ON KARUNARATNE'S METHOD
OF CHECKING
HOURLY TIDAL HEIGHTS

by A.S. FRANCO [*]

1. INTRODUCTION

KARUNARATNE's (1980) Harmonic Component Fit Method, HCFM, is in fact a very efficient and inexpensive method of checking hourly tidal heights. It has been in use in Brazil almost since its publication. However some simplifications have been introduced which have proved to be extremely useful. Thus it seems interesting to explain the way HCFM is used in Brazil.

2. THEORETICAL BASIS

Let the tidal height at instant t be expressed in terms of harmonics M_1 to M_Q by

$$\zeta (t - t_0) = \sum_{j=-Q}^{Q} c_j \exp [i\omega_j(t - t_0)] \tag{2a}$$

where

$$c_j = \delta_j R_j \exp(-i r_j) \qquad \delta = \begin{cases} 1 & j = 0 \\ \frac{1}{2} & j \neq 0 \end{cases} \tag{2b}$$

and R_j and r_j are, respectively, the amplitude and the phase at $t = t_0$ of constituent M_j with frequency ω_j. In addition the following convention will be used :

$$\begin{aligned} r_j &> 0 \text{ and } \omega_j > 0 \text{ for } j > 0 \\ r_j &= \omega_j = 0 \qquad \text{for } j = 0 \\ r_j &< 0 \text{ and } \omega_j < 0 \text{ for } j < 0 \end{aligned} \tag{2c}$$

(*) Instituto de Pesquisas Tecnológicas do Estado de São Paulo S.A. - IPT, Caixa Postal 7141, 05508 São Paulo, Brasil.

The problem consists in finding c_j using the values $\zeta(t - t_0)$ for $t \neq t_0$ and computing $\zeta(0)$ in terms of c_j.

From expression (2a), for $t = t_0$ and (2b), we obtain :

$$\zeta(0) = \sum_{j=-Q}^{Q} \delta_j R_j \cos r_j$$

or   (2d)

$$\zeta(0) = \sum_{j=0}^{Q} R_j \cos r_j$$

Thus the problem is completely solved when $R_j \cos r_j$ is known.

3. HOW TO FIND $\zeta(0)$

If we make

$$t - t_0 = k\Delta t \quad \text{with } k = 0, \pm 1, \pm 2, \ldots \tag{3a}$$

expression (2a) can be written as follows :

$$\zeta(k\Delta t) = \sum_{j=-Q}^{Q} c_j \exp(i\omega_j k\Delta t) \tag{3b}$$

If we add the values of $\zeta(k\Delta t)$ symmetrically, with respect to $\zeta(0)$, we can write :

$$\zeta_k + \zeta_{-k} = \sum_{j=-Q}^{Q} c_j [\exp(i\omega_j k\Delta t) + \exp(-i\omega_j k\Delta t)]$$

$$= \sum_{j=-Q}^{Q} c_j [2 \cos(\omega_j k\Delta t)]$$

$$= 2 c_0 + \sum_{j=1}^{Q} (c_j + c_{-j}) [2 \cos(\omega_j k\Delta t)]$$

or, according to (2b)

$$\zeta_k + \zeta_{-k} = \sum_{j=0}^{Q} R_j \cos r_j [2 \cos(\omega_j k\Delta t)]$$

If N is the number of heights before and after $\zeta(0)$ and

$$k = 1, 2, \ldots N$$

the following system can be formed:

$$\{\zeta_k + \zeta_{-k}\} = 2 \parallel \cos(\omega_j k\Delta t)\} \parallel \{R_j \cos r_j\}$$

If $N = Q + 1$, we can solve the system to obtain

$$\{R_j \cos r_j\} = \frac{1}{2} \parallel \cos(\omega_j k\Delta t) \parallel^{-1} \{\zeta_k + \zeta_{-k}\} \tag{3c}$$

If we make

$$\parallel \cos(\omega_j k\Delta t) \parallel^{-1} = A_{jk} \tag{3d}$$

and add all the elements of the vector $\{R_j \cos r_j\}$ and the elements of equal k in matrix A_{jk} in (3c), we obtain from (3c) and (3d)

$$\sum_{j=0}^{Q} R_j \cos r_j = \frac{1}{2} \parallel \sum_{j=0}^{Q} A_{jk} \parallel \{\zeta_k + \zeta_{-k}\} \qquad (k = 1, 2, \ldots Q + 1)$$

or, according to (2d)

$$\zeta(0) = \frac{1}{2} \| \sum_{j=0}^{Q} A_{jk} \| |\zeta_k + \zeta_{-k}|$$ (3e)

where

$$\frac{1}{2} \| \sum_{j=0}^{Q} A_{jk} \|$$

is a row vector which, multiplied by the column vector $|\zeta_k + \zeta_{-k}|$ gives $\zeta(0)$.

4. NOTES ON COMPUTATION

The elements of matrix

$$\| \cos(\omega_j k \Delta t) \|$$

are calculated for

$$\omega_j = j\omega_1$$

where

$$\omega_1 = 14.492052°/h$$

In routine work we use j and $k-1$ from 0 to 6 which permits us to compute $R_j \cos r_j$ for M_0 to M_6. The row vector is:

$$\frac{1}{2} \| \sum_{j=0}^{Q} A_{jk} \| =$$
$$\| 0.90580 \quad -0.66997 \quad 0.39841 \quad -0.18476 \quad 0.06319 \quad -0.01426 \quad 0.00160 \|$$

This row vector works exactly as the former interpolation formulae and gives a fast and accurate check of hourly heights. Some trials have been done with $Q = 4.5$ and 6 and experience showed that best results were obtained for $Q = 6$.

A computer program was prepared to allow for the tolerance and print only the days when doubtful heights occur. A sample of the output can be seen in figure 1.

The upper figures in each double row are the heights computed with (2d) and the lower figures are the observed heights. The first pair of rows give the heights from 0 h to 11 h and the second pair the heights from 12 h to 23 h. The star preceding a lower figure indicates a probable anomalous height. At the end of the second pair of rows appear the number of the day in the year and the year itself.

The figure as given represents the *observed* tide from seven hours before 0 h of day 135 to seven hours of the next day.

It is interesting to note how important the extension of the figure is. In fact we see that, as indicated by the asterisk, the height corresponding to 23 hours would be wrong, according to the adopted tolerance (15 cm). However the figure shows that an anomalous height is not there, but at 2 h of the next day. After the correction of such height no more anomalous heights appeared in day 135.

SANTOS: REGISTRO INICIADO EM 1/5/1972

| 146 | 150 | 177 | 172 | 167 | 148 | 137 | 120 | 99 | 77 | 75 | 79 | | |
| 137 | 158 | 171 | 176 | 165 | 149 | 136 | 121 | 98 | 79 | 73 | 81 | | |

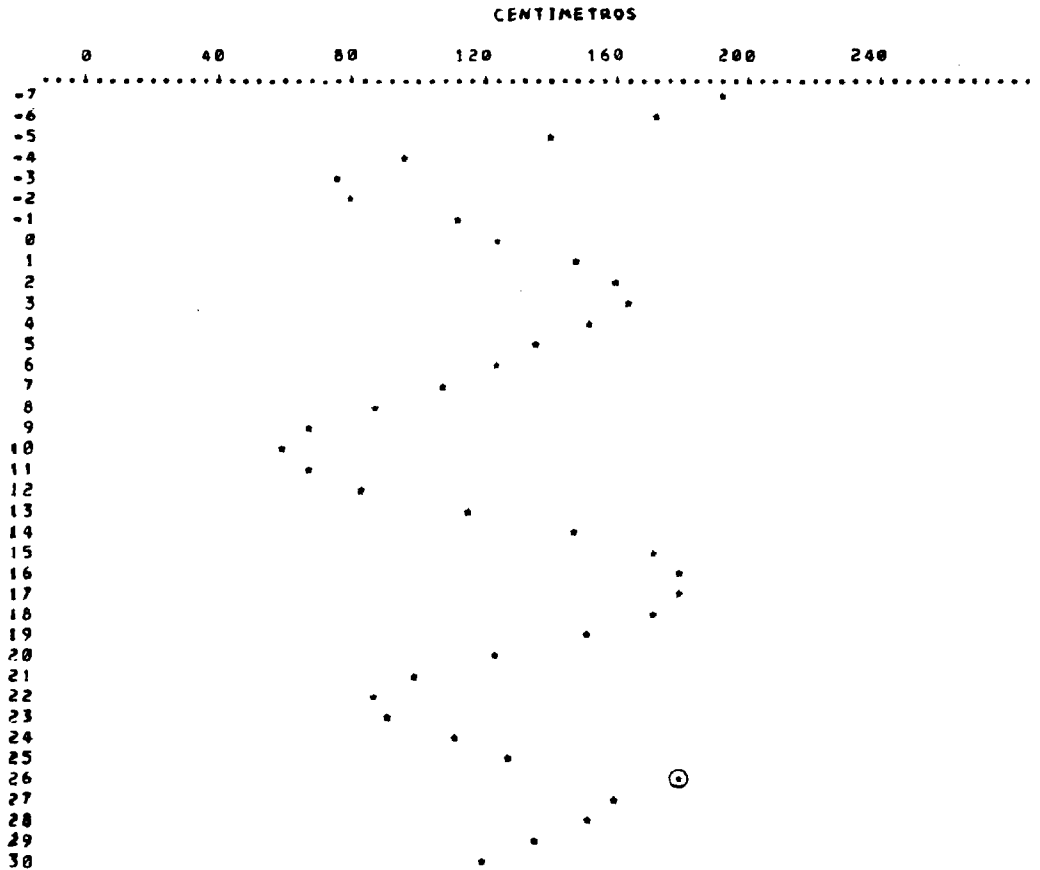| 99 | 126 | 168 | 185 | 191 | 193 | 181 | 165 | 133 | 114 | 92 | 121 | | |
| 97 | 127 | 160 | 183 | 193 | 190 | 183 | 163 | 135 | 110 | 100 | 105 | 1351972 |

FIG. 1. — Computer output.

5. CONCLUSION

The method was tried with mixed tides in southern Brazil as well as the semidiurnal shallow-water tides in the Amazon estuary. The results were good enough to justify adoption of the present version of the KARUNARATNE method.

Acknowledgement

REFERENCES

KARUNARATNE, D.A. (1980): An improved method for smoothing and interpolating hourly sea level data. *Intern. Hydrog. Review*, LVII (1), January, pp. 135-148.

LENNON, G.W. (1965): The treatment of hourly elevations of the tide using an IBM 1620. *Intern. Hydrog. Review*, XLII (2), July, pp. 125-148.