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QUALITY CONTROL OF OFFSHORE POSITIONING SURVEYS

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ABSTRACT

This paper discusses the need for a quality control method for offshore positioning surveys and then its actual use. It limits the discussion to radio locationing systems only, but this restriction does not imply that Satellite Navigation or Underwater Acoustic Positioning Systems do not require such quality control.

The subject is divided into two parts. Firstly the calibration methods necessary to determine the zero/delay settings for the survey equipment and to establish the propagation velocity required in the survey. Secondly, the positioning of the survey vessel with the necessary, but often missing, "simultaneous quality control" of this positioning.

The methods described in this paper are compared with field results, from which can be seen the often limited value of the most commonly used, and sometimes time-consuming and expensive, calibration methods.

It will be shown here that the generally accepted principle of the location offshore of vessels and structures using only two position lines not only lacks reliability but can often result in costly resurveys.

The lack of data redundancy in offshore surveys stands in sharp contrast to the practice in land surveying where data redundancy in observation and control surveys is common practice.

With the increasing practice of having extra positioning equipment on stand-by—especially in remote areas—and with the increase in cost of offshore surveys, Shell's Topographic Survey Department has developed a method based on redundancy of data. This method provides a continu-

ous quality control of the offshore positioning while surveying, and avoids unpleasant surprises after the survey vessel has left the area.

INTRODUCTION

Positioning in marine surveys for the oil industry makes use of a wide variety of techniques ranging from sextant and theodolite measurements for inshore work to satellite navigation, integrated with long-range navigational aids and sonar doppler for the remote areas beyond 300 km from the coast. The in-between zone is covered by a wide range of radiolocation (survey) systems, whereas for some special projects underwater acoustic systems are employed. The applications for these techniques vary from the positioning of seismic shotpoints, the placing of mobile drilling rigs on previously shot seismic lines, the horizontal control of airborne or seaborne gravity/magnetometry surveys, the positioning control of pipeline route surveys and the laying of these pipes thereafter, the shallow geophysical surveys and subsequent accurate siting of large production structures as well as the control of large-scale engineering and hydrographic surveys.

Since the majority of these activities still take place in the zone between the inshore visual-control area and the more remote sea areas an effort is being made to identify the present problems in radiopositioning and to try to improve upon performance.

This paper deals with the need for a reliable standard method of calibration prior to offshore surveys and a real-time quality control during such surveys, with the aim of reducing errors and ambiguities and at the same time giving confidence to the survey party on board and providing subsequent scrutiny and correction possibilities to the remote onshore processing department.

The radio systems are categorized by the type of geometric pattern they generate, and this determines the calibration procedures to be used.

There are three distinct types of pattern in radiopositioning : hyperbolic, circular and linear.

Hyperbolic patterns

These patterns are generated by two pairs of beacons. The equipment on the survey vessel records a signal that is the result of either a measured phase difference or a measured time difference between the signals transmitted by each of the two beacons in a pair (Hi-Fix, Syledis systems).

The formula for the calculation of the hyperbolic pattern is :

$$P = (\mathrm{Dist}\,(MS) + \mathrm{Dist}\,(MV) - \mathrm{Dist}\,(SV)) \cdot F/V + Z$$

in which :

P = pattern value

F = transmission frequency
Z = zero/delay correction
V = propagation velocity of the radiowave
Dist (MS) = distance between the shore beacons (Master and Slave)
Dist (MV) = distance Master beacon to vessel
Dist (SV) = distance Slave beacon to vessel

Circular patterns

These patterns are each generated by one shore station which is triggered by the transmitter on board the survey vessel. The observed signals are either time intervals measured between the vessel's outgoing signal and the returning one retransmitted by the shore beacon, or the phase difference between the two signals (Maxiran, Argo systems).

The formula for the calculation of the circular pattern is :

$$P = \text{Dist (MV)} \cdot 2F/V + Z$$

Linear patterns

These patterns are generated by shore beacons transmitting the angular value of horizontal rotation of their aerials relative to a Reference Object (R.O.). (Artemis system). Theodolite stations generate similar linear patterns.

The formula for the calculation of the linear pattern is :

$$P = R + A + Z$$

in which :

R = reference bearing to R.O.
A = angular value of aerial rotation.

Each of the above three types of pattern gives one geometric position line (LOP). The position of the vessel is the intersection of two of such lines of position. This statement, however, is the classical misconception of the positioning problem, as will be seen later.

There is no mariner or surveyor in the world who in conventional navigation does not use a check bearing just to make sure. Why then does one, with radiopositioning systems, use only two LOPs ?

CALIBRATIONS

A. PRESENT METHOD OF CALIBRATION

The hyperbolic and circular types both have the propagation velocity of the radiowaves (at the comparison frequency), the transmission frequency and the zero/delay constants in the equipment directly related to their type. The main parameter governing the propagation velocity of systems in the LF and MF is the electromagnetic conductivity of the surface over which the radiowaves are transmitted, while for the systems working in the HF and UHF band, the propagation velocity is governed by the tropospheric refractive index.

Calibration methods hitherto varied, depending largely upon the equipment, skill, experience and improvisation talents of the survey personnel. Unfortunately, however, too often insufficient time has been allocated for this all-important exercise. Many operators nowadays still consider this part of the survey as not necessary—partly due to ignorance or overselling by the equipment manufacturers, but more often due to the demand for high production figures since calibration is not considered to be productive. All types of pattern, however, have a basic element in common : the coordinates of the shore-based radiobeacons and the consistency of the geodetic network in the area. Without these, the actual position of the LOPs cannot be correctly computed.

Frequency calibration was, and still is, not normally carried out in the field, due to the need for delicate specialized equipment. This calibration is, hopefully, done in the manufacturer's workshop. It is, however, possible to correct for a change in frequency during the computation as part of the velocity term.

Recent experience shows an urgent requirement for the ability to measure the absolute or relative signal strength in order to ensure accuracy of signals above a rapidly deteriorating minimum. Signal-to-noise ratio is also an important factor for accurate radio positioning. These aspects have been neglected by the majority of positioning survey contract firms.

The method of calibration is dependent upon pattern type.

Hyperbolic patterns

Until recently only two pairs of LOPs were generated. Baseline extensions (maximum and minimum values) were observed during the lanecount. Pattern values were checked at known points. The observed differences were either entered into the calculation as C – O (calculated minus observed) values or else minimised by the more experienced surveyor by the adoption of a new propagation velocity bearing in mind the physical properties of the various types of soil/water/air at the transmitted frequency. In some cases, however, a value supplied by the manufacturer was used.

It is obvious that a check on the consistency of the station coordinates from this calibration method is very difficult indeed. It takes a great deal of skill and patience to (a) determine one or more stations to be "out", and (b) pinpoint the culprit.

Circular patterns

These patterns were normally calibrated over a long baseline, the length of which depended on local circumstances—preferably being equivalent to the average distance from base station to survey area. The observed difference on this baseline was either entered into the calculation as a zero/delay correction or mechanically fed into the equipment. As for the case of hyperbolic calibrations, the best known velocity value was assumed or else the value supplied by the manufacturer was used (particularly in cases where the equipment readout is in units of length).

The more experienced man however insisted upon presetting a known or calculated delay into the equipment and had it checked by internal station calibration. Thereafter a minimum of two known baselines were measured. This facilitated a check on the consistency of the coordinates, if (as was normally done) the equipment was erected on the base stations to be used for the survey, and provided a better assumption of the propagation velocity. An additional check on zero/delay errors was carried out, circumstances permitting, by crossing the baselines both internally and externally. From the difference between the value obtained internally (an addition) and the external one (a subtraction) the zero/delay error can be directly calculated for the beacon on whose side the extension was crossed. The value obtained for the other beacon consists of that beacon's delay error and the velocity error over double the baseline distance, as can be seen from the following formulae :

$$Pm1 + Zm + Ps1 + Zs = b \cdot V/2F$$
$$Pm2 + Zm - Ps2 - Zs = b \cdot V/2F$$
$$Pm1 + Pm2 + 2Zm + Ps1 - Ps2 = 2b \cdot V/2F \text{ (addition)}$$
$$2Zm = 2b \cdot V/2F - (Pm1 + Pm2 + Ps1 - Ps2)$$
$$Pm1 - Pm2 + Ps1 + Ps2 + 2Zs = 0 \text{ (subtraction)}$$
$$2Zs = Pm2 - (Pm1 + Ps1 + Ps2)$$

It must be stated that, for various reasons, this method was unfortunately not always used.

Linear patterns

The problem of propagation velocity is irrelevant to this type of pattern. However, station coordinates and R.O. are as important as in the other types. Hardly any calibration for this type of pattern has been carried out in the past. One relied completely on the information supplied by the manufacturer and on the "wisdom" of the engineer setting up the equipment.

B. PROPOSED METHOD OF CALIBRATION

Many of the surveys for the oil industry are in remote areas of the world. Positioning equipment should arrive in the area well before the survey vessel, and in this period the equipment can be tested and calibrated.

A new method of calibrating circular pattern type equipment has been developed which avoids costly logistic support and at the same time offers a maximum of checking possibilities.

In this procedure we must establish the following :

a) check that the equipment is working properly;
b) determine the zero/delay values for each set and each exchangeable part of the equipment, including all spares;
c) define the best average propagation velocity of the radiowaves for the survey area, including changes due to local conditions.

The procedure has been established as follows.

On arrival all the equipment is checked at the operations base. The surveyor in charge must ensure that all pieces are tagged with an identification number. He must carefully note that the method of putting in zero/delay settings is not to be changed. In some cases (Maxiran system) we found it necessary to request sealing of the correction dials as they are very apt to be accidentally altered.

In the calibration procedure we insist that the determination of the zero/delay value is also carried out at the operations base. To effect this a shore baseline of 800-1 500 metres is laid out by simple tape measurement, for instance along a jetty or a quiet road. Its minimum length is governed by the manufacturer's specifications, since swamping by the beacon's transmitter can occur at very close range. Use of an attenuator is in most cases advisable.

Since the uncertainty in the propagation velocity is less than 0.1 %, the error on a 1 500 m baseline is negligible for the purpose of determining the delay setting.

The procedure has several advantages : it is easily adaptable to all sets, there are no communication problems, and it allows the surveyor to follow the complete procedure and to record the results on the spot. This procedure for basically checking the zero/delay error has now been applied in many surveys (in Portugal, Brazil, Tunisia, the North Sea, and Japan) and some facts have come to light. The cables of the Syledis aerial, for instance, had been assumed to have a certain constant delay per metre, but it was found that in some cases the value was nearly double that figure. With Maxiran equipment it was generally accepted by operators that the linear amplifiers and other parts of the equipment had a constant delay factor, as determined in the workshop (this is often a very misleading statement). However, with the procedure described above, errors of up to 30 metres were found.

The other factor, the propagation velocity, cannot be determined over the short baseline. The simplest way to determine this factor is the measurement of ranges over a long baseline, preferably over sea water and close to the area to be surveyed. Often such a baseline is difficult to find, or else the absence of adequate logistic support prevents this measurement. As an error in the third factor—the station coordinates—may exist, one must be careful in accepting these values, particularly if the velocity found differs considerably from the expected one. In that case the surveyor is then compelled to observe a second baseline.

Baseline crossings with the survey vessel (both internal and external crossings) whilst en route to the "prospect"—the industry's term for survey area—serve the same purpose as the long shore-baseline calibration.

Here the additonal feature of being able to check base station coordinate consistency should not be neglected.

If, in the worst case, neither of these velocity determinations can be made, then this factor can be computed from a series of at least four stationary (either drifting or at anchor) threeway fixes widely spread over the area of coverage. An improved estimate of the propagation velocity can be computed from the redundant observations by the least squares method. By executing this same procedure on a number of selected points gathered during the survey this initial value can be improved. This last subject will be discussed further in the section dealing with onboard quality control of positioning data.

The method of calibrating the hyperbolic type of pattern is basically the same as that described earlier. The additional compulsory third pattern line, however, offers a more realistic possibility of obtaining a mean value for the propagation velocity. The method is similar to the one described in the paragraph on circular patterns. It has been tested extensively on a Shoran-Hi-Fix/3 system in Borneo with success.

The main problem remaining to be solved lies in the as yet unaccountable anomalies occuring in different parts of the coverage area. Thought is being given to a solution whereby for each of the patterns (lines of position) a variable velocity will be used which is a function of the surface over which the wave passes, or else to a solution which treats non-uniform velocities. The same problem exists for the circular systems when working at extreme capacity limits, since a drop in signal strength gives an effect which is comparable to a change in velocity.

The principal problems in the calibration of line pattern systems are the setting up of the equipment, and the entering of the R.O. value and the coordinates of the stations, but in the context of this paper they can be considered as basic surveying procedures. A check by the surveyor is essential, in particular one on the sealing of the R.O. setting. To enable the surveyor to check this setting during the course of the survey it would be preferable that the manufacturer include this reference value, together with the rotation in angle, in the coded transmission.

In the calculation, the bearing should be calculated from the coordinates of the R.O., and the difference with the reference value should be used as a zero correction instead of using the reference value directly in the computation, as is being done now. A further discussion is given in the section on quality control.

The improved calibration procedures will have a direct impact on the results obtained with the quality control package.

QUALITY CONTROL

During the course of a survey normally only limited control is carried out. Phase comparison hyperbolic systems seldom have redundant LOPs. Their lane values are ambiguous and prone to lane loss due to atmospheric disturbances, radio noise, power loss, or skywave effects. Frequently this is only checked by repeating readings at previously determined points.

Circular patterns are very often used in the two-range mode, and even if three beacons are available many systems can only display two values simultaneously. The recording of the third pattern value can then only be effected either before or after the survey of a line as the equipment has to be switched over to the third beacon. If the checks are taken as so-called "running" three-way fixes (i.e. non-simultaneous), then the ship's movement can considerably affect the validity of the check. During the initial period of offshore exploration a few systems only monopolized the industry; by always using the same positioning system a high degree of repeatability was achieved, enabling a vessel or drilling rig to return to a seismic "event" for either a follow-up survey or spudding a well. No real emphasis was placed on "absolute" positions that could be reproduced with other systems. The need for the latter arose when more and better systems became available; this requirement became a "must" by reason of factors such as frequency licensing limitations and the specifications for international and concession boundary determinations.

Long after the survey had been completed and the vessel had left the area errors were sometimes discovered or suspected from "misties" in seismic interpretation. However, to prove the interpreting department right or wrong in their accusations was either very difficult or impossible, since only scanty, or no, redundant observations were available. In cases where different surveys positioned with different systems are being correlated, if the discrepancies reported prove genuine, the absence of redundant data could become serious indeed. It is difficult to estimate how many of the surveys carried out—particularly those in the early years of offshore exploration—contain large positioning errors. The cost involved in resurveying or in drilling a well in the wrong place is considerable.

This is one of the main reasons why onboard positioning quality control is vital. To prove a survey correct, redundant field data in the form of one or more extra position lines should be recorded and analysed in real time. It is only with the help of these redundant observations that real confidence in the survey positioning can be obtained. The acquisition of redundant survey data has always been a basic principle in land surveying.

Around 1974 the topographical department of Shell Expro London, which is directly involved in the UK's North Sea surveys, made an effort to force survey contractors to include the recording of at least one more pattern. At the same time a software package for the table-top HP 9830 calculator was developed to enable instantaneous position computation aboard ship utilising from two to eight positioning patterns of the hyperbolic, circular and/or linear type, or any combinations thereof. The calculation is based on the method of variation of coordinates (least squares adjustment), and the derived standard deviation for the single observation is the tool and criterion for the reliability of the vessel's position.

In 1978 a revised program was written for the more powerful HP 9845 computer, the revision being based on Shell Expro's experience. This quality control method is now being implemented in all Shell-controlled surveys.

Calculation of a position fix

The observed pattern values are first corrected for the zero/delay error and for the propagation velocity. The following formulae are applicable :

Hyperbolic : $U_i = (P_i - Z) \cdot V/F - \text{Dist (MS)}$ (in metres)

Circular : $U_i = (P_i - Z) \cdot V/2F$ (in metres)

Linear : $U_i = \text{Azi (MS)} + (P_i - R - Z) \cdot \pi/180$ (in radians)

where :

U_i = corrected pattern value
P_i = pattern reading
Z = zero/delay correction
V = corrected propagation velocity
F = transmission frequency
Dist (MS) = distance Master to Slave beacon
R = reference bearing to R.O. in degrees, as set in the equipment
Azi (MS) = calculated bearing to R.O. in radians

The calculation of the position should be made on the spheroid since long distances can be involved. Such a computation would take 4 - 6 seconds per point on the HP 9845, which is considered too long in view of the time interval of data acquisition on board. Therefore, a quicker solution was found consisting of correcting the metre pattern values for the Transverse Mercator projection scale factor and carrying out the position iteration on the TM grid. This method is four times faster, and the error between the spheroidal calculation and the TM grid is less than one metre at a distance of 300 km if appropriate formulae for the scale factor are applied to the lines. The iteration is repeated until the vector of the differences is less than one metre.

The standard deviation can now be calculated from the differences between the corrected observed pattern values and the values computed from the final coordinates :

$$\delta = \sqrt{(\Sigma\, V_1\, V_2)/(n - 2)}$$

n being the number of patterns observed.

It is stressed that the value of the standard deviation, especially when using a small number of redundant data, is only an indication of the measure of disagreement between the pattern values.

The least squares subroutine, as used in the HP 9845 program, is given as Appendix A to this paper. The program is written in the BASIC language. Appendix B contains a three-range position calculation program for the pocket calculator HP 97. The same techniques as described above have been used.

In the method of calculation the type of pattern is irrelevant. Theoretically, a fix can be calculated from any combination of the three patterns from different systems. However, only when the coordinates of the shore stations are based on the same geodetic datum and when the accuracy of the different systems can be considered comparable—and only then—can two or more different systems be directly used for the quality control program. No weighting has yet been included.

A remark must be made with regard to the calculation of a linear pattern. Normally, it has been the practice to accept the reading setting on the master beacon as being the correct bearing of the reference line. In the case of a wrong setting it is not possible to detect the error at a later stage as this bearing does not appear in the logged observations. In the method of calculation described above, the calculated value (Azi) and the beacon value (R) are used. This reduces the possibilities of error. The program thus serves as a check on positioning equipment generating linear patterns.

FUTURE DEVELOPMENTS

Further investigation is needed into some elements of the above method which are still to a certain extent affecting the results.

(1) The propagation velocity has been assumed to be uniform throughout the computation method. We know that this is not true, but the problem is firstly what computational model to accept for different velocities, and secondly what factors are contributing to the changes in velocity (i.e. types of soil, the salinity and temperature of water, the seasonal effects on ground conductivity, the signal strength to mention but a few).

(2) Pattern geometry and pattern stability both have an effect on the standard deviation as calculated by this method, but they have so far not been taken into account.

The program is, however, already being used for navigation of a vessel during surveys. Its usefulness to the quality control man on board to detect errors on the spot has been proven, since any gradual or sudden change in the standard deviation value will serve to alert him. It also gives the survey party the opportunity to remedy the majority of these errors whilst still in the area, in particular errors caused by wrong base station coordinates, changes in equipment, pattern drifts observed at a monitor station but not compensated for, etc.

When post-processing, the data can be more carefully scrutinized, and velocity calculations can be repeated in order to achieve a better result since the proposed logging procedure ensures that all data are available.

A program is under development to derive the best fitting propagation velocity and zero/delay corrections from the redundant data. The solution will also use the least squares principle.

```
2770 Leastsquares:  REDIM X(Maxpatt,2),Y(2,Maxpatt),Z(2,2),A(2),T(2),U(Maxpatt),
V(Maxpatt),W(Maxpatt)
2775      MAT U=ZER
2780      I9=0
2785      W1=Y
2790      W2=X
2795      FOR S1=1 TO Maxpatt
2800      IF R(S1)*K(S1)=0 THEN 2840
2805      ON ABS(P(S1,9)) GOTO 2810,2820,2830
2810      U(S1)=ABS(ABS(R(S1))+D(S1)+G(S1))*(C(S1)/F(S1))-B(S1)
2815      GOTO 2835
2820      U(S1)=ABS(R(S1)+D(S1)+G(S1))*C(S1)/(2*F(S1))
2825      GOTO 2835
2830      U(S1)=B(S1)+(ABS(R(S1))-D(S1)+G(S1))*(360/C(S1))*(PI/180)
2835      I9=I9+1
2840      NEXT S1
2845      IF I9<2 THEN 3185
2850      I9=1
2855      FOR S2=1 TO 10          !  Maximum number of iterations=10
2860      MAT X=ZER
2865      MAT V=ZER
2870      W4=(W1-Y0)/A+Q0
2875      W9=(A/(1-E2*SIN(W4)^2))^2*(1-E2)
2880      FOR S1=1 TO Maxpatt
2885      IF R(S1)*K(S1)=0 THEN 3030
2890      ON ABS(P(S1,9)) GOTO 2895,2895,2970
2895      W4=SQR((W2-P(S1,11))^2+(W1-P(S1,10))^2)
2900      W5=(W2-X0)^2+(W2-X0)*(P(S1,11)-X0)+(P(S1,11)-X0)^2
2905      W5=(1+(1+W5/(36*W9))*W5/(6*W9))*K0
2910      ON ABS(P(S1,9)) GOTO 2915,2950
2915      W6=SQR((W2-P(S1,13))^2+(W1-P(S1,12))^2)
2920      W7=(W2-X0)^2+(W2-X0)*(P(S1,13)-X0)+(P(S1,13)-X0)^2
2925      W7=(1+(1+W7/(36*W9))*W7/(6*W9))*K0
2930      X(S1,1)=(W1-P(S1,12))/W6-(W1-P(S1,10))/W4
2935      X(S1,2)=(W2-P(S1,13))/W6-(W2-P(S1,11))/W4
2940      V(S1)=U(S1)-W6/W7+W4/W5
2945      GOTO 3030
2950      X(S1,1)=(W1-P(S1,10))/U(S1)
2955      X(S1,2)=(W2-P(S1,11))/U(S1)
2960      V(S1)=U(S1)*W5-W4
2965      GOTO 3030
2970      CALL Grdbrg(P(S1,10),P(S1,11),W1,W2,W5,W8)
2975      W7=W8*W8*60*4.848E-6
2980      X(S1,1)=(P(S1,11)-W2)/W7
2985      X(S1,2)=(W1-P(S1,10))/W7
2990      V(S1)=U(S1)-W5
2995      IF V(S1)>-PI THEN 3010
3000      V(S1)=V(S1)+2*PI
3005      GOTO 2995
3010      IF V(S1)<PI THEN 3025
3015      V(S1)=V(S1)-2*PI
3020      GOTO 3010
3025      V(S1)=V(S1)*60*180/PI
3030      NEXT S1
3035      MAT Y=TRN(X)
3040      MAT Z=Y*X
3045      MAT A=Y*V
3050      IF DET(Z)=0 THEN 3165
3055      MAT Z=INV(Z)
3060      MAT T=Z*A
3065      W3=SQR(T(1)^2+T(2)^2)
3070      W1=W1+T(1)
3075      W2=W2+T(2)
3080      IF W3>1E6 THEN 3165
3085      IF W3<1 THEN 3100
3090      NEXT S2
3095      GOTO 3165
3100      I9=0
3105      W5=W6=0
3110      FOR S1=1 TO Maxpatt
```

```
3115    IF R(S1)*K(S1)=0 THEN 3130
3120    W5=V(S1)^2+W5
3125    W6=W6+1
3130    NEXT S1
3135    S=0
3140    IF W6=2 THEN 3150
3145    S=SQR(W5/(W6-2))
3150    Y=W1
3155    X=W2
3160    GOTO 3190
3165    IF (X=X7) AND (Y=Y7) THEN 3185
3170    X=X7
3175    Y=Y7
3180    GOTO 2775
3185    I9=1
3190    RETURN
```

The following parameters are passed on from the main program :

X = first approx. value for Easting
Y = first approx. value for Northing
R (i) = Range/Hyperbolic/Linear pattern value for pattern i
K (i) = Pattern on/off array (0 or 1)
D (i) = Zero or delay correction for pattern i
G (i) = Fixed zero or delay value for pattern i
C (i) = Propagation velocity (or factor) for pattern i
F (i) = Comparison frequency for pattern i
B (i) = Baseline distance pattern i
P (i, 9) = type of pattern : 1 = hyperbolic, 2 = range, 3 = linear
P (i, 10) = Northing of Slave beacon
P (i, 11) = Easting of Master beacon
P (i, 12) = Northing of Master beacon
P (i, 13) = Easling of Master beacon
YO = False Northing
XO = False Easting
QO = Latitude Origin (radians)
A = Semi-major axis of spheroid
E2 = Excentricity to power 2
KO = Scale factor at central meridian

Appendix B

User Instructions

THREE RANGE POSITIONING - PREP

1 Station: N, E, nr (3x)	velocity factor (C-O) 1,2,3	a, f K_o, E_o	approx. pos. φ_m, N_m, E_m	PRINT WRITE DATA CARD

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	LOAD PROGRAM CARD				
2	STATION DATA: Northing	N_i	↑		
	Easting	E_i	↑		
	Pattern number	i	A		
3	Repeat '2' for other stations				
4	Input (C-O) data: pattern 1	$(C-O)_1$	↑		
	pattern 2	$(C-O)_2$	↑		
	pattern 3	$(C-O)_3$	B		
5	Spheroid/Projection data: semi major axis	a	↑		
	inverse flattening	f	↑		
	scale factor at origin	K_o	↑		
	false Easting	E_o	C		
6	Optional: propagation velocity factor (if not used = 1)	p			
	Also conversion factor from range unit to meters.		f	B	
7	Aprox. position (mid area): Latitude	φ_m	↑		
	Northing	N_m	↑		
	Easting	E_m	D		
	START OF CALCULATION.				
8	To write data card needed in				
	program: CALC. Insert blank card.		E		card
9	To print list of input parameters		f	E	list

Program Description

Program Title THREE RANGE POSITIONING - PREP
Name ir. J.G. Riemersma / C. Pryce Date 1.3.79
Address Shell Internationale Petroleum Maatschappij EP/12
City the Hague State Netherlands Zip Code

Program Description, Equations, Variables, etc.

Purpose: to calculate T.M. coordinates from 2-3 pattern ranges.

Input: Station coordinates (in meters), (C-O) values, semi major axis spheroid, inverse flattening spheroid, scale factor at origin, false Easting, approx latitude for centre of area, northing and easting of approx. pos. to start iteration. Propagation velocity factor (if not used = 1) can also be used to convert ranges to meters.

Output: The produced data card is required in program: THREE RANGE POSITIONING - CALC

Formulae: 'Geodesy' by G. Bomford. Third edition p. 564-565

$$b = a.(1 - 1/f) \qquad e^2 = (a^2 - b^2)/a^2$$

$$R = \sqrt{\rho . \nu} = a . \sqrt{(1-e^2)} \; / \; (1 - e^2 . \sin^2 \varphi_m)$$

The values K_0, E_0 and $6R^2K_0^2$ are transferred on the data card to the computation program, together with station data, (C-O) corrections and propagation velocity factor.

Operating Limits and Warnings The program is based on the use of Transverse Mercator coordinates, but could be modified for use with other projection system coordinates.

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Input station coord	057	X^2	53	
002	DSP2	-63 02		058	RCL0	36 00	
003	4	04		059	X^2	53	
004	x	-35	computation of	060	-	-45	
005	1	01	store location	061	RCL6	36 06	
006	0	00		062	X^2	53	
007	+	-55		063	÷	-24	
008	STOI	35 46		064	STO0	35 00	$e^2 = (a^2 - b^2)/a^2$
009	R↓	-31	store coordinates:	065	1	01	
010	STO(i)	35 45	N, E	066	ENT↑	-21	
011	DSZI	16 25 46		067	RCL0	36 00	
012	R↓	-31		068	-	-45	
013	STO(i)	35 45		069	√X	54	
014	RTN	24		070	RCL6	36 06	
015	*LBLB	21 12	Input (C-O) values	071	x	-35	
016	P⇄S	16-51		072	ENT↑	-21	
017	STOD	35 14		073	1	01	
018	R↓	-31		074	P⇄S	16-51	
019	STO9	35 09		075	RCL4	36 04	
020	R↓	-31	store (C-O):	076	P⇄S	16-51	
021	STO5	35 05		077	SIN	41	
022	P⇄S	16-51		078	X^2	53	
023	RTN	24		079	RCL0	36 00	
024	*LBLb	21 16 12	Input velocity factor	080	x	-35	
025	STO3	35 03		081	-	-45	
026	RTN	24	ρ	082	÷	-24	$R^2 = a.\sqrt{(1-e^2)}\,/$
027	*LBLC	21 13	Input spheroid /	083	X^2	53	
028	P⇄S	16-51	projection data	084	6	06	$(1 - e^2.\sin^2\varphi_m)$
029	STOE	35 15	E_0	085	x	-35	
030	R↓	-31		086	RCL1	36 01	
031	STO1	35 01	K_0	087	X^2	53	
032	R↓	-31		088	x	-35	
033	STOA	35 11	f	089	STO2	35 02	
034	R↓	-31		090	P⇄S	16-51	$= 6.R^2K_0^2$
035	STO6	35 06	a	091	RCL3	36 03	
036	P⇄S	16-51		092	P⇄S	16-51	
037	RTN	24		093	ENT↑	-21	
038	*LBLD	21 14	Input approx. pos.	094	RCLE	36 15	← 10÷ (correction)
039	STO6	35 06	E_m	095	+	-55	
040	R↓	-31		096	STO0	35 00	
041	STO5	35 05	N_m	097	P⇄S	16-51	$= \frac{E_0}{10} + \rho$
042	R↓	-31	RCL3	098	RTN	24	
043	STO4	35 04	φ_m — $X \neq 0$	099	*LBLE	21 15	
044	1	01	GTO 0	100	WDTA	16-61	WRITE DATA CARD
045	STO3	35 03		101	RTN	24	
046	*LBL0	21 00	CALCULATION	102	*LBLe	21 16 15	
047	P⇄S	16-51		103	1	01	PRINT DATA
048	1	01		104	3	03	
049	ENT↑	-21		105	STOI	35 46	
050	RCLA	36 11		106	1	01	
051	1/X	52		107	STO0	35 00	
052	-	-45		108	*LBL1	21 01	
053	RCL6	36 06		109	RCL0	36 00	
054	x	-35		110	PRTX	-14	
055	STO0	35 00	$b = a.(1 - 1/f)$	111	RCL(i)	36 45	
056	RCL6	36 06		112	PRTX	-14	

REGISTERS

0	1	2	3	4	5	6	7	8	9
			ρ	φ_m	N_m	E_m			

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
$b, e^2, \frac{E_0}{10} + \rho$	K_0	$6R^2.K_0^2$	N_1	E_1	$(C-O)_1$	a	N_2	E_2	$(C-O)_2$

A	B	C	D	E	I
f	N_3	E_3	$(C-O)_3$	E_0	indirect store counter

Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	ISZI	16 26 46	
114	RCLi	36 45	
115	PRTX	-14	
116	ISZI	16 26 46	
117	RCLi	36 45	
118	PRTX	-14	
119	SPC	16-11	
120	1	01	
121	ST+0	35-55 00	
122	ISZI	16 26 46	
123	ISZI	16 26 46	
124	RCL0	36 00	
125	4	04	
126	X=Y?	16-33	
127	GTO2	22 02	
128	GTO1	22 01	
129	*LBL2	21 02	
130	P⇄S	16-51	
131	RCL6	36 06	
132	PRTX	-14	
133	RCLA	36 11	
134	PRTX	-14	
135	DSP9	-63 09	
136	RCL1	36 01	
137	PRTX	-14	
138	DSP2	-63 02	
139	RCLE	36 15	
140	PRTX	-14	
141	P⇄S	16-51	
142	SPC	16-11	
143	RCL3	36 03	
144	PRTX	-14	
145	SPC	16-11	
146	RCL4	36 04	
147	PRTX	-14	
148	RCL5	36 05	
149	PRTX	-14	
150	RCL6	36 06	
151	PRTX	-14	
152	SPC	16-11	
153	RTN	24	
154	R/S	51	
160			

INPUT:

```
5563852.22 ENT↑
 449172.98 ENT↑
      1.00 GSBA
5576774.09 ENT↑
 386410.93 ENT↑
      2.00 GSBA
5541866.31 ENT↑
 349334.41 ENT↑
      3.00 GSBA
      0.00 ENT↑
      0.00 ENT↑
      0.00 GSBE
6378388.00 ENT↑
    297.00 ENT↑
     .9996 ENT↑
 500000.00 GSBC
     60.00 ENT↑
5480000.00 ENT↑
 420000.00 GSBC
           GSBE
```

COMMENTS

TEST DATA PRINT OUT:

```
                GSBe
       1.00  ***
 5563852.22  ***
  449172.88  ***
       0.00  ***

       2.00  ***
 5576774.09  ***
  386410.93  ***
       0.00  ***

       3.00  ***
 5541866.31  ***
  349334.41  ***
       0.00  ***

 6378388.00  ***
     297.00  ***
0.999600000  ***
  500000.00  ***

       1.00  ***

      60.00  ***
 5480000.00  ***
  420000.00  ***
```

Register print

```
       4.00  0
       0.00  1
       0.00  2
       1.00  3
      60.00  4
 5480000.00  5
  420000.00  6
       0.00  7
       0.00  8
       0.00  9
     297.00  A
 5541866.31  B
  349334.41  C
       0.00  D
  500000.00  E
      25.00  I

             P⇄S
             PREG

   500021.05  0
        1.00  1
2.447295801-14  2
  5563552.22  3
   449172.88  4
        0.00  5
  6375386.00  6
  5576774.09  7
   386410.93  8
        0.00  9
```

LABELS					FLAGS
A STATION COORDINATES	B (C-O)	C SPHEROID / PROJECTION	D APPROX. POS.	E WRITE DATA CARD	0
a	b velocity factor	c	d	e PRINT DATA	1
0 START CALCULATION	1	2	3	4	2
5	6	7	8	9	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☐	DEG	☑	FIX	☑
1	☐	☐	GRAD	☐	SCI	☐
2	☐	☐	RAD	☐	ENG	☐
3	☐	☐			n 2	

User Instructions

THREE RANGE POSITIONING - CALC			
RANGES 1, 2, 3	NEW VELOCITY FACTOR	NEW APPROX POSITION	NEW (C-O) 1, 2, 3

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	LOAD DATA CARD FROM 'PREP'			
	OR			
	RUN PROGRAM PREP			
2	LOAD PROGRAM CARD: CALC			
3	INPUT: RANGE 1 or 0		↑	
	RANGE 2 or 0		↑	
	RANGE 3 or 0		A	
4	OUTPUT: Residual 1			r_1
	Residual 2			r_2
	Residual 3			r_3
	Standard Deviation			$\hat{m}$
	Northing			N_p
	Easting			E_p
	or 0 if no solution.			
5	Optional: NEW Velocity factor	p	B	
	Last calculation will be repeated			
	with new value			
6	Optional: NEW approx position	N_m	↑	
	to start iteration. Last calculation	E_m	C	
	will be repeated with new value.			
7	Optional: NEW (C-O) values.	$(C-O)_1$	↑	
	Last calculation will be repeated	$(C-O)_2$	↑	
	with new value.	$(C-O)_3$	D	
	OUTPUT TEST DATA:			
	57803.80 ENT↑			
	86124.10 ENT↑			
	77354.90 GSB-			
	1.14 ***			
	-1.42 ***			
	1.40 ***			
	1.34 ***			
	5510575.35 ***			
	414576.57 ***			

Program Description

Program Title THREE RANGE POSITIONING - CALC
Name ir J.G. Riemersma / C.Pryce Date 1.3.79
Address Shell Internationale Petroleum Maatschappij EP/
City the Hague State Netherlands Zip Code

Program Description, Equations, Variables, etc.

Purpose : to calculate TM coordinates from 2-3 ranges, together with the standard deviation in single obsv.

Method : Least squares using variation of coordinates method.

Input: Data card from program: PREP and 2-3 ranges. Optional: change in propagation velocity, (C-O) and new iteration start point.

Formulae: 'Geodesy' by G. Bomford, Third edition, p.219

R = corrected 'observed' distance PS
d = calculated grid distance PS (P = point, S = station)
$R = (\text{Range} + (C-O)) \cdot p \cdot K$
Linescale factor K is calculated from:

$$K = K_0 \cdot (1 + ((E_p-E_0)^2 + (E_p-E_0)(E_s-E_0) + (E_s-E_0)^2)/6 \cdot R^2 \cdot K_0^2$$

Observation equations :

$$((N_p-N_s)/d) \cdot \delta N_p + ((E_p-E_s)/d) \cdot \delta E_p = (R-d) + r$$

Normal equations :

If the observation equation is $a \cdot \delta N + b \cdot \delta E = \varepsilon$ then the normal equations are :

$$\sum aa \cdot \delta N + \sum ab \cdot \delta E = \sum a \cdot \varepsilon$$
$$\sum ab \cdot \delta N + \sum bb \cdot \delta E = \sum b \cdot \varepsilon$$

from which δN and δE can be solved.

Standard deviation : $\hat{m} = \sqrt{\sum rr/(n-2)}$

r = residual , n = nr of patterns.

Operating Limits and Warnings

1. Computation time 1 to 2 minutes per point.
2. Calculation is based on TM projection and can be modified for other projections. The false Easting must be a 0 or a multiple of 10000. To save storage the false Easting is stored together with the velocity factor.
3. Computation has been used succesfully with ranges of 300 Km.
4. If no solution, try a new approximation.

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Input ranges:	057	GTO3	22 03	
002	STOE	35 15	R_3	058	RCL3	36 03	
003	R↓	-31		059	ST+5	35-55 05	
004	STOA	35 11	R_2	060	RCL4	36 04	
005	R↓	-31		061	ST+6	35-55 06	Update stored
006	P⇄S	16-51		062	RCL2	36 02	N_m, E_m
007	STO6	35 06	R_1	063	1	01	
008	P⇄S	16-51		064	X≤Y?	16-35	if vector > 1 m
009	CF0	16 22 00		065	GTO0	22 00	repeat calc.
010	*LBL0	21 00	START COMPUTATION:	066	SF0	16 21 00	
011	0	00		067	GTO0	22 00	
012	STO0	35 00		068	*LBL4	21 04	Branch to end.
013	STO1	35 01		069	RCL1	36 01	
014	STO2	35 02	normal equations	070	2	02	
015	STO3	35 03	store cleared	071	X=Y?	16-33	
016	STO4	35 04		072	GTO5	22 05	
017	1	01		073	RCL0	36 00	calculate standev
018	6	06		074	√X	54	
019	STOI	35 46	Path 1 obsv eq.	075	PRTX	-14	$\hat{m}$
020	GSB1	23 01		076	*LBL5	21 05	Print results
021	2	02		077	RCL5	36 05	
022	0	00		078	PRTX	-14	
023	STOI	35 46	Path 2 absv. eq.	079	RCL6	36 06	N_p
024	GSB1	23 01		080	PRTX	-14	
025	2	02		081	SPC	16-11	E_p
026	4	04		082	RTN	24	
027	STOI	35 46	Path 3 obsv eq.	083	*LBL3	21 03	No solution
028	GSB1	23 01		084	CLX	-51	
029	F0?	16 23 00		085	RTN	24	
030	GTO4	22 04	Branch if residual	086	*LBL1	21 01	Return if no range.
031	RCL0	36 00	comp only	087	RCLi	36 45	
032	RCL1	36 01		088	X=0?	16-43	
033	÷	-24		089	RTN	24	
034	ST×2	35-35 02	Solve normal eq.	090	DSZI	16 25 46	
035	ST×4	35-35 04	δ_N and δ_E in	091	DSZI	16 25 46	
036	RCL1	36 01	store 3 and 4	092	RCLi	36 45	
037	ST-2	35-45 02		093	GSB2	23 02	$E_s - E_o$
038	RCL3	36 03		094	-	-45	
039	ST-4	35-45 04		095	X²	53	
040	RCL2	36 02		096	LSTX	16-63	
041	ST÷4	35-24 04		097	RCL6	36 06	
042	RCL1	36 01		098	GSB2	23 02	$E_p - E_o$
043	RCL4	36 04		099	-	-45	
044	×	-35		100	×	-35	
045	ST-3	35-45 03		101	LSTX	16-63	
046	RCL0	36 00		102	X²	53	
047	ST÷3	35-24 03		103	+	-55	
048	RCL3	36 03		104	X⇄Y	-41	
049	RCL4	36 04		105	X²	53	
050	→P	34		106	+	-55	
051	STO2	35 02		107	P⇄S	16-51	$K = K_o(1+((E_p-$
052	PSE	16 51	vector display	108	RCL2	36 02	$E_o)^2+(E_p-E_o)(E_s-E_o)+$
053	1	01		109	÷	-24	$(E_s-E_o)^2)/6.R^2K_o^2$
054	EEX	-23		110	1	01	
055	6	06		111	+	-55	
056	X≤Y?	16-35	Branch if vector > 10^6	112	RCL1	36 01	

REGISTERS

0	1	2	3	4	5	6	7	8	9
Σaa	Σab	Σbb	ΣaE	ΣbE	N_m	E_m	R_1-d_1	R_2-d_2	R_3-d_3
S0 E_o+p	S1 K_o	S2 $6R^2K_o^2$	S3 N_1	S4 E_1	S5 $(C-O)_1$	S6 Range 1	S7 N_2	S8 E_2	S9 $(C-O)_2$

A	B	C	D	E	I
Range 2	N_3	E_3	$(C-O)_3$	Range 3	indirect store counter

Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	×	-35	
114	P⇄S	16-51	
115	ISZI	16 26 46	
116	RCLi	36 45	
117	ISZI	16 26 46	
118	RCLi	36 45	R=(range+(C-O))
119	+	-55	
120	GSB7	23 07	·p·K.
121	×	-35	
122	×	-35	
123	DSZI	16 25 46	
124	DSZI	16 25 46	
125	RCL6	36 06	
126	RCLi	36 45	
127	-	-45	
128	STO8	35 08	δe
129	DSZI	16 25 46	
130	RCL5	36 05	
131	RCLi	36 45	
132	-	-45	
133	STO7	35 07	δN
134	→P	34	
135	ST÷7	35-24 07	
136	ST÷8	35-24 08	
137	R↑	16-31	
138	X⇄Y	-41	(R-d)
139	-	-45	
140	F0?	16 23 00	
141	GTO6	22 06	Branch if residual
142	RCL7	36 07	comp only
143	X⇄Y	-41	
144	×	-35	
145	ST+7	35-55 07	Σaε
146	LSTX	16-63	
147	RCL8	36 08	
148	×	-35	
149	ST+4	35-55 04	Σbε
150	RCL7	36 07	
151	X²	53	
152	ST+0	35-55 00	Σaa
153	RCL8	36 08	
154	X²	53	
155	ST+2	35-55 02	Σbb
156	RCL7	36 07	
157	RCL8	36 08	
158	×	-35	
159	ST+1	35-55 01	Σab
160	RTN	24	
161	*LBL6	21 06	Print residuals
162	PRTX	-14	r_1, r_2, r_3
163	X²	53	
164	ST+0	35-55 00	Σrr
165	1	01	
166	ST+1	35-55 01	n
167	RTN	24	
168	*LBLB	21 12	

STEP	KEY ENTRY	KEY CODE	COMMENTS
169	ENT↑	-21	Store new velocity
170	GSB2	23 02	factor
171	P⇄S	16-51	← 10÷ (correction)
172	STO0	35 00	
173	R↓	-31	
174	ST+0	35-55 00	
175	P⇄S	16-51	← CLFØ (correction)
176	GTO0	22 00	
177	*LBLC	21 13	Store new approx
178	STO6	35 06	position
179	R↓	-31	
180	STO5	35 05	← CLFØ (correction)
181	GTO0	22 00	
182	*LBL2	21 02	Retrieve false
183	P⇄S	16-51	Easting
184	RCL0	36 00	
185	1	01	
186	EEX	-23	
187	x 3	0x 3	(correction)
188	÷	-24	
189	INT	16 34	
190	1	01	
191	EEX	-23	
192	4	04	
193	×	-35	
194	P⇄S	16-51	E_0
195	RTN	24	
196	*LBL7	21 07	Retrieve velocity
197	P⇄S	16-51	factor
198	RCL0	36 00	
199	1	01	
200	EEX	-23	
201	4	04	
202	÷	-24	
203	FRC	16 44	
204	1	01	
205	EEX	-23	
206	4	04	
207	×	-35	p
208	P⇄S	16-51	
209	RTN	24	
210	*LBLD	21 14	Store new (C-O)'s
211	P⇄S	16-51	
212	STOD	35 14	
213	R↓	-31	
214	STO9	35 09	
215	R↓	-31	
216	STO5	35 05	
217	P⇄S	16-51	← CLFØ (correction)
218	GTO0	22 00	
219	R/S	51	
220			

LABELS

A ENTER RANGES	B NEW VELOCITY	C NEW APPROX	D NEW (C-O)	E
a	b	c	d	e
0 START COMP	1 OBJV EQ. SUBROUTINE	2 EXTRACT E_0	3 NO SOLUTION	4 BRANCH TO END
5 BRANCH 3 RANGES	6 RESIDUAL CALCULATION	7 EXTRACT p	8	9

FLAGS

0	used to print residuals
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☐	DEG	☑	FIX	☑
1	☐	☐	GRAD	☐	SCI	☐
2	☐	☐	RAD	☐	ENG	☐
3	☐	☐			n	2