

**ADMIRALTY METHOD OF TIDAL PREDICTION
(FORM NP 159)
REFINEMENTS AND INCREASE IN ACCURACY**

by Lieut. Commander A.G. MERRIMAN, M.N.I., R.N. (*)

This paper is intended to supplement the paper 'Admiralty method of tidal prediction, NP 159' by Commander N.C. GLEN, R.N., published in the *International Hydrographic Review*, LIV(1), January 1977 [1], and should be read in conjunction with that paper.

The Admiralty Form NP 159 [2] is a simplified graphical method of obtaining a predicted tidal curve which gives a good approximation to that produced by a full prediction using far more data. Although at first sight it may appear rather tedious, no other method of comparable accuracy approaches its simplicity; thus, although any gain in accuracy is welcomed, any such gain must be weighed against simplicity of application if it is not to destroy the essential character of the method. This criterion has been carefully taken into account when designing these improvements. The first part of the paper concerns the simple application of an additional 5 constituents, while the second part shows how a programmable calculator can increase not only the ease of operation but also the accuracy of prediction by removing some approximations which have to be made in order to keep the drawing task within reasonable proportions.

ADDITIONAL CONSTITUENTS

Seasonal corrections to M_2 and S_2

Well over 400 tidal harmonic constituents have been identified, but a very large number of these have insignificant effect on the tidal level and are not commonly used in prediction. Among those thought to fall into this category are two with speeds equal to that of $M_2 + S_a$ and $M_2 - S_a$. Recent analyses have

(*) Hydrographic Department, Taunton, Somerset TA1 2DN, U.K.

shown that at some ports the amplitudes of these constituents are far greater than their astronomical magnitude would suggest and they therefore appear to have a large shallow water element. These constituents have been given various names by different workers in this field. The United Kingdom Hydrographic Department and the Institute of Oceanographic Sciences have mutually agreed to call them MA_2 (Speed 28.94 deg/hr) and MB_2 (Speed 29.03 deg/hr), see an unpublished paper 'On the definition of the annual modulation of the M_2 tide' by D.E. CARTWRIGHT [3]. The combined effect of these two constituents is to give an apparent seasonal variation of M_2 in both g and H , and it is possible to tabulate it as such.

Two other constituents exist which have the same effect on S_2 . These are T_2 (29.96 deg/hr) and R_2 (30.04 deg/hr). Unlike MA_2 and MB_2 these two have significant astronomical magnitudes, and they are allowed for in the calculation of A and F for S_2 (see *Admiralty Tidal Handbook* No. 3 - NP 122(3), para 4 [4], and Commander GLEN's paper, table I). However, recent analyses have again shown that the amplitudes are in some cases greater than the astronomical magnitude would suggest, and thus they also appear to have a large shallow water element. As with MA_2 and MB_2 the effect is to cause seasonal variations in S_2 and these can again be tabulated, but in this case the purely astronomical effect has to be subtracted before tabulation as it is already allowed for in A and F .

From 1980 onwards *Admiralty Tide Tables* [5] will tabulate seasonal values of M_2 and S_2 for those ports where the size of the variation justifies it. This will be Table VI, and the ports affected will carry a symbol referring to this table in the appropriate columns in Part 2. The total number of ports affected in all three volumes is about 20, and the largest variation is about 60° in g (though this is associated with a very small value of H) and about 0.3 m in H . It must be appreciated however that the actual effect on predicted heights may be larger than these figures suggest as :

- a) both M_2 and S_2 may be affected;
 - b) the value of F by which H is to be multiplied may be as large as 1.3;
- and
- c) shallow water corrections are obtained by multiplying the square and cube of the semi-diurnal tide by their respective multipliers.

Hence the resultant effect may be as large as 0.5 m.

The change in predicted height may therefore be significant, and as the application of the correction is simple the criterion for its use is fulfilled.

Fortnightly variation in Mean Sea Level

There are several constituents with a speed of about 1 deg/hr (i.e. 1 cycle a fortnight). Analysis shows that the most likely to be significant is the shallow water harmonic MSf. $(E + u)$ of MSf is equal to $(E + u)$ of $S_2 - (E + u)$ of M_2 , and thus it moves through a complete cycle as this difference moves from 0 to 0 : as this phase relationship is the same as that which governs the spring/neap cycle it is possible to tabulate the effect of MSf relative to the day of springs. From 1981

onwards this will be done in *Admiralty Tide Tables* as Table VI(a) which will include those ports where :

- a) the effect is appreciable;
- b) the day of springs can be identified for a nearby port in Part I with sufficient accuracy.

The maximum effect found so far gives a variation of over 1 m between the extreme values of mean level (ML), and this would justify the inclusion of quite complicated processes to obtain the necessary correction. The application by means of this table is however so simple that any port with a known variation greater than 0.1 m has been included. As with the seasonal correction to M_2 and S_2 , the ports concerned will carry a reference in the appropriate column in Part 2. Inclusion of this table has made practical the inclusion of Harmonic Constants for certain ports, which without the table gave only very approximate predictions.

P.H. LE BLOND in his paper 'Forced fortnightly tides in shallow rivers' [6] produces mathematical reasoning why g of MS_f should also equal g of $S_2 - g$ of M_2 , and analysis shows that this relationship applies in a large number of cases. Where this is so the Mean Level will attain a maximum at springs and a minimum at neaps, and if MS_f is large enough it will account for the apparent anomaly that occurs at the head of certain estuaries, namely that LW neaps falls below the level of LW springs.

USE OF PROGRAMMABLE POCKET CALCULATORS

The remainder of this paper concerns the use of programmable pocket calculators to replace the graphical portion of Form NP 159. The full method of prediction is itself a simple formula, being :

$$f.H.\cos [(E + u) - g]$$

repeated for each constituent and for each time required. Although the calculation of this formula is well within the capabilities of many small calculators, a large number of constituents must be used if a reasonable degree of accuracy is to be obtained. This means either that large amounts of data must be entered repeatedly (with the accompanying chance of error) or that a considerable number of data stores are required, and this limits its use to large computers and the bigger desk top calculators. Additionally, the calculation of $(E + u)$ and f , though not complicated, is lengthy for a large number of constituents and will be outside the economical range of use of pocket calculators. It may also be that paucity of data means that NP 159 is the best available method of prediction, in which case the following may be taken to apply to computers, but it is not written with them in mind.

Anybody who is at all conversant with his own pocket calculator should be able to write a program reproducing the graphical steps of Form NP 159, with the accuracy slightly increased by the removal of random errors inherent in a graphical solution. The purpose of this section is however to delve more deeply

into the reasoning behind NP 159, and thus enable one to increase accuracy by removing certain approximations that had been included in order to keep the drawing task within manageable limits.

The basic tenets of the method are :

- a) that constituents close to each other in speed will have approximately equal phase lags (g), and their amplitudes (H) will be approximately proportional to their theoretical magnitudes. They can thus be grouped together under the heading of a major constituent, as described in *Admiralty Tidal Handbook* No. 3 [4] and Commander GLEN's paper [1];
- b) that the $1/4$ and $1/6$ diurnal tides are proportional to the square and cube respectively of the semi-diurnal tide, and that these move at twice and three times the rate of that tide.

These two assumptions are fundamental to the method, and are the means of allowing for up to about 35 constituents while only using data for 6 (in this context counting f_4 and f_6 as constituents). We are here concerned with using the properties of a calculator to make best use of these assumptions.

(In what follows, M_2 is to be taken as meaning 'the M_2 group', etc.).

NP 159 performs the following operations :

- (i) obtains the phase and amplitude for the M_2 tide at one time during the day;
- (ii) obtains the phase and amplitude of the S_2 tide for the same time;
- (iii) hence by vectorial addition obtains a phase and amplitude for the combined semi-diurnal tide;
- (iv) from this it derives a phase and amplitude for each of the $1/4$ and $1/6$ shallow water constituents using the theory in (b) above;
- (v) by advancing the combined semi-diurnal tide at 29 deg/hr and the $1/4$ and $1/6$ diurnal tides at approximately 2 and 3 times this rate, it produces a combined semi-quarter-sixth diurnal tide for 12 hours;
- (vi) by assuming that this tide repeats itself 12 1/2 hours before and after, it generates the curve for the remaining 12 hours of the day;
- (vii) it repeats steps (i), (ii), (iii) and (v) using K_1 and O_1 and a rate of 14 1/2 deg/hr to generate 12 hours of the diurnal curve;
- (viii) it assumes that the diurnal curve repeats itself with the sign changed 12 1/2 hours before and after;
- (ix) by summing the curves in (vi) and (viii), it produces a combined predicted curve for the whole 24 hours.

The following approximations are used in these steps :

- (a) it assumes that the combined semi-diurnal tide moves at 29 deg/hr. Even if A of M_2 changed at exactly this rate this would still be only an approximation, owing to change of phase between M_2 and S_2 . In fact, examination of consecutive values of A shows that the rate may vary from this figure by enough to give appreciable errors at the end of the day;
- (b) it makes a similar assumption concerning the diurnal tide;
- (c) it assumes that the values of F remain constant throughout the day.

Examination of consecutive values shows that this may change by as much as 6 % in one day;

- (d) it assumes the precise repetition of the curve at 12 1/2 hr intervals (inverted in the case of the diurnals);
- (e) the effect of the above assumptions on the semi-diurnal tide may be amplified in the case of the 1/4 and 1/6 diurnals.

All the above approximations can be removed as follows. This uses the essential property of calculators which is that they excel in repeating the same calculation over and over again using different data :

- (1) by making the basic calculation for each time for which a prediction is required, using individual rates for each constituent, thus allowing for change in phase rate due to S_2 and O_1 ;

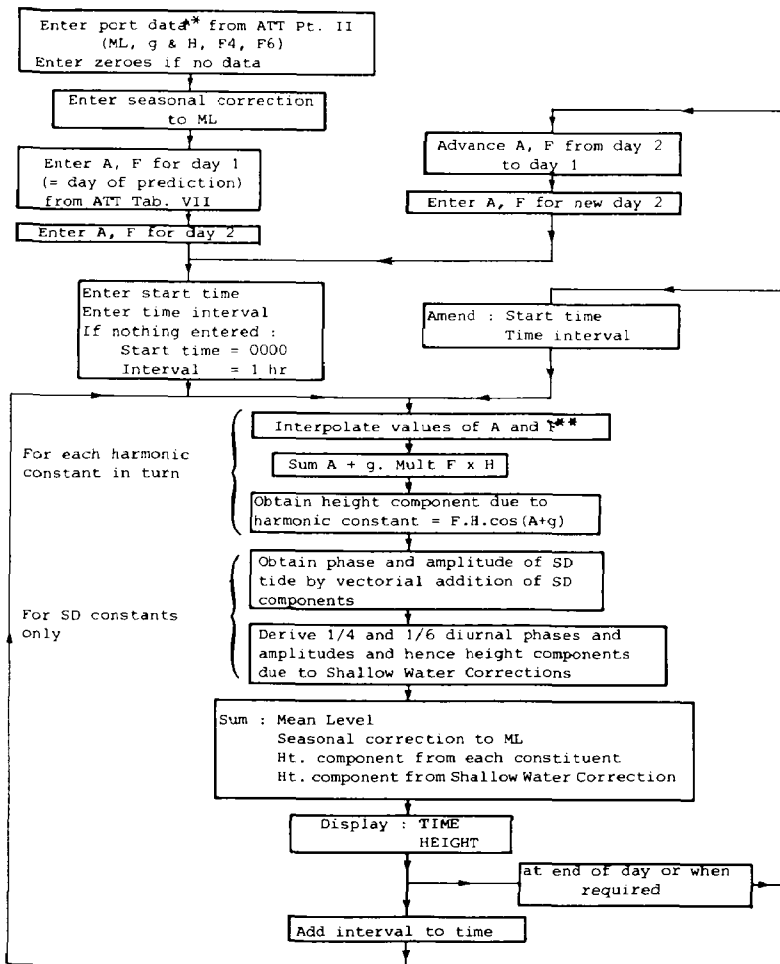


FIG. 1. - The Admiralty Method of Tidal Prediction - NP 159.

* If port data is entered by pre-recorded card, provision must be made for entering values from Tables VI and VI(a) if required.

** See Note 1 concerning interpolation of A.

- (2) although it is possible to use the astronomical rates of the 'Name Constituent' of each group, greater accuracy can be obtained by interpolation between tabulated values of A if the number of steps/stores allows this;
- (3) again subject to the availability of steps/stores, interpolated values of F should be used in preference to a daily constant.

The above steps concern the accuracy of such a program but, subject to availability of steps and stores and general versatility of the calculator, efforts should be made to increase the flexibility of the program to reduce the data input. The following are suggested :

1. Reduction of the data entry by :
 - (a) recording and re-entry of data for ports which are predicted frequently; (Re-entry should allow for amendments, as required by Tables VI and VI(a)).
 - (b) enabling subsequent days' predictions without re-entry of port data or daily data already in store;
 - (c) enabling prediction of more than one port on the same day without re-entry of daily data.

2. Automatic production of hourly predictions throughout the day.

3. Ability to repeat portions of the curve commencing at chosen times and using a chosen interval (e.g. 10 minute intervals) over HW and LW. Although individual heights can also be produced by these steps, they are generally of little value.

(It should also be possible to write a program so that in addition to the above it will produce both the times and heights of HW/LW, and to indicate the time at which a particular height is reached, but it is considered that in the type of port where prediction is most likely to be used such figures may well be ambiguous and it is generally better to draw a small portion of the curve).

Figure 1 shows a possible flow chart which performs the suggested functions.

Notes :

1. Great care must be exercised in the interpolation of the hourly rate of change of A. It must be noted that the final step in the derivation of A is to subtract from 360° ([1], p. 75). Thus although $(E + u)$ increases with time, A decreases. Also the calculated rate must approximate to the astronomical rate of 30° per hour for the semi-diurnal constituents and 15° per hour for the diurnal. Thus, if A_1 is the value for the first day and A_2 the value for the next, then $(A_2 - A_1)/24 \simeq -30^\circ$ or $(A_1 - A_2)/24 \simeq 30^\circ$ for the semi-diurnal constituents and similarly, using 15° , for the diurnals. The actual steps to obtain the correct result will depend on the calculator in use, but a convenient method is to ensure that, for the semi-diurnals, $A_1 - A_2$ is greater than 600° by the addition of the minimum number of 360° . In the case of the diurnals, use 300° instead of 600° . Then the result of the calculation $(A_1 - A_2)/24$ gives the hourly rate to be used for the day.

2. If there is a shortage of data stores it is usually possible to store more than one figure by placing them (after applying a suitable multiplier) either side of the decimal point. Strangely, such a method often seems to reduce the number of steps required as well as the storage.

3. In view of the interpolation used to obtain values of the rates of change for A and F for the particular day, arrangements should be made to prevent the continuation of the program to the next day without the insertion of new values for A and F and a re-calculation of the rates of change.

4. It may be of interest to note that 24 hour predictions using this method were compared with a full prediction using 60 constituents. The differences were comparable with those found when using all the 25 constituents which can be obtained from one month's analysis. In fact, analysis over such a short period may well have errors in the constituents which are greater than those caused by the assumptions made in the calculation of A and F. Therefore it may well be that, unless a long period analysis is available, the predictions obtained by using the method of N.P. 159 and a calculator are not only the simplest to obtain but also the most accurate.

REFERENCES

- [1] N.C. GLEN (1977): The Admiralty method of tidal prediction - NP 159. *Intern. Hydrog. Review*, LIV(1), pp. 73-85.
- [2] Admiralty method of tidal prediction - Form NP 159 (1975).
- [3] D.E. CARTWRIGHT (1977): On the definition of the annual modulation of the M_2 tide. (Unpublished manuscript).
- [4] *Admiralty Tidal Handbook*, No. 3 - NP 122(3).
- [5] *Admiralty Tide Tables*, Volumes I, II and III.
- [6] P.H. LE BLOND: Forced fortnightly tides in shallow rivers. Submitted to 'Atmosphere-Ocean', November 1978.