# **ANALYSIS OF FORTNIGHTLY AND MONTHLY TIDES**

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## ABSTRACT

The fortnightly and monthly tidal constituents can be extracted from tidal records if care is taken in the analysis. One year of observations may be insufficient to resolve these constituents from the meteorologically forced background noise. In this paper, I describe a procedure to reduce the background noise, and to determine the amplitude, phase and uncertainties of these constituents. Analysis is performed on records from four ports in western Canada, at a latitude where air pressure changes and winds cause significant sea level changes.

# ANALYSIS OF FORTNIGHTLY AND MONTHLY TIDES

When water level records are analyzed with harmonic analysis schemes to extract tidal constituents, there are several constituents of periods near two weeks and one month which are often included. The four largest are listed in Table 1.

Constituent	Frequency	Period	Equilbrium	Shallow		
	(cycles/day)	(days)	potential "	water tide		
MSm	0.03143	31.81	$-0.00673$	$\lambda_2$ – M <sub>2</sub>		
Mm	0.03629	27.55	$-0.03518$	$N, -M,$		
MSf	0.06773	14.77	$-0.00583$	$S_2 - M_2$		
Mf.	0.07320	13.66	$-0.06663$	$K_2 - M_2, K_1 - O_1$		

Table 1 Fortnightly and monthly tidal constituents

(\*) Canadian Hydrographic Service, Pacific Region, Institute of Ocean Sciences, Sidney, B.C. V8L 4B2, Canada.

 $(**)$  Ref. [2] CARTWRIGHT and EDDEN, 1973.

The equilibrium potential is proportional to the height of the tide in an ocean which is always in equilibrium with the tidal forcing function. However, the tide at any one port depends strongly upon the response of the adjacent sea, which limits the usefulness of the equilibrium potential to comparisons between constituents of similar frequencies. In Table 1, the equilibrium potential was the basis of elimination of smaller long period constituents not listed. The equilibrium potential of  $M<sub>2</sub>$ , the largest constituent, is 0.63, a factor of ten larger than Mf, the strongest long period tide.

A standard procedure for analysis of ocean tides is to run a harmonic analysis program on one year of an hourly height time series of sea levels. Harmonic analysis is a least squares fit of cosine functions with periods equal to the main equilibrium tidal periods, and to the appropriate sum and difference periods for the shallow water tides. A recent scheme, written by FOREMAN  $(1977)$  [5] has 45 astronomical constituents and 24 shallow water constituents, with options for up to 77 additional shallow water constituents. The optimal length of record is one year; the results can be improved by running separate analyses on successive years of data and averaging the computed constituents.

Results of 19 one-year analyses at Victoria and 14 at each of Prince Rupert, Tofino and Vancouver are shown in Table 2. Variations from year to year in the long period tides are very large in both amplitude and phase. Vector averages and standard deviations are computed according to the following formulae. If the amplitude is  $A_i$  and the phase is  $\theta_i$ , then

$$
\bar{A}_x = \frac{1}{N} \sum_{j=1}^N A_j \cos \theta_j
$$
  

$$
\bar{A}_y = \frac{1}{N} \sum_{j=1}^N A_j \sin \theta_j
$$
  

$$
\bar{A} = (\bar{A}_{x^2} + \bar{A}_{y^2})^{1/2}
$$
  

$$
\sigma = \left[ \frac{\sum_{i=1}^N (A_j \cos \theta_j - \bar{A}_{x^2})^2 + \sum_{j=1}^N (A_j \sin \theta_j - \bar{A}_{y^2})^2}{N-1} \right]^{1/2}
$$
  

$$
\theta = \tan^{-1} (\bar{A}_{y} / \bar{A}_{y^2})
$$

where N is 19 for Victoria, 14 for the remaining ports.

Table 2

V ector average amplitudes, phases and standard deviations of four long period tides at British Columbia ports. Amplitudes and standard deviations are in millimetres, G is the Greenwich epoch expressed in degrees

	Mf			MSf		Mm			MSm			
	lA mp	G	$\sigma$	Amp	$\mathbf{G}$		$\sigma$  Amp  G		$\sigma$	Amp	$\mathbf G$	σ
Victoria   8.7   169   16.9   8.3   200   20.7   8.3   180   23.8   4.7   288   22.9   Vancouver . 16.8 132 16.4 6.5 186 24.2 3.7 91 28.6 4.1 282 26.5   Prince Rupert. 21.4   179   17.0   3.2   336   23.4   9.2   143   33.2   1.9   320   25.1												

Only for the Mf constituent at several ports is the standard deviation of the amplitude less than the amplitude itself, and for Mf the two are of similar magnitude. These results show that Mm, Mf, MSm, MSf should not be included in the harmonic analysis of one-year tidal records of these ports.

One hopes to employ an analysis scheme which shows the relative contributions of tides and weather to sea level records. This is most easily done with a Fourier analysis covering the entire data record at once. The weather which contam inates the tides has a continuous spectrum, while the tidal signals have line spectra. By choosing longer time series, the noise continuum is spread over more Fourier coefficients, and the amplitude of the tidal lines does not diminish. In theory, then, these tidal constituents can be resolved by choosing a record sufficiently long.

The time series of hourly heights at Victoria, Vancouver, Tofino and Prince Rupert were low passed with an  $A_{24}A_{25}A_{26}$  filter (GODIN, 1972) [6] to remove the diurnal and semi-diurnal tides. The record at Tofino was free of gaps, and this low passed record sufficed to fill gaps in the low passed records at other stations (57 days at Victoria, 108 days at Vancouver, 16 at Prince Rupert). Data points at 12 hour intervals were chosen from the low passed records, and series of 9918 points  $(13.58 \text{ years})$  were selected, starting at 0100 PST January 4, 1963. Mf and Mm dominate the fortnightly and monthly tides, and  $13.58$  years comprise 363.01 cycles of Mf and 179.97 of Mm. Fast Fourier Transforms (FFT) of the series were computed, and the amplitudes of the first 500 coefficients of the Prince Rupert record are plotted in figure 1b.

Records of hourly sea surface air pressures supplied by the Atmospheric Environment Service for Vancouver, Tofino and Prince Rupert airports were similarly analyzed. No gaps existed in these records. The record at Vancouver Airport served to compare with both Victoria and Vancouver tidal records. Fast Fourier transforms of 9918 points of the air pressure records, for the same time period beginning at 0100 PST, January 4, 1963, were computed; the amplitudes for Prince Rupert are plotted in figure la. The units of the sea level records are millimetres, while the air pressure units are tenths of millibars, the two units being equivalent to within 1% for seawater. The amplitudes of sea level fluctuations predominate over the air pressures, particularly at the annual period. Only those tidal lines whose amplitudes clearly rise above the background level can be evaluated and predicted. The 363 rd Fourier coefficient, w hich lies closest to the Mf period, clearly stands out from the neighbouring values, but the 180th coefficent (Mm) does not.

It is possible to use the relationship between sea levels and air pressures to reduce the background noise within which the tidal lines in figure 1b are embedded. Studies of water level changes in the deep sea show the ocean responds largely as an inverted barometer to air pressure changes ( $CRAWFORD$ , RAPATZ and HUGGETT, 1981)[3]. An increase in air pressure of 1 millibar lowers the sea surface by one centimetre. Near shore over continental shelves, the effects of wind are more strongly felt (ADAMS and BUCHWALD, 1969)[1] and sea levels may rise more than one centimetre if the alongshore wind and air pressure act together. If the background signal in the sea level record was due to an inverted barometer response of sea levels, one could add the vector representing the



FIG. 1.  $-$  Amplitude periodogram of a) air pressure, b) sea level and c) adjusted sea level at Prince Rupert. Units are millimetres for sea level and tenths of millibars for air pressure.

amplitude and phase of the spectral coefficients of air pressure to that of sea level. For a purely inverted barometer response, these coefficients are out of phase by  $180<sup>o</sup>$  and cancel upon addition. Along the west coast of North America north of California, and any coast line with an appreciable continental shelf, the effect of the wind is also felt. Because we lack long term measurements of winds over the shelf, one cannot compensate easily for its effect upon sea level  $(C_{\rm RAW}$ . FORD, 1980)[4].

It is possible to improve upon the inverted barometer compensation to reduce background noise in the sea level record. This is done by determining the actual nature of the relation between sea levels and air pressures at each port, by use of the admittance function  $(GODIN, 1976)[7]$  and the coherence. Then one could subtract from the transform values of sea level, plotted in figure 1b, the portion which is coherent with the air pressures, and so reduce the background signal. Because the tidal energy in the air pressures is insignificant at fortnightly and monthly periods, the contribution of the tides is not affected by this adjustment. Therefore the noise level is reduced while the tidal signal is not affected.

The coherence is a measure of the amount of association between two time series over a band of frequencies, while the admittance is the phase difference and ratio of amplitudes of the coherent portion of the two time series. In the



case of sea level and air pressure, an inverted barometer response would give a coherence of 1.0, admittance amplitude of 1.0 and phase of  $180^\circ$ . Coherence is normally denoted by  $\gamma$ .

To determine coherence between the sea level and air pressure time series, the mean and any linear trend present in each of 8,192 data points, beginning 0100 PST 4 January 1963, were removed. A one-tenth cosine bell filter was run over each time series. The values of admittance and phase, plotted in figure  $2$ , were each computed over bands of 64 neighbouring frequencies, and are plotted up to the frequency where the squared coherence falls below  $0.5$ . At the frequencies near the Mm and Mf tides, values of  $\gamma^2$  were between 0.74 and 0.90. The Mf tide in the sea level records has sufficient energy to shift the admittance for the entire spectral band centered at 12.8 days (log frequency  $-1.489$ ) and this band could not be used to form the spectral corrections for sea level data. The Mm tide with lower amplitudes does not appreciably influence the admittance.

The dashed line in figure 2 was assumed to be the spectral relationship between sea level and air pressure, to remove the effect of air pressure from the sea level records. For example, the vector representing the amplitude and phase of Prince Rupert air pressures at the Mm frequency was rotated 198°, multiplied by  $1.39$  (the values of phase and amplitude of the admittance shown in figure 2) and subtracted from the vector of sea level fluctuations at Prince Rupert at the Mm frequency. Similar adjustments to the transform values of sea levels at all four ports at all frequencies were made. The resulting adjusted sea level periodogram at Prince Rupert is plotted in figure 1c. Tidal lines now rise clearly above the background noise. It remains to compute their amplitude and phase.

# **COMPUTATION OF AMPLITUDE AND PHASE OF LONG PERIOD TIDES**

A lthough these tides appear in the amplitude spectra as single lines, they are more precisely a cluster of tidal lines as shown in Table 3, but only the principal coefficients within each group have sufficient amplitude to penetrate the background noise. To compute the amplitude and phase of each tidal line, a time series was generated of the equilibrium tide of all the MSm, Mm, MSf and Mf groups listed in Table 3 for the period January 4, 1963 to July 24, 1976 and this series was subjected to the same Fourier analysis. A hydrographer with access to the admittance program for tides (ZETLER, CARTWRIGHT and BERKMAN, 1979)[10] could generate the time series of equilibrium long period tides to achieve the same results.

Although the principal tides of Mf and Mm  $(Mf_4$  and Mm<sub>4</sub>) are very close to pure harm onics of the 13.58 year time series, the neighbouring tidal lines are not. They differ in frequency from the principals by multiples of one cycle in  $18.61$  years and/or one cycle in  $8.85$  years, the periods of rotation of the lunar node and lunar perigee respectively. The num ber of cycles difference is given by the fourth and fifth Doodson numbers in Table 3. Because these neighbouring

#### **ANALYSIS OF TIDES**

# Expansion of MSm, Mm, MSf, Mf tidal constituents Constituent Doodson number | Equilibrium potential  $MSm$ ,  $\begin{bmatrix} 0 & 1 & -2 & -1 & -2 & 0 \\ 0 & 1 & -2 & -1 & -1 & 0 \end{bmatrix}$  0.00002<br>  $MSm$ ,  $\begin{bmatrix} 0 & 1 & -2 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$  $MSm_2$   $\begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 1 & -2 & -1 & -1 & 0 & 0.00007 \ \hline MSm_3 & 0 & 1 & -2 & 1 & -1 & 0 & 0.00048 \ \hline \end{array}$  $MSm_3$   $\begin{bmatrix} 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$  $-0.00673$ MSm,  $\begin{vmatrix} 0 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix}$  1 0 0.00044  $Mm_1$  0 1 0 -1 -2 0 0.00003<br>  $Mm_2$  0 1 0 -1 -1 0 0.00231  $\text{Mm}_2$  0 1 0 - 1 - 1 0<br>  $\text{Mm}_3$  0 1 0 - 1 0 0  $\text{Mm}_3$   $\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \end{bmatrix}$   $\begin{bmatrix} -0.03518 \\ 0.00229 \end{bmatrix}$  $Mm_1$  0 1 0 - 1 1 0 0.00229<br>  $Mm_1$  0 1 0 1 0 0 0.00188  $\mathsf{Mm}_\mathfrak{z}$  |  $\mathsf{0}\,\,\mathsf{1}\,\,\mathsf{0}\,\,\mathsf{0}\,\,\mathsf{0}\,\,\mathsf{1}\,\,\mathsf{0}\,\,\mathsf{0}\,\,\mathsf{0}\,\,\mathsf{1}\,\,\mathsf{0}\,\,\mathsf{0}\,\mathsf{0}\,\mathsf{1}\,\mathsf{8}\,\mathsf{8}$ Mm,  $\begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$  0.00077 M m , 0 1 0 1 2 0 0.00021 MSf<sub>1</sub>  $\begin{vmatrix} 0 & 2 & -2 \\ 0 & 2 & -2 \end{vmatrix}$  0 - 1 0 - 0.00042<br>MSf<sub>1</sub>  $\begin{vmatrix} 0 & 2 & -2 \\ 0 & 2 & -2 \end{vmatrix}$  0 0 0 - 0.00583 MSf<sub>2</sub>,  $\begin{vmatrix} 0 & 2 & -2 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{vmatrix}$   $\begin{vmatrix} -0.00583 \\ -0.00583 \end{vmatrix}$  $MSF_1$  | 0 2 - 2 0 1 0 | 0.00038 MSf,  $\begin{vmatrix} 0 & 2 & -2 \\ 2 & 2 & 0 \\ 0 & 0 & 0 \end{vmatrix}$  0.00004 Mf<sub>1</sub>  $\begin{vmatrix} 0 & 2 & 0 & -2 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$  = 0.00015  $0 \quad 2 \quad 0 \quad -2 \quad 0 \quad 0 \quad -0.00288$  $Mf_3$   $0 2 0 -2 10$  0.00019<br> $Mf_4$   $0 2 0 0 0 0$  -0.06663  $M f_4$  | 0 2 0 0 0 0 0 | -0.06663 Mf<sub>s</sub>  $\begin{vmatrix} 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 &$  $M f_6$   $\begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 2 & 0 & 0 & 2 & 0 & -0.00258 \\ M f_7 & 0 & 2 & 0 & 0 & 3 & 0 & 0.00006 \hline \end{array}$ Mf<sub>7</sub>  $\begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 2 & 0 & 0 & 3 & 0 & 0.00006 \hline \end{array}$



lines are not exact harmonics of the data record, when a Fourier transform is conducted some of their energy will "leak" into the main constituent and either augm ent or diminish its amplitude, depending upon the relative phases of the two.

If it is assumed that the response of the ocean to the equilibrium tidal forcing is uniform across the cluster of tidal lines forming each of the groups, then the Fourier transform values of the equilibrium tide will accurately represent the total relative tidal contribution at each Fourier frequency.

Consider the Mf group. The principal line has equilibrium amplitude  $0.06663$ , and the amplitude of Fourier coefficient (F.C.) 363 of the equilibrium tide is  $0.07466$ . The amplitude of F.C. 363 of the observed tide must be reduced by the ratio  $0.06663/0.07466$  to produce the amplitude of the Mf<sub>4</sub> constituent alone. Most of the modulation of  $M f_4$  is by  $M f_5$ , a tidal line differing in frequency by one cycle in 18.61 years, having an amplitude of  $0.41$  of Mf<sub>4</sub>. The phase lag of  $Mf_4$  (referred to as G in harmonic analysis notation) is the difference

in phase between  $F.C. 363$  of the observed sea levels, and that of the equilibrium tide.

Background noise introduces uncertainties into the amplitudes and phases, and biases the amplitude upwards, but not the phase. Fourier coefficients closest to the tidal frequencies have amplitudes E, due to contributions from the tidal lines H, and from the noise  $\eta$ , which are related by

$$
\mathbf{E}^2 = \mathbf{H}^2 + \eta^2
$$

Values of  $\eta^2$  can be estimated from neighbouring Fourier coefficients. Here, averages of  $\eta^2$  over 17 to 19 coefficients (away from satellite tidal lines) were computed. Unbiased values H are given by

$$
H=(E^2-\eta^2)^{1/2}
$$

The uncertainty in amplitude is given by WUNSCH (1967)[9] as  $\eta^2$  for one standard deviation. WUNSCH approximated a formula for phase error given by MIDDLETON  $(1948)[8]$  and WUNSCH's phase uncertainties were applied to the British Columbia constituents.

The total expected error is the sum of :

- 1) error in admittance amplitude and phase used to correct the sea level spectra ;
- 2) uncertainty due to residual background noise, noted above.

The former is given by GODIN  $(1976)[7]$  as :

$$
e(\omega) = \left[ \left( \frac{1 - \gamma^2(\omega)}{\gamma^2(\omega)} \right) \left( \frac{1}{(1 - P)^{1/n}} - 1 \right) \right]^{-1/2}
$$

where e is the error,  $\omega$  is the frequency,  $\gamma$  is the coherence,  $\eta$  is the number of Fourier coefficients in the band average, and P is the probability that

$$
|Z| - e \le |Z'| \le |Z| + e
$$

$$
Arg (Z - e) \leq Arg Z' \leq Arg (Z + e)
$$

where Z' is the computed estimate of the true complex admittance Z. For a confidence of one standard deviation, P is 0.68.

At all four ports, the minimum value of  $\gamma$  surrounding the Mf and Mm frequencies is 0.86 giving

$$
e = 0.07 \text{ for } |Z|
$$
  
= 4° for arg Z

The estimated values of amplitude and phase for the fortnightly and monthly tides, together with the rms average of the uncertainties are listed in Tabl

Mf and Mm have been treated as gravitationally forced tides, not shallow water tides, an assumption which is not strictly true. To examine the two effects, we can compare the relative amplitudes of the equilibrium potential of Mf and MSf  $(= 11.4)$  with the observed relative amplitudes of these tides  $(= 2.7 \text{ at } 1.4)$ Tofino, 3.9 at Prince Rupert). The relative enhancement of MSf at these two ports is likely due to shallow water effects in MSf. Table 1 shows the constituents which interact to generate shallow water tides. The expected strengths should be in proportion to the amplitude of the constituent interacting with  $M_2$ , and for Tofino these are

MSS  
\n
$$
\begin{array}{lll}\n & 1.0 & (S_2 - M_2) \\
Mm & 0.73 & (N_2 - M_2) \\
Mf & 0.27 & (K_2 - M_2) \\
0.03 & (\lambda_2 - M_2)\n\end{array}
$$
\n(1.0 (S<sub>2</sub> - M<sub>2</sub>)

If we take the amplitude of MSf at Tofino and Prince Rupert as entirely due to shallow water effects, then the expected shallow water Mf tide is  $0.27$  of MSf, equal to 1.5 mm in amplitude, attributed to the  $K_2 - M_2$  interaction. It is a contribution one tenth as strong as the observed Mf, which can then be assigned to direct gravitational forcing.

Both the direct and shallow water Mf tides have the same frequency, so the assignment of the observed tide to either source matters only for modulation of the Mf tide, which is controlled by  $Mf<sub>5</sub>$  for direct gravitational forcing and by the modulation of  $M_2$  and  $K_1$  for shallow water forcing. Because the direct forcing dominates,  $Mf_s$  will dominate the modulation.

By similar reasoning, one can see that the expected shallow water amplitude of Mm is  $0.73$  of MSf, which is 4 mm, an amplitude slightly smaller than that observed at three of the four ports. Again, the origin of a tidal line affects only the modulation. The nodal modulations of Mm, N<sub>2</sub> and M, are  $\pm 13\%$ ,  $\pm 4\%$ ,  $\pm$  4%, respectively; the expected error due to modulation will then be no more than 14% . This error has not been included in the uncertainties in Table 4. The results show that Tofino and Prince Rupert are similar in behavior of Mf and Mm. The Mf amplitude is significantly lower at Victoria, and the Mf phase is less at Vancouver than found at the remaining three stations.

The phase difference of Mf between Vancouver and the other three ports is large, even if the effect of shallow water terms is included. The phase difference



# **Table 4** Amplitudes and phases of fortnightly and monthly tides

of sea level fluctuations between Tofino and Vancouver at frequencies near Mf, in other words, the meteorologically forced portion of the record, indicates a  $4^\circ$ shift, equivalent to four hours, far short of the observed Mf phase difference of 43° between Tofino and Vancouver. Any effect due to a local response of the Strait of Georgia or Vancouver Harbour which could affect the phase of the Mf tide should also affect the sea levels at neighbouring frequencies, yet the two behave in a very different way, for which we have no explanation.

These results show that although these four long period tides should not be included in a one-year harmonic analysis of a port, a vector average of Mf and M m over a longer time period will give better values of amplitude and phase for these two constituents.

## **CONCLUSIONS**

A comparison of the amplitudes and phases in Table 4 with the vector averages found in Table 2 shows that most agree to within the errors noted in Table 4, although the amplitudes of the MSf tides at Prince Rupert and Tofino given by the harmonic analyses are far out of line. The high standard deviations noted in Table 2 indicate that none of the four constituents should be determined from a one-year analysis. Averaged values of amplitudes and phases from successive one-year harmonic analyses on many years of data are more accurate.

It is possible to determine from the Fast Fourier Transform values whether a record is sufficiently long to resolve a particular constituent. Figure lb indicates that the Mf tidal line is sufficiently strong to penetrate the background noise, but that Mm is not. In such a case the Mm constituent cannot be determined with any reliability. W ith large peaks in the background signal surrounding the smaller peak at the Mm frequency, in figure 1 b, it is not possible to assign the source of this smaller peak to the Mm tide or to background noise. A better value for this constituent is derived by subtracting the portion of the background signal which can be attributed to meteorological forcing. The Fourier transform values of the residual signal left after meteorological effects are removed, illustrated in figure 1 c, indicate that the Mm constituent rises above the noise level and can be determined with confidence.

Figures 1b and 1c illustrate the two advantages of the FFT analyses. The ability to distinguish tidal signal from background noise is easily seen in a plot of transform amplitudes. The removal of background meteorological noise from the sea level record is best done through the admittance function computed from the FFT values. Once the analysis reaches the state illustrated by figure 1c one can proceed through the analysis described in this paper, or compute the corresponding time series by running an inverse FFT on the coefficients to obtain a clean time series, on which harmonic analysis can be performed. The two approaches should give similar results, provided the satellite constituents are handled in a similar manner.

The principal difference between the results listed in Tables 2 and 4 is in the treatment of the background noise. That the phase of the Mm constituent for

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Prince Rupert changes by only  $7^\circ$  between these two tables indicates that the major contributor to the Fourier transform value in figure lb closest to the Mm tide is the Mm tide itself. Note that without computing the cleaned periodogram, illustrated in figure lc, it would not be possible to determine the Mm constituent with such accurate confidence limits.

A lthough the procedure described in this paper is lengthy, it may not be necessary to undertake all of it. A long time series in a less noisy region may give accurate values of the long period constituents w ithout the need to reduce the background noise. A simple FFT of the time series could determine such a low background noise level.

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### CHAPTER 43

The bay which (leads) to Barugaza being narrow, it is difficult for those coming in from the sea to enter, whether they happen to come by the right or left (passage), though the latter approach is better. For on the right hand near the mouth of the bay there lies a rough and rocky strip called Herone, opposite the village of Kammoni; and on the left, opposite this, the headland before A stakapra called Papikë, w here there is a bad anchorage ow ing to the current w hich surges round it and because the anchor-cables are cut by the roughness and rockiness of the sea-bottom. And even if one gets through into the bay, the mouth of the river by Barugaza is difficult to find because the country is low, and nothing can be observed with certainty till one is nearer; and even when it is found, the entrance is dangerous because of the shoals in the river round about it.

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Because of this, the royal fishermen of the district round the entrance go up with fully-manned long ships called trappaga and kotumba as far as Surastrene to meet (incoming ships) which they pilot to Barugaza. For they lead them straight from the mouth of the bay with (their own) crews through the shoals and tow them to berths already appointed, taking them up when the tide begins to rise, and at high tide mooring them at their berths and basins. The basins are the deepest parts of the river up to Barugaza, this being on the river upstream, about 300 stades from the mouth.

### CHAPTER 45

The whole land of India has a great many rivers, and there is a great ebb and flow of tides, the high tides increasing at new moon and at full moon for three days, lessening during the intervening periods of the moon. At Barugaza the (alternation) is much greater, so that of a sudden the sea-bottom can be seen and parts of the land are dry where a little before (ships) were sailing; and when the tide comes in from the sea the water of the rivers is forced back more strongly than in the normal flow, for many stades.

### **CHAPTER 46**

For this reason, the approach and departure of ships is dangerous for the inexperienced who are coming into the mart for the first time. Because the violent movement (of the water) when the tide is already rising cannot be withstood, and anchors do not hold, ships are caught by its force and are turned sideways by the violence of the current, driven on to the shoals, and wrecked. The smaller (boats) indeed are capsized. Those that have turned into the creeks during the ebb of the tide, unless they are propped upright, are filled with water from the first head of the current when the rising tide suddenly returns. For the sea water comes in with such violence at the new moon, especially if the flood is at night, that, even if (a ship) begins its entry when the sea is still calm, all at once there is borne in from the mouth (of the bay a sound like) the shouting of an army heard from afar, and immediately the sea rushes in over the shoals with a hissing roar.

> Extract from "The Periplus of the Erythraean Sea", translated from the Greek by G.W.B. HUNTINGFORD and published by the Hakluyt Society of London (British Library, Great Russell Street, London, WC1B 3DG) in 1980. This work, one of the earliest known Pilot books, was probably compiled by a Greek maritime trader in the first or second century A.D. The chapters quoted refer to the entrance to the Narmada River in the Gulf of Khambhat (Cambay) (see Indian Chart No. 254).