UNDERWATER ACOUSTIC POSITIONING SYSTEMS: STATE OF THE ART AND APPLICATIONS IN DEEP WATER

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Summary

For many underwater location and navigation requirements, and especially those in the deep ocean, acoustic methods are likely to remain the principal solution. A wide range of examples and their main features are reported. Basic design considerations and the environmental and operational factors limiting performance are discussed. Relative performance may be good but absolute geodetic accuracies are limited by the accuracies of navigation at the sea surface.

1. INTRODUCTION

If there is one feature which characterises underwater positioning at the present time it is probably the variety of implementations, almost to the point that each user operates a different system. This is particularly apparent in the deep ocean, though in industrially active areas like the North Sea some contractors have encountered confusion due to multiple use of identical systems. The underwater situation contrasts with the well established, publicly available methods of radio navigation above the surface. Underwater technology is probably advanced enough that a range of systems could be conceived and built for

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general use, but even on the continental shelf a viable demand for this just does not exist, nor is there a public department with a mandate to provide such a service; in the deep ocean the area to cover is vast with even fewer numbers of workers. Thus underwater operators are invariably faced with the installation of private, portable or expendable systems for relatively short missions.

The variety of applications is reflected in the variety of underwater location methods, ranging from the simplicity of a pressure gauge indicating the depth of an instrument relative to a vessel from which it is towed or to which it is tethered, to a comprehensive network capable of tracking a number of units simultaneously in three dimensions and in real time. It is well known that poor and unreliable visibility in the seas and the negligible penetration and the lack of propagation of radio waves into the sea rule out electromagnetic waves for remote positioning systems underwater. Fortunately, acoustic waves propagate well and can be exploited within fairly well understood limits.

There are several ways one could categorise the methods of applying underwater acoustics; Table 1 illustrates the present choice, based upon passive listening source location or active interrogation location methods. Some examples with their operational features will be described briefly, drawing contrasts with radio aids. Then the major environmental factors which limit performance will be described and finally prospects for future progress will be considered.

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2. ACTIVE SOURCE
WITH PASSIVE LISTENING LOCATION METHODS

The history of the location of underwater explosions with widely spaced listening hydrophones began in World War I, but developed little until World War II. The subsequent technological advances in transducers, cables, batteries and particularly transistors and computers enabled wider applications of the basic principles, and by the early 1960's practically all techniques had been tried. Perhaps the first serious use [1] of acoustic navigation underwater was by downed aircraft, which ejected a charge to detonate near the axis of the Sofar channel, the explosion being picked up oceanwide by strategically placed hydrophones; by comparing arrival time-differences between two or more hydrophone-pairs the aircraft was located and rescue services initiated. The method was later extended for missile impact location [2].

In modern source location systems the transmitter source often, though not necessarily, will be a simple ping or mono-frequency pulse. Usually the object to be located carries the pinger, whose signal is then received on a number of fixed hydrophones suspended down from the surface or up from the seabed; the hydrophone signals pass by cables to a multiple channel receiver where time-differences of arrival of the pinger pulse at different pairs may be measured. If the position is to be located in a three dimensional frame a minimum of 4 hydrophones is needed, or one less for a two dimensional fix. The relative positions of all hydrophones must be known beforehand: one convenient way of determining these is to visit each hydrophone in turn with the pinger to obtain base-line lengths for trilateration of the complete grid. Further work is necessary for geodetic tie-up. The locus of a constant time difference is a hyperboloid of revolution with the two hydrophones as foci, and for two dimensional geometry pre-computed loci for different pairs may be drawn on a chart in precisely similar form to a Decca chart. Lane identification is automatic and the time resolution is determined by the pulse rise time and signal-to-noise ratio at the hydrophone amplifier output. Absolute accuracy is rarely limited by the timing clock, but invariably by variations in the speed of sound, discussed later. Data reduction by computer rather than manual plotting is now the norm, particularly in three dimensional applications.

When a fixed object is to be relocated it is common practice to employ one hydrophone only at successive different positions, by steaming the ship, which is equipped with surface navigation, until enough fixes for adequate position determination are obtained. This method is widely used in oceanographic research where in-situ recording instruments have been designed to measure, for example, currents, temperatures, tidal pressures, seismic events. In I.O.S. designs the location 'pinger', or 'acoustic beacon' as it is sometimes called, may be integral with the acoustically commanded release which separates the buoyant instrument package from the anchor weight at the end of mission; in other cases the release and beacon may be separated vertically for improved first detection range. At 10 kHz operating frequency maximum slant ranges reach 10 km. With the earlier
applications, drift of pinger rate meant that the rate of change of ping arrival time could only indicate approaching or receding courses, but with modern crystal stabilised pingers and synchronised recorders the point of closest approach is also easily observed. Swallow [3] used this technique for the first time to track neutrally buoyant floats, to ranges of a few km, thereby beginning the Lagrangian measurement of deep ocean currents. More recently Webb & Rossby [4], [5] have extended ranges of this technique beyond 1 500 km using Sotar channel propagation and an acoustic frequency near 300 Hz to give daily fixes from Sotar hydrophones; critical to this development has been the design of an organ pipe type of transucer, 1.8 m long and 0.3 m diameter. Float battery life is in excess of 2 years. Autonomous listening stations have now been developed increasing deployment flexibility outside Sotar hydrophone networks. A number of floats are used simultaneously, differentiated by their precise frequency and transmission time. This large development in Lagrangian current measurement has played a major part in MODE, and verified in a dramatic way the presence in the ocean of mesoscale eddies, having spatial scales around 100 km, time scales around 2 months, and accounting for 95 % of the kinetic energy in the oceans. As to be seen later, long ranges such as these can only be obtained with floats at or near the depth of the Sotar channel axis. French workers have optimised their designs for ranges to 250 km using frequencies around 1.8 kHz, with a swept frequency pulse and a signal correlation receiver.

Another example on a much smaller scale, though no less dramatic, is described by Hawkins et al. [6] who attached small (4.6 cm long by 2.6 cm O.D.) cylindrical pingers operating at 41 kHz to live cod released into an enclosed Scottish loch, instrumented with four hydrophones around 200 m apart. Fixing errors in the order of 0.4 m were small enough to follow the fish movements and demonstrate differences in behaviour after release, and then between day and night time. With their array, Hawkins et al. also tracked a fishing trawl across a noise range. Tape recordings were made and analysis mainly performed later in the laboratory.

None of the scientific examples quoted have required location and plotting in real time. Tracking experimental torpedoes on special ranges is an application where real time detection and plotting is required and the added expense can be justified. Signal processing using digital computers also has an enormous role to play in the location of noise-like sources such as submarines and ships by passive sonar. This is a large subject beyond the scope of this paper. The art would appear to lie in detecting and correlating the same source at different sites, following which the correlation time differences may be used in the conventional way to estimate source location. An interesting example of location by listening is that of Watkins and Schevill [7], who were able to locate the position of sound producing whales with hydrophones slung overside from a quiet ship. The whale sounds are not simple pulses but are sufficiently recognisable to be identified on pairs of hydrophones. A pinger lowered from the ship was used to calibrate the otherwise loose geometry of the hydrophone network.

Underwater and/or land based recording seismometer stations are used to locate earthquake epicentres in much the same way, with the added complication that arrivals via different geological strata travel with widely varying speeds. The use of controlled explosions along navigated tracks provides data on the sound
velocity profiles and hence contributes towards defining the geophysical structures.

Variations on the simple pinger and hydrophones arrangement are possible. For example, the pinger signal can be coded with information from a pressure sensor to give depth directly. Or by making the pinger repetition period extremely accurate, to around 1 part in $10^9$, one hydrophone less can be employed, since ranges to each hydrophone can be determined directly without recourse to time-difference pairs. As an alternative to this high clock precision, Dunbar [8] has proposed the use of pingers synchronised to OMEGA radio transmissions, in applications where connection to a surface buoy with aerial and radio receiver is permissible. Another variation of the pinger-hydrophone combination, attracting increasing attention as oil exploration extends into deeper water, is active drill ship position stabilisation. In this 'inverted' situation the pinger is placed on the seabed very near the required drill site; hydrophones are distributed around the hull of the vessel and cabled to an onboard computer, which determines position and then controls thrusters [9] or anchor winches to maintain station over the site, despite winds and tide. In this multiparameter control system it is not surprising to find [10] the application of real-time Kalman filtering within the control loop.

The majority of the undersea systems mentioned above employ pulsed sound, somewhat in contrast to most common radio aids such as Omega, Decca and Hifix, for example, which essentially measure phase differences between signals received from widely spaced, land-based, master and slave continuous wave transmitters. From figure 1 it can be observed that, for a given frequency, the maximum underwater range is on the order of $5 \times 10^4$ wavelengths; this compares with typically 100 in the radio case. Underwater the relative speeds between a source and receiver vary from 0.01 % to 1 % of the speed of sound for neutral floats to the fastest ships respectively, whereas the fastest aeroplanes barely reach 0.0001 % of the speed of radio waves. The implications of these facts are that an analogous CW underwater system would have many 'lanes', whose indication would change rapidly due to receiver motion and due to environmental effects, discussed below.

Marine radio navigation aids operate in two dimensions, either by line-of-sight, or by ground wave propagation, largely unaffected by surface irregularities smaller than the wavelength. Underwater the third dimension of depth is often required and the surface and seabed boundaries are both troublesome [11], generally to a greater extent than radio sky-wave interference. Pulsed methods are probably easier for an alert observer to interpret in the presence of multipath propagation. However, programming this interpretive skill into an automatic system may not be easy and CW systems may have advantages in some cases.

This is certainly indicated by the remarkable performance achieved by Porter et al. [12], [13] using CW beacons near 12 kHz at 5 000 m depth on the seabed to determine the drift of a near surface hydrophone with a resolution of a few cms ($\lambda/4$) at ranges up to 8 km. Though absolute accuracy is limited by the ability to survey-in the beacons relative to a geodetic reference, there is little doubt that this system is near the ultimate for the near vertical geometry used by the authors, and is particularly well suited for short duration, relative navigation.
3. ACTIVE INTERROGATION
WITH ECHO OR TRANSPONDER REPLIES

In this category range is obtained directly from the round path travel time following transmission or interrogation. The use of echo-sounders and sonars for navigation is often in the nature of a secondary aid, since they have usually been designed for other purposes. As an example, in the era before satellite navigation was available to ships, I.O.S. were working in the area of the Meriadzek Terrace on the North East edge of the Bay of Biscay and could receive Main Chain Decca, but the poor angle of cut gave large uncertainties roughly in the SW to NE direction; seabed contours, however, run roughly NW to SE in that area, so
that the combination of topography and Decca was quite effective to better than a mile for relocation of sites. Within the era of satellite navigation, there have been examples of bad fixing being recognised as such because the sounding at the time of the fix disagreed with the chart at the nominal fix position. Of course topography matching, analogous to airborne terrain following, cannot be universal and it does presuppose that the charts were prepared with adequate navigational precision in the first place. The use of side-scan sonar imagery matching is an extension of the idea, useful in areas of recognisable, seabed topographic patterns; the latter may look quite different when observed from different aspects, however.

All forms of sonar can be used with reflecting or transponding targets. For example, sonars with good bearing resolution can interrogate and, by beaming or scanning onto the transponder reply, can provide the angles of azimuth and elevation necessary for the location of the transponder in spherical coordinates. This technique has been used with considerable success by the Ministry of Agriculture, Fisheries and Food, Lowestoft, using a sector-scanning sonar to locate fish tagged with a miniature transponder. A notable application by Greer Walker et al. [14] has demonstrated the ability of plaice to make use of tidal currents by rising into the water on one half of the cycle and remaining on the seabed during the other half cycle. Bearing directed sonar is often attractive for locating divers or free or tethered submersibles relative to the support ship, especially for operational and safety purposes rather than for precision of fixing. The resolution of a sonar in range is generally good but it is expensive in arrays, steering and stabilisation to achieve high angular discrimination. Variations on this theme include using different interrogate and transpond frequencies to allow high discrimination against sonar reverberation, and using digital sonar techniques suitable for the large signal and single reply situation.

The Global Marine and Edo Western companies developed a high resolution sonar mechanically scanned at the end of the Glomar Challenger bit string, to enable bit relocation into the entry cone on the deep ocean bed. Three passive reflectors are attached to the cone and by observing the shipboard PPI display the ship and string are guided to place the bit centrally prior to entry. This has been in use for over 9 years, enabling bit replacement and so greater drill penetration.

The use of two-component acoustic doppler logs should be mentioned, since continuous vector integration of vessel speed with gyro heading gives the navigated track. Some doppler logs use a high frequency over a short path length relying on scattering from planktonic and particulate matter in the water and therefore measuring speed through the water. Speed resolution is limited by doppler spreading due to plankton motions and it is doubtful whether performance can compete with a good two-component electromagnetic log. However, some doppler logs use a low enough frequency to reach the seabed, at least in shallow continental shelf areas, thus measuring ground speed. Some submersibles have taken advantage of this facility for near seabed work.

The most common form of underwater navigation consists of a network of fixed transponders, buoyed up from or on the seabed, relative to which the position of an interrogator/receiver can be located. The geometrical solution
required is that of three intersecting spheres, or circles in the planar simplification sometimes possible when depths of transponders and interrogator are known. A good number of commercial versions of this technique are now marketed, applications including the navigation of divers [15], deep [16] and shallow submersibles [17], tethered submersibles, towed bodies and surface vessels. When transponder spacings are the same order as the maximum range they are sometimes referred to as long-baseline systems, and positioning resolution and accuracy can be quite high. When transponder range is an order or more greater than transponder spacing the term short-baseline system is applied, for which precision tends to be lower. In fact the latter system is equivalent to a range bearing system, the bearing resolution being determined by the number of range resolutions in the baseline length in the same way that the number of wavelengths in a normal array define beamwidth. In an analogous way improved angular resolution can be achieved if the signal-to-noise ratio is high by techniques of wave front curvature determination, or by repeated observations if the geometry is slowly changing.

4. TRANSPONDER DESIGN CONSIDERATIONS

In view of the important place transponder navigation has, it is worth looking at some design features in detail. The principal advantages of transponders over pingers are, first, that the source saves power by transmitting only when requested; second, range is determined directly, and third, the two-way transmission path largely cancels out sound speed errors due to water current. In order to retain these advantages the whole transponder system design is a good deal more critical than that of a pinger; in particular, it should not respond falsely too often to noise, nor fail to reply correctly to the interrogation; there is little point in the transponder replying if it cannot be received. The problem is that of optimising the detection thresholds. Assuming that interrogation and reception are by the same transducer at one end of the path, and listening and responding use one transducer at the other end of the path, the situation is generally as follows: the propagation loss is the same in both directions; the transponder has a low noise level, but needs a high threshold relative to the r.m.s. noise and has a limited reply power; the interrogator receiver, usually close to the vessel, probably has a higher noise level, but has a lower detection (display) threshold, and a higher power availability. Then, assuming Gaussian noise statistics, a transponder reception bandwidth B, and a threshold voltage R decibels above the r.m.s. noise, the probability of false triggering (PFT) in time T can be simply calculated, table 2.

<table>
<thead>
<tr>
<th>R(dB)</th>
<th>11.8</th>
<th>13.0</th>
<th>13.9</th>
<th>14.6</th>
<th>15.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.F.T./BT</td>
<td>10^{-3}</td>
<td>10^{-4}</td>
<td>10^{-5}</td>
<td>10^{-6}</td>
<td>10^{-7}</td>
</tr>
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</table>
Since maximum range (Fig. 1) will usually correspond to \( T = 10^1 \) pulse resolution intervals, \( 1/B \), it follows that a 12 dB threshold leaves an unacceptably high probability of false triggering around once per transmission, whilst a 14 dB threshold gives an acceptable P.F.T. without reducing the maximum range too much. Having set such a transponder threshold and chosen the maximum power in each reply for the battery life available, it then follows that the interrogator power level must exceed the transponder power level by the difference in noise levels less the difference in detection thresholds. Since noise levels at the transponder can vary over as much as 30 decibels it is usually advisable to precede threshold detection by an automatic gain control (a.g.c.) amplifier. At the interrogator a switched gain receiver may be adequate with an attended analogue visual display, but an a.g.c. receiver is again recommended for unattended automatic detection.

With more complex (wide-band) pulse waveforms and matched filter detection, pre-detection clipping simplifies the receiver at the expense of about 2 dB in signal-to-noise performance, which can usually be tolerated. The criterion for threshold level at the receiver is usually different from that at the transponder; having sent one interrogation pulse a single reply is expected and defining maximum range where there is the same probability of correct detection as of false, noise-induced detection, a threshold set at 12 dB is tolerable for this single reply. With the integration from visual displays it is possible to work at much lower signal-to-noise ratios, providing 'n' interrogations and transponder replies are made, though the equivalent improvement in detection threshold is nearer 2 dB per doubling in practice than the theoretical 3 dB per doubling (10 \( \log_{10} n \)). With computer based digital signal processing and detection, the 3 dB per doubling improvement can be approached more closely, but the software to account for an unknown range move-out during the 'n' transmission intervals gets cumbersome. It may be thought that instead of 'n' transmissions the transponder power level should be increased 'n'-fold and the same energy expended in a single pulse; this has the benefit of simplicity, but since the transponder is not always at maximum range there will be a waste of energy overall. The flexibility to choose 'n' depending upon the signal-to-noise situation can thus be valuable.

When more than one object carrying a transponder is to be located, some means of identifying the transponder arrivals is necessary. This can be by transpond frequency diversity (TFD), by interrogate frequency diversity (IFD) or by pulse length or pulse diversity (PD). PD is popular, but performance can suffer due to multipaths and pulse spreading in the bounded medium. IFD requires only a single channel receiver, but does not allow simultaneous ranging to all transponders: TFD, which requires a multi-channel receiver, was chosen by I.O.S. to simultaneously track up to 18 medium range (50 km) neutrally buoyant floats [18] from one ship, advantage being taken of the ship's speed to move from one position to another to interrogate the floats before they have moved significantly. In addition to floats, two or more TFD channels have been allocated to bottom transponders on a long baseline to navigate the ship. As an alternative to long ship runs between fixes, one or more remote slave interrogators, triggered by TFD from the master interrogator on the ship, have been used successfully.
5. THE EFFECTS OF AN IMPERFECT ENVIRONMENT ON PERFORMANCE

Errors in position determinations underwater arise from errors in timing, in sound velocity and from operational procedures, including the surveying-in phase, discussed in Section 6. The sea surface and seabed boundaries reflect signals, generating confusing multipaths.

5.1. Timing errors

Timing errors due to the master clock should not be significant with the high stability and precision of available oscillators. However, the presence of noise introduces variance into the transponder and the receiver detection processes. There are reviews of ambient underwater noise energy spectral levels [19], though the prediction of levels can be very difficult in areas of shipping or industrial activity. The system may have to cope with variations in noise levels approaching 40 dB. There is a dearth of carefully documented experience on the effect of real noise on both false indications and timing errors. Impulsive noise may be particularly troublesome for simple mono-frequency pulse detection whilst the more complex (wide-band) waveforms may be less susceptible for the same pulse energy. The nominal resolution of all except the CW beacon system is given by $B^{-1}$, but it is possible to do better than this in good signal-to-noise conditions. The simplest analysis using Gaussian noise indicates that r.m.s. timing errors will be around $0.7 B^{-1} (S/N)^{-1/2}$, when $S/N > 1$, where $S/N$ is the signal-to-noise energy ratio in the signal band $B$. This suggests that timing errors at the limiting detection thresholds discussed earlier should amount to about one seventh of the nominal resolution. Towards maximum range the timing errors increase, but will not exceed $B^{-1}$ whilst the pulse remains detectable.

5.2. Sound speed structure

There are three principal effects arising from variations in sound speed: the most obvious is the calibration of range from travel time with seasonal variations in the mean speed and with spatial variations, especially with depth, giving rise to a range dependent speed; spatial variation causes sound refraction which allows multiple path arrivals at different times; the formation of shadow zones is also a result of spatial variations in sound speed. Short term temporal variations can also be important especially over long horizontal paths.

The total range of variations in sound speed in the ocean is about 5 %, but this can be reduced to less than 1 % by knowledge of temperature (T) which is often the dominating factor, though sound speed (C) is also a function of salinity (S) and pressure or depth (D). Polynomial equations linking these have been available for many years, but it should be added that these are still being refined.
by current absolute determinations [20] to 1 part in $10^5$ over the practical ranges of T, S and D. For example, Del Grossos equation differs from the older equation of Wilson by around 0.6 m/s at surface pressures, whilst the pressure coefficients at depth of Wilson fit some data better than those of Del Grosso. Measurements of sound speed with a resolution of 1 part in $10^4$ using a single- around 'Velocimeter' are convenient in the field, but the instrument must be calibrated under controlled and known S, T, D, conditions against one of the formulae. For relative, short term navigation an exact knowledge of the mean speed may not be too important, but for absolute positioning and long term repeatability it is clearly necessary.

Typical deep water sound speed and temperature profiles are shown in Fig. 2. For ray paths from the vertical out to about 50°, ray paths are straight lines, or nearly straight, and the appropriate slant range speed to use is the harmonic mean sound speed (H.M.S.S.) between source and receiver depth [21]. Also plotted on Fig. 2 is the surface to depth H.M.S.S. The near surface part of the profile is the most variable from day to day and seasonally. The depth of the sound velocity minimum tends to reduce with latitude, so that an extreme North Atlantic winter profile might have a monotonically increasing speed down from the surface. Correct use of H.M.S.S. for steep paths should enable errors in sound speed to be kept below 0.02%.

![Fig. 2. - Depth profiles of Temperature, T°C and Sound Speed, C m/s. Also the Harmonic Mean Sound Speed, H, from surface to depth.](image-url)
Fig. 3. – Measured and predicted phase fluctuations and recorded amplitude samples from a 7.8 km one-way transmission experiment in shallow water at 2 kHz. (From Reference 22). Note the tidal stream component and the larger drift due to seasonal cooling.

For nearly horizontal ray paths, generalities are more difficult to make. As a long range example of one-way travel path variability in shallow water, Fig. 3, taken from Westen et al. [22] demonstrates the importance of the tidal stream component and the seasonal temperature in well mixed isothermal water. Over the 7.8 km path at 2 kHz the 100 radians increase of phase represents a decrease in sound speed of 0.1596, of which 0.0396 is due to the ±0.5 knot component of the tidal stream. For a different experiment, over 210 km at 406 Hz using Sofar channel propagation in the deep ocean, Spinell et al. [23] reported fluctuations of 100 radians representing a 0.026 % fluctuation in sound speed; in this case the fluctuations were attributed largely to internal waves on the main thermocline. It is important to note that these two CW experiments both demonstrated amplitude fluctuations of between 20 dB to 30 dB, and with a more noise-like character, whilst the phase stabilities simultaneously were high, despite the sound channelling boundaries being so different in the two cases.

Examples of sound ray propagation for a stratified deep ocean and from one source above and one below the sound speed minimum are given in Fig. 4A.
and B for the profile of Fig. 2. Vertical exaggeration is tenfold and to avoid confusion the traces have been arbitrarily stopped at the surface and bottom boundaries. Out to the first turning point around 25 km there is only one direct path and the harmonic mean between source and receiver depths is quite a good speed to use. Daintith [24] has described an improved method using the statistics, mean and variance of the speed profile, valid to the first turning point. At longer ranges multiple refracted paths to the same reception point are observed. The ray which has travelled furthest arrives first because the increased speed more than compensates for the extra distance. For propagation from a source at the axis (sound speed minimum) to a receiver at the same depth the last arrival travels at the sound minimum speed. The actual range/arrival time relationships for all refracted paths can be pre-computed, the small differences being indistinguishable in Fig. 4, and in principle it does not matter which is chosen. In

Fig. 4. Ray diagrams for sound sources at depths of 500 m and 3 000 m near 28°N, 69°W. Each ray is marked at 1 second intervals of travel time (from Reference 18).
practice the presence of later reflected arrivals, generally of weaker strength, often
biasses preference to first arrivals. The use of a constant speed value near the
profile minimum for all ranges may be convenient for first order calculations but
does lead to ranging errors around 1 \% [18].

When the sound increases with depth from the sea surface a near surface

\[ \text{DEPTHS} \]

\[ \text{AFI} \]

\[ \text{IRY} \]

\[ \text{20 25 30 35 40 45 50 55 60 65 70} \]

\[ \text{ Overall} \]

\[ \% \]

\[ \text{NO DATA} \]

\[ \text{HORIZONTAL RANGE INTERVALS (Km)} \]

\[ \text{Fig 5. - Percentage success of interrogations from a source array at 500 m and at}
\]

\[ \text{3000 m to transponder floats at 500 m, 1500 m, 3000 m and 4000 m as a function of}
\]

\[ \text{range (from Reference 18).} \]
duct is formed, propagation loss being very dependent on sea state. In this case the surface speed should be used.

The existence of quiet or shadow zones can be inferred from areas in Fig. 4 where rays are sparse or absent; this has been confirmed, Fig. 5 showing the percentage success of interrogation to transponders at a variety of depths and ranges [18]. Below the main thermocline the ocean tends to become isothermal and the principal depth dependence of sound speed amounts to 0.018 m/s per metre. Sound rays in the lower ocean therefore are concave upwards. This means that a sound source close to a flat seabed cannot be heard on the seabed beyond a certain critical range, given approximately by $460 (h)^{1/2}$ metres where $h$ is the height of the source above the bottom in metres. This is the reason transponder beacons are buoyed up as much as 30 to 100 m; calculations indicate that the positional errors due to lean-over in bottom currents around 0.2 m/s are a few per cent of $h$. As a comparison, the line of sight range above the ocean surface is approximately $3600 (h)^{1/2}$ metres, or put another way, the deep ocean sound refraction distortion makes the seabed horizon appear as though the earth was $1/60$ th of its real size. Above the main thermocline sound speed gradients ($g$) may be larger, the arcs tighter, thus reducing by $(g^{-1/2})$ the limiting ranges to a surface shadow zone from a lower depth.

To obtain really long ranges, the Sofar channel must be used, avoiding reliance on surface reflections. The source depth must therefore be between the surface and that depth below the axis with the same sound speed as the surface. Whilst the lower refraction cycle is reliable in the absence of topographic obstruction, the upper refraction cycle is very dependent on the upper ocean stability. The Sofar channel is insonified more fully as the source starting speed gets closer to the axis speed.

5.3. Boundary reflections

Reflections from boundaries are most troublesome when they overlap the direct path signals [11]. If source or receiver are within one range pulse resolution ($\delta R$) of the boundary, overlap will occur at all separations. Overlap will be avoided at all ranges if

$$\frac{2h_1 h_2}{R_{\text{max}}} > \delta R,$$

where $h_1$ and $h_2$ are source and receiver separations from the boundary. If $R_{\text{max}} = 10^3 \times \delta R$ and if $h_1 = h_2 = h$, then $h > 23 \times \delta R$ to avoid overlap. For $\delta R < h < 23 \times \delta R$ there will be overlap of pulses to some degree from the maximum range inwards. Though there is generally some loss on reflection from the seabed, gross topographic features can prevent accurate prediction of arrival time of reflected pulses and it may be difficult to distinguish them from totally refracted arrivals. Mixed reflected/refracted arrivals are also present at times.
6. SURVEYING-IN AND OPERATIONAL PROCEDURES

Whether a source location or transponder location method is used the coordinates of hydrophones or transponders must be determined either relative to each other or in an absolute way referenced to another frame. Positioning the network of hydrophones is an important phase preceding operations. Sometimes this is straightforward as with drillship hydrophones, or is known from geodetically surveyed stations as with Sofar stations. In many cases a source such as a pinger placed in a few known positions can be used to calibrate the net, as demonstrated by the whale positioning example quoted earlier. However, more generally in a three dimensional net of $H$ hydrophones each transmission from an unknown position yields $(H - 1)$ range-differences, so $N$ different independent positions yields $N(H - 1)$ range-differences whilst the number of unknown coordinates is $3H + 3N$; but one position must be defined to set a co-ordinate origin, so that the problem is solvable providing

$$N(H - 1) > 3(H + N - 1) \text{ or } N > \frac{3(H - 1)}{(H - 4)}$$

which means $H$ must be 5 or more and the minimum value of $N$ must be between 12 and 4. This minimum value for $H$ may be reduced to 4 and the minimum number of $N$ to between 6 and 3 if all depths are known. In two dimensions only the minimum value of $H$ is 3, so this technique needs at least one known position in addition to the reference position and frequently this is obtained by placing the pinger at one or more of the hydrophone locations. With all these schemes, more than the minimum number of hydrophones $H$ and of locations $N$, can be used with advantage to improve confidence in the shape of the network, but it is important to note that the orientation of the network in azimuth is unknown. To know this, another reference frame such as an optical or radar fix on the land or a radio navaid or satellite fix for two or more of the hydrophone or test pinger positions are required. Sometimes an approximate orientation from the vessel course between successive positions is adequate.

Similar conditions apply to the surveying-in of a transponder net with the difference that $T$ transponders generate $T$ ranges for each of $I$ interrogator positions so that $I > \frac{3(T - 1)}{(T - 3)}$. Knowing depths, and allowing a two-sided ambiguity, a minimum of two transponders can suffice. Using a number of $I$ interrogator positions in excess of the minimum but with some errors, programmes have been written to derive least mean square error estimates of the network geometry [21], [25], [26]. Other techniques include crossing baselines and obtaining the minimum sum of the distances to the transponders, or more complicated steaming patterns to obtain adequate data in the shortest time. A method used successfully by I.O.S. is as follows: the transponder positions are initially assumed to be at the known ship position when deployed overside; then, at each new satellite fix for the ship carrying the interrogator, an acoustic fix by transponders is obtained simultaneously. The transponder positions are then iteratively adjusted to minimise the sum of the squared differences between satellite fixes and transponder fixes; finally the individual Northing and Easting differences
Fig. 6. - Differences in Northings and Eastings between acoustic and Transit Satellite fixes. The dashed circle represents the r.m.s. radial difference of 360 m. Note the bad satellite fixes marked with Day No./Time which are well separated from the major distributions shown in histogram form.

may be plotted as in the example of Fig. 6, from a two transponder net of baseline 10.697 km in water 4,350 metres deep. The r.m.s. radial error of 360 metres encloses 34 out of the 44 fixes; from the distributions the five labelled fix differences are seen to be larger than is reasonable. From other observations at similar (7-20 km) ranges using high resolution automatic timing, the r.m.s. short term acoustic ranging variation was 7 m, due, it is believed, to sound speed fluctuations caused by internal waves, plus perhaps a small component due to noise induced timing errors. Also, in good LORAN C areas, tighter difference plots are obtained, so that one concludes that the five differences are excessive because they are bad satellite fixes, though they passed other quality control checks. Removing these five reduces r.m.s. radial error \( \delta R \) to 253 metres, shifting the mean 75 m to the South and 39 m to the East.

The expected r.m.s. ranging error \( (\delta R) \) can be partitioned according to

\[
(\delta R)^2 = (\delta R_s)^2 + (\delta R_c)^2 + (\delta R_t)^2
\]

where the subscripts s, c and t refer to surveying-in, sound speed and timing errors respectively. As discussed below, the total r.m.s. positional error will be \( a(\delta R) \) where \( a(>1) \) is a geometrical factor just under 3 for the above example, suggesting \( \delta R = 85 \) m. Great care was taken to use an accurate harmonic mean sound speed though errors in this are very difficult to isolate at distant deep sites. A conservative assessment for \( \delta R_c (14 \) m) and for \( \delta R_t (3 \) m) lead to the conclusion
that for this system $\delta R$ is dominated by $\delta R_s$, and that the internal relative acoustic navigational errors (14 m) were one sixth those due to the geodetic reference needed to make it an absolute system.

Figure 7 summarises in approximate form contributions to ranging errors to be expected with range. At very short ranges refraction is not so important with speed fluctuations as low as $(\delta c/c) = 3 \times 10^{-4}$. At long ranges only Sofar channel propagation is available, the mean sound speed changes but slowly and is thus well known, spatial fluctuations are smoothed out to some extent and $(\delta c/c)$ tends to be about $10^{-3}$. In the medium ranges, allowance for refraction must be made to avoid unnecessarily large errors. Since $\delta t < B^{-1} < \frac{R_{\max}}{c} \times 10^{-3}$, it follows that noise induced errors $(\delta t/t)$ will be of order $10^{-4}$, and generally not significant. The surveying aid error depends upon which system is used, the single fix values indicated being usefully reduced by multiple observations where possible as in the example above.
Having a surveyed network of hydrophones or transponders the system can be employed for its principal purpose of navigation and a new operational geometrical error factor comes into effect. This has been summarised clearly by Lowenstein and Mudie [27] from which the following section is extracted.

Only occasionally will the ranges to a pair of transponders intersect at right angles, thus the uncertainty in position will be magnified by the geometry of the situation. The ratio 'a' between the range error and the total positional error (δP) is dependent only on θ, the angle of intersection of the ranges at the navigated cycle. Writing δP = aδR it can be shown that

\[ a = \frac{\sqrt{2}}{\sin \alpha} \]

By basic geometry the locus of equal intersection angle, α, is the circumferences of two coaxal circles of radius R where:

\[ R = \frac{D}{2 \sin \alpha} \]

and D is the transponder spacing. As the usable navigational area is limited by the maximum range limitation, only the shaded area in Fig. 8 is usable.

The useful area for navigation has been evaluated for different values of ‘a’, the positional precision factor, and D/R_{max}, the normalised transponder spacing (i.e. the actual spacing divided by the maximum range). The results (Fig. 9) are given in terms of η, the area over which the imprecision is less than ‘a’, divided by the sum of the maximum areas covered by each transponder (2πR_{max}^2).

It can be seen that as the transponder spacing is reduced the area available for low precision navigation increases at the expense of area available for high precision navigation. Very close spacing is not recommended as there is a very rapid reduction of useful area with decreasing transponder spacing.

![Diagram](https://via.placeholder.com/150)

**Fig. 8.** Within the shaded areas of the coaxal circles the r.m.s. position error is less than 'a' times the r.m.s. ranging error for location by two transponders, whose maximum ranges are R_{max}, and which are positioned at A and B, a distance D apart, where \( a = \frac{\sqrt{2}}{\sin \alpha} \). Figure to scale for D = 0.3 R_{max} and a = 5 (from Reference 27).
For most work, a spacing of 0.2 of the maximum range is ideal as one will then obtain 85% of the maximum coverage and still have a maximum imprecision of less than 50 times the ranging error. For precise work the transponders would be placed 0.5 of the maximum range apart. A coverage of $\eta = 0.3$ (60% of the maximum) will be achieved in which the maximum imprecision will be 5 times $\delta R$.

If three transponders are placed in an equilateral triangle with each side of length 0.28 $R_{\text{max}}$, then the diameter of the coaxal circles is equal to the maximum range and accurate navigation ($a = 5$) exists over a large nearly symmetrical area equal to $2.7 D^2$.

If an infinite number of transponders are available then the best technique would be to plant a hexagonal lattice in which the transponder spacing is the maximum range. In this system one would be within range of three transponders at all times, and within range of four when crossing from one region to the next.

It is important to recognise that the operational geometry introduces a multiplying factor to the ranging error. There is always, therefore, a pressure to keep the ranging error low with sometimes, in addition, a pressure to employ
more and closely spaced hydrophones or transponders; the extent to which increasing the latter can help to reduce the error factor is limited by economic and logistical considerations such as the extra time required to survey them in.

Algorithms, of varying complexity [11], [25], [26], have been written for computer calculation of fix positions for practically all geometries and combinations of hydrophones, sources and transponders. It is not sufficient just to solve the idealised geometry if a rugged processor is required, since safeguards against false detections, missed detections, incorrect multipath recognition and noise need to be built in. As an example, a negative ranging error may cause two range circles not to intersect, or could prevent three range circles from giving a closed 'cocked' hat, yet a good fix may still be obtained, in these cases by using the intersections of principal axes.

7. FUTURE PROSPECTS FOR DEEP NAVIGATION

The characterisation of underwater positioning systems in this paper may be arbitrary but the characteristics of sound propagation in the deep ocean are fundamental and divide range coverage naturally into three regions. In all regions absolute precision would be improved by better surface navigation aids; the introduction of Navstar within a few years will bring great benefit in underwater work too. Most deep ocean positioning will be able to combine Satellite Navigation at the surface with good short range (<20 km) underwater systems having a resolution of a few metres. The civil use of the medium and long range systems relying entirely on underwater fixing will probably be limited to oceanographic research requirements.

Technological advances may not generate much improvement in systems deployed for short missions, where most overall improvement is likely to arise from more flexible and more rugged operating software. However, there are technological limitations for really long deployments, exceeding two years; improved batteries and non-corroding materials for pressure cases and load bearing are needed in particular. With one notable exception [12] presently employed systems are pulsed; the latter are not optimum from an information theory point of view and designers beginning from scratch should seriously consider CW methods capable of finer resolution; sound speed variations affect CW and pulsed methods alike, whilst multipath problems may be different for the two methods and may need the application in-situ of low power, microprocessor-based signal processing.

REFERENCES


(*) Author's note: The acronym ANGUS in references [16] and [17] refer to different submersible systems.

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ELLIPSOID OF REFERENCE - OLD STYLE

'As to the supposition it self, which our Seamen make, in the allowing but 60 Miles to a degree, I am not ignorant how much this hath been canvased of late years especially, and that the prevailing Opinion hath been that about 70, or upwards, should be allowed. But till I can see some better grounds for the exactness of those trials, that have been made on Land by Mr. Norwood and others, considering the inequality of the Earth's Surface, as well as the obliquity of the way: in their allowing for which I am somewhat doubtful of their measures. Upon the whole matter, I cannot but adhere to the general Sea-calculation, confirmed as to the main by daily experience, till some more certain Estimate shall be made, than those hitherto attempted. For we find our selves, when we sail North or South, to be brought to our intended place, in a time agreeable enough with what we expect upon the usual supposition, making all reasonable allowance, for the little unavoidable deviations East or West: and there seems no reason why the same Estimate should not serve us in crossing the Meridians, which we find so true in Sailing under them. As to this Course of ours to Guam particularly, we should rather increase than shorten our Estimate of the length of it, considering that the easterly Wind and Current being so strong, and bearing therefore our Log after us, as is usual in such cases; should we therefore, in casting up the run of the Log, make allowance for so much space as the Log it self drove after us (which is commonly three or four Miles in 100, in so brisk a gale as this was) we must have reckoned more than 125 degrees; but in this Voyage we made no such allowance: (though it be usual to do it) so that how much soever this Computation of mine exceeds the common Draughts, yet it is of the shortest, according to our Experiment and Calculation.'