy of numerical and computer models to predict offshore conditions. This, in turn, would require improved models and, in this specific case, a better method of estimating the phase of the tide. In any event, future deployment of shelf gages and DSTG's will be required for the verification of shelf models and global tide models.

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AUSTRALIAN HYDROGRAPHER AT SEA

Captain M. CALDER, the Hydrographer, Royal Australian Navy, took part in the Admiral's Cup Fastnet Race in August 1979. He was a member of the crew of HMTY *Kalisana*. The following interview appeared in the Australian Navy News.

"Of the 306 yachts that started in the Fastnet, only 86 were listed by the Race Control Centre as having finished. Some 177 were thought to have "completed the course but half of them were considered to have subsequently retired because of the use of engines. Twenty-three yachts were abandoned and of these six were presumed sunk. Dozens of yachts limped into British or Irish ports with severe damage. Nineteen lives were lost — fifteen from among the competitors and four from a yacht following the race. In Division Four in which *Kalisana* raced, 58 yachts started and only six finished. *Kalisana* was fourth in its division and 80th overall".

Captain CALDER said that in his mind the tragedies and confusion of this year's Fastnet were caused by two major factors. Firstly, for the past three Fastnet races (over six years) he understood the weather had been kind and there had been a steady build-up in the size of the fleet with many competitors lacking real heavy weather experience. In general terms it must be assumed that the tragedies were caused by inexperience as evidenced by the number of people who either failed to shorten sail early enough, or took off too much and could not control their boats, and then abandoned boats that were later found sound and often without major damage and crews had taken to flimsy inflatable liferafts.

There is no doubt that during the night of 13/14 August the Irish Sea was no place for the inexperienced. The sea is relatively shallow — 70 to 100 metres — and there are strong tidal streams, particularly near the Cornish coast and the Scilly Isles. These streams, combined with the storm force winds, quickly whipped up a sea that was heavy, steep, short and what was probably most important, unpredictable. The latter made it very difficult to pick a line through a wave and would have tested the best of helmsmen.