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NAUTICAL ASTROFIX IN RECTANGULAR COORDINATES
AND ANALYTICAL SHIFTING (TRANSFER) FOR SPACED OBSERVATIONS

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SUMMARY

In this article we give a useful linear solution to the problem of nautical star fixes by using rectangular coordinates.

An original analytical shifting of the altitude observed in keeping with the course of the ship completes the note.

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With the more widespread use of micro-computers, there is a tendency to solve in a purely analytical way the problem of astronomical fixes in navigation. In this case, it is necessary to carry out, also in an analytical way, the transfer of observations for the course of the ship in the intervals between fixes (spaced observations).

The most common and simplest formula for analytical and easily programmable determination of a nautical star fix is one which is given in rectangular coordinates (VASSALLO, 1980). Rectangular coordinates make possible generalised linear programming. By referring to a trihedral OXYZ, where O is the centre of the celestial sphere, OZ the axis of rotation of the Earth, and OXY the plane of the Equator, we may easily write the equation of the plane of the circle of equal altitude for the star A :

$$XX_A + YY_A + ZZ_A = \sin h_A \qquad (1)$$

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whereby

$$\begin{cases} X_A = \cos\delta_A \cos\hat{P}_A \\ Y_A = \cos\delta_A \sin\hat{P}_A \\ Z_A = \sin\delta_A \end{cases}$$

δ_A = declination of the star observed

$\hat{P}_A$ = $(\alpha_A - t_{01})$ = angle at the pole of the star

α_A = right ascension of the star

t_{01} = local sidereal time of the observer as a function of estimated longitude λ_0

h_A = observed and corrected altitude

With two observed and corrected altitudes at the times t_{01} and t_{02} we will have the following solving system :

$$\left.\begin{aligned} XX_A + YY_A + ZZ_A &= \sin h_A \\ XX_B + YY_B + ZZ_B &= \sin h_B \\ XX_0 + YY_0 + ZZ_0 &= 1 \end{aligned}\right\} \quad (2)$$

where the third equation represents the plane tangent to the celestial sphere at the estimated zenith (Φ_0, λ_0); there is, therefore :

$$\begin{cases} X_0 = \cos\Phi_0 \\ Y_0 = 0 \\ Z_0 = \sin\Phi_0 \end{cases}$$

System (2) solved in relation to X, Y, Z gives :

$$\begin{cases} \hat{P}_x = \tan^{-1}(Y/X) \\ \Phi = \sin^{-1}(Z) \end{cases}$$

where $\hat{P}_x$ is the correction of the estimated longitude to obtain the true longitude.

With three observed and corrected altitudes, we have the solving system :

$$\left.\begin{aligned} XX_A + YY_A + ZZ_A &= \sin h_A \\ XX_B + YY_B + ZZ_B &= \sin h_B \\ XX_c + YY_c + ZZ_c &= \sin h_c \end{aligned}\right\} \quad (3)$$

from which we extract

$$\left.\begin{aligned} \hat{P}_x &= \tan^{-1}(Y/X) \\ \varphi &= \sin^{-1}(Z/K) \end{aligned}\right\} \quad (4)$$

with

$$K = (X^2 + Y^2 + Z^2)^{1/2}$$

The values in (4) obtained from System (3) are independent of any systematic errors in the altitudes observed.

ANALYTICAL TRANSFER OF THE INITIAL OBSERVATION TO THE TIME OF THE LAST OBSERVATION TO ACCOUNT FOR THE DISTANCE m COVERED BY THE OBSERVER IN THE INTERVAL

The course of the ship between one observation and the next results in a difference in $\Delta\Phi$ and $\Delta\lambda$ of the observer. To obtain the transfer of the observation to allow for this movement, we shift the star observed simultaneously with the difference $\Delta\Phi$ and $\Delta\lambda$, so as to maintain unchanged the observed zenithal distance in relation to the ship's new position. The shifting of the star ($\Delta\alpha$ and $\Delta\delta$) in right ascension α and declination δ, is easily obtained with the formulae which, in astronomy give the variations $\Delta\alpha$ and $\Delta\delta$ for the phenomenon of precession in α and δ.

If we take the meridian of the observer as the origin of the spherical coordinates, we can write the formulae of the general precession in α and δ (I.I. MUELLER, 1969) as follows :

$$\left.\begin{aligned} \Delta\alpha &= \Delta\lambda + \Delta\Phi \sin \hat{P} \tan\delta \\ \Delta\delta &= \Delta\Phi \cos \hat{P} \end{aligned}\right\} \tag{5}$$

where $\Delta\alpha$ and $\Delta\delta$ are respectively the corrections to the coordinates of the star A (for instance) before calculating the coefficients X_A, Y_A, Z_A, which we have in systems (2) and (3).

We point out that : $\hat{P}$ = angle at the pole; may be East or West and must be considered always positive; furthermore, we establish :

$$\Delta\lambda \begin{cases} (+)\ \underline{\text{East}} \text{ if the ship is moving East} \\ (-)\ \underline{\text{West}} \text{ if the ship is moving West} \end{cases}$$

$$\delta \ \{ \text{always with its own sign}$$

$$\Delta\Phi \begin{cases} (+) \text{ for ship's movement to the North} \\ (-) \text{ for ship's movement to the South.} \end{cases}$$

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