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THE ADVANTAGES OF STEREOGRAPHIC PROJECTION: CHARTS 2812 AND 2813 OF THE "DEUTSCHES HYDROGRAPHISCHES INSTITUT" (DHI)

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1. CONTENTS AND PURPOSE

Charts 2812 and 2813 show the Northern and Southern hemispheres of the Earth respectively in polar stereographic projection, the chart face being restricted to the approximate coastlines. The charts may be used for global representations of all kinds, for example for hydrographic and meteorological purposes. The peculiarity of stereographic projection is, however, shown to particular advantage if it is intended to represent circles, since circles on the Earth's surface also appear on the chart as circles (or straight lines) if it is permissible to regard Earth approximatively as a sphere. This is to be taken as a presupposition for the following remarks. Furthermore, all lines on the chart intersect at the same angle as in nature: the projection is "conformal".

Automation was employed in producing the charts. The coastline data are stored in digital form and may be recalled for any desired projection and for any scale.

2. STEREOGRAPHIC PROJECTION

The Earth's surface is projected from a point of projection located on it onto a plane of projection in contact with the Earth's surface at a point opposite the point of projection. In polar stereographic projection, the point of contact is one geographical pole (the North Pole on chart 2812), and the point of projection

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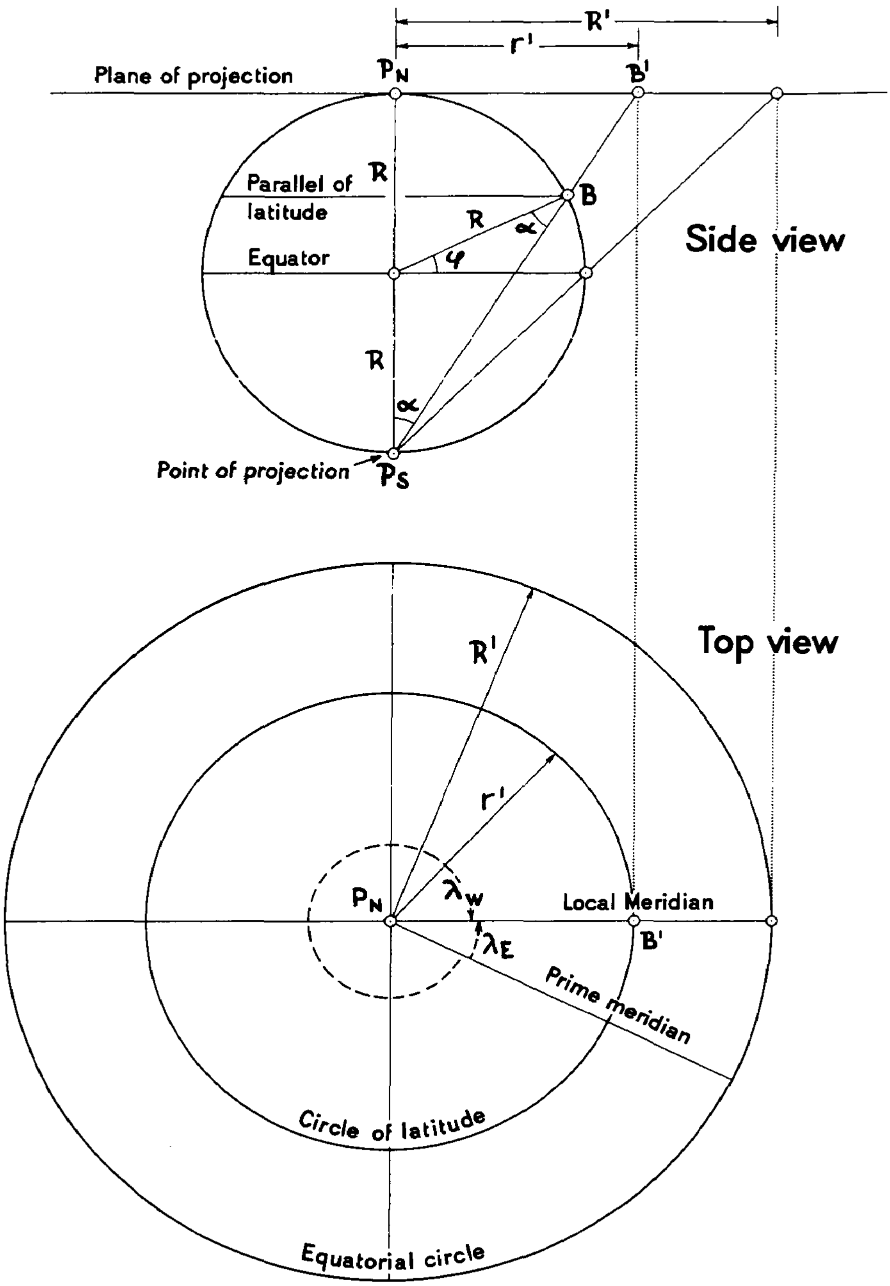

FIG. 1. – Projection of the surface of the Earth, assumed to be spherical in shape, on a plane in polar stereographic projection.

is the other geographical pole (in the example, the South Pole; refer to fig. 1). Here the pole chosen as the point of contact is at the centre of the projection. The images of the meridians radiate from the chosen pole, and the images of the parallels of latitude form circles around this pole. In principle, the entire surface of the Earth may thus be projected onto one plane; however, increasing distance from the central point leads to increasing distortions, and the opposite pole is represented at an infinite distance. It is therefore appropriate to restrict oneself to one hemisphere at a time.

The angle of geographical longitude, which is measured at the pole as the spherical angle between the prime meridian and the local meridian, appears in its true size as an angle between the half line for the prime meridian and the half line for the local meridian. The scale at the outer edge of Charts 2812 and 2813 shows the geographical longitude, counting in a westward direction from 000° to 360° ($\lambda_w = 360° - \lambda_E$). The half line representing the local meridian can thus easily be annotated on the chart.

The following equation results from figure 1 as the radius of a circle of latitude in the projection plane:

$$r' = R' \tan \frac{90° - \varphi}{2},$$

where $R' = 2R$, representing the radius of the equatorial circle (R: radius of the Earth). When the equator is crossed, φ changes its sign. r' and R' stand for a given geographical latitude in a constant ratio, which is retained even when the scale is reduced. Therefore, the equation

$$r(\varphi) = r(0) \tan \frac{90° - \varphi}{2} \qquad (1)$$

applies to these charts, $r(0)$ being the chosen radius of the equatorial circle and $r(\varphi)$ the radius of the circle of latitude pertaining to φ.

In Table 1 the radii of several circles of latitude are listed, as they appear on the charts being described [$r(0) = 150$ mm]. For the sake of clarity the circles of latitude are marked at intervals of only 5° on the charts. For other geographical latitudes and a different radius of the equatorial circle the pertinent radii can, however, easily be determined in accordance with equation (1), after $r(0)$ has been measured on the chart being used (paper distortion!).

Thus each point on the hemisphere in question given in terms of latitude and longitude may be annotated on the chart with compass and ruler.

Example : Chart 2812

DHI Hambourg : $\varphi = 59°\ 32.9'$ N

$\lambda = 009°\ 58.2'$ E $= 350°\ 01{,}8'$ W $\approx 350°$ W.

For $r(0) = 150$ mm, equation (1) yields $r(\varphi) = 49.4$ mm.

49.4 mm is marked off on the 350° half line (referring to the scale at the edge of the chart) from the midpoint (the geographical North Pole), whence one obtains the point representing the location of the DHI.

Table 1
Stereographic projection

φ	$\frac{r(\varphi)}{r(0)}$	$r(\varphi)$ for $r(0) = 150.0$ mm mm
00°	1.0000	150.0
05°	0.9163	137.4
10°	0.8391	125.9
15°	0.7673	115.1
20°	0.7002	105.0
25°	0.6371	95.6
30°	0.5774	86.6
35°	0.5206	78.1
40°	0.4663	69.9
45°	0.4142	62.1
50°	0.3640	54.6
55°	0.3153	47.3
60°	0.2679	40.2
65°	0.2217	33.3
70°	0.1763	26.4
75°	0.1317	19.7
80°	0.0875	13.1
85°	0.0437	6.5
90°	0	0

3. CIRCLES

Circles on the surface of the Earth also appear on the chart as circles (or straight lines). When a circle is described on the chart, however, it should be noted that the centre of the circle on the chart does not generally correspond with the representation on the chart of the centre of the circle in nature.

Example :

A circle with a spherical radius of 1200 nautical miles, corresponding to 20°, is to be described around a point at geographical latitude 60° N. On the local meridian this corresponds to a difference of latitude of 20° to each side. The circle thus extends northwards to 80° N and southwards to 40° N.

From Table 1 results

$$r(80°) = 13.1 \text{ mm}$$
$$\text{and } r(40°) = 69.9 \text{ mm}.$$

The mean value is thus

$$\bar{r} = (69.9 \text{ mm} + 13.1 \text{ mm})/2 = 41.5 \text{ mm}.$$

This value determines the centre of the circle on the local meridian on the chart, while r(60°), determining on the chart the circle's centre in nature, is 40.2 mm.

The radius of the circle is

$$(69.9 \text{ mm} - 13.1 \text{ mm})/2 = 28.4 \text{ mm}$$

4. APPLICATION TO SATELLITES

4.1. Horizon circle

The horizon circle of a satellite S surrounds all points on the Earth's surface from which the satellite can be "seen" (fig. 2). Its spherical centre is the subsatellite point S', which is the point where the line connecting the satellite and the centre of the Earth intersects the Earth's surface. The spherical radius ρ_0 of the circle is given by the equation

$$\rho_0 = \text{arc cos} \frac{R}{R + H} \tag{2}$$

where

R : radius of the Earth

H : height of the satellite above the surface of the Earth.

4.2. Circles of equal altitude

All observers who can "see" a satellite at the same time at the same altitude h are on a circle of equal altitude. The spherical centre of such circles is the

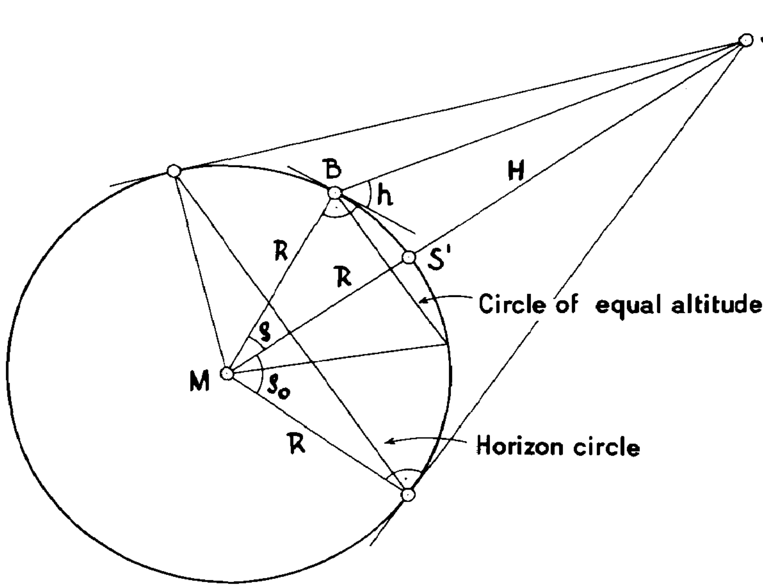

FIG. 2. – In nature : Circles of equal altitude and horizon circle of a satellite.

subsatellite point S'. The spherical radius ρ of the circle, as seen in figure 2, is given by the following equation:

$$\rho = \text{arc cos } \left(\frac{R}{R+H} \cos h\right) - h \tag{3}$$

For $h = 0°$, equation (3) changes into equation (2).

4.3. Geostationary satellites

Each geostationary satellite orbits the Earth eastwards above the equator at an altitude of $H \approx 35\,790$ km, so that its period corresponds to the period of rotation of the Earth about its axis and it remains stationary above one point of the equator, relative to the Earth.

If one takes a mean value $R = 6\,371$ km for the radius of the Earth, one obtains from equation (2) for the radius of the horizon circle $\rho_0 = 81°\ 18.5'$. From equation (3) one obtains the values of the spherical radius ρ given in Table 2 for various circles of equal altitude.

Symmetrical to the equator, a circle of equal altitude reaches a maximum geographical latitude which corresponds to its spherical radius ($\varphi_m = \pm\rho$). The radii of the circles of latitude touching the circle of equal altitude in the stereographic projection for $r(0) = 150$ mm are yielded by equation (1) and are likewise given in Table 2.

As in nature all circles around a point on the equator intersect the equator at right angles, this applies also to the representations of these circles in stereographic projection. The differences in longitude between the points of intersection and the midpoint correspond to the spherical radius.

Table 2

Circles of equal altitude for a geostationary satellite in polar stereographic projection.
$r(0) = 150$ mm

h	$\rho = \lvert\varphi_m\rvert$	$r(\rho)$ mm	$r(-\rho)$ mm	$\bar{r}$ mm
0°	81° 18.5'	11.4	1973.9	992.6
5°	76° 20.5'	18.0	1252.5	635.3
10°	71° 26.5'	24.5	918.1	471.3
20°	61° 50.2'	37.6	598.0	317.8
30°	52° 28.8'	50.9	441.6	246.3
40°	43° 21.2'	64.7	347.9	206.3
50°	34° 25.6'	79.0	284.7	181.8
60°	25° 40.0'	94.3	238.5	166.4
70°	17° 02.2'	110.9	202.9	156.9
80°	8° 29.8'	129.3	174.1	151.7
90°	0°	150.0	150.0	150.0

Example :

Meteosat $\varphi = 00°$ $\lambda = 000°$.

The circle of equal altitude for $h = 70°$ corresponds to $\rho = 17°\ 02'$ and it therefore extends as far as

$$\varphi = 17°\ 02'\ \mathrm{N}\ [r(\rho) = 110.9\ \mathrm{mm}]$$
$$\text{and } \varphi = 17°\ 02'\ \mathrm{S}\ [r(-\rho) = 202.9\ \mathrm{mm}].$$

The circle of equal altitude intersects the equator at $\lambda_1 = 017°\ 02'$ W and at $\lambda_2 = 017°\ 02'$ E $= 342°\ 58'$ W.

On the chart, the centre of the circle of equal altitude is found on the meridian for $\lambda = 000°$ either by marking off $\bar{r} = 157$ mm from the pole or as the point of intersection with the tangent at the equatorial circle in one of the points of intersection mentioned above (refer to fig. 3).

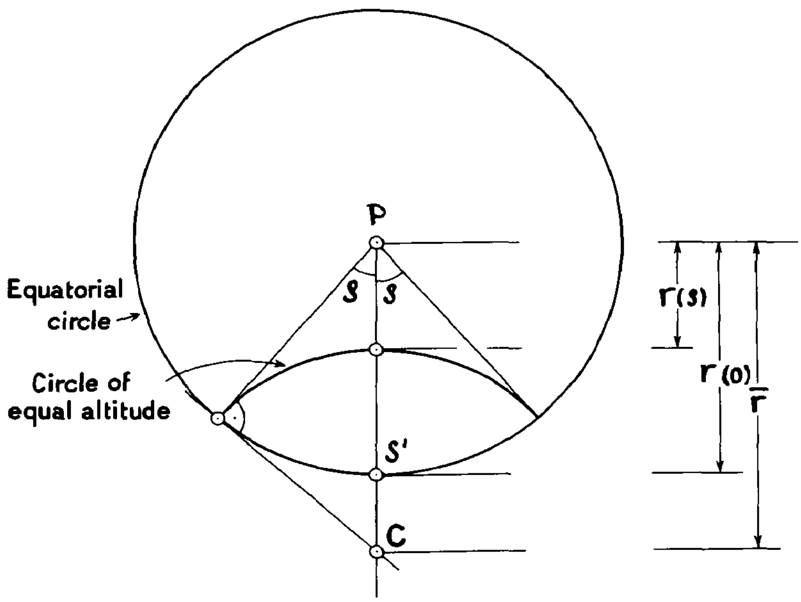

FIG. 3. – On the chart: Construction of a circle of equal altitude for a geostationary satellite.

Example :

For the three geostationary MARISAT satellites

Atlantic	015.0° W
Pacific	176.5° E = 183.5° W
Indian Ocean.............	073.0° E = 287.0° W

the area of coverage is annotated on the Northern and Southern hemispheres respectively in figures 4a and 4b. Each individual circle of equal altitude of $h = 5°$ has been chosen as a boundary line.

Figure 5 shows these circles of equal altitude as well as several lines of equal azimuth (in this case not in circular form !) for any geostationary satellite.

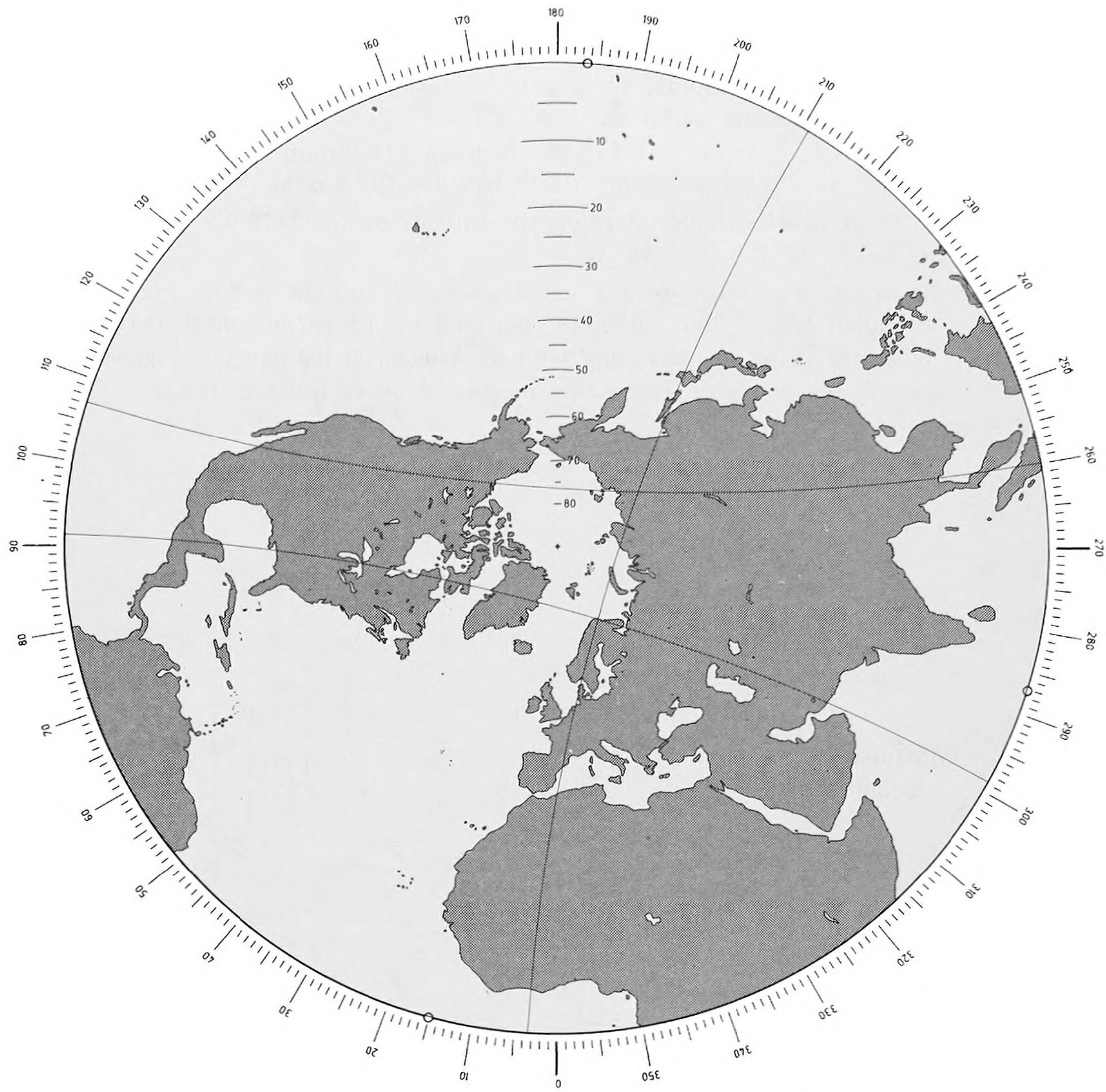

FIG. 4a. – Coverage area of the three geostationary MARISAT satellites (h = 5°) : Northern hemisphere.

A transparent sheet suitable to be laid upon the charts being described may be obtained from the DHI: this sheet can be aligned with any desired geostationary satellite. This diagram serves the purpose of taking the altitude and the azimuth of the satellite for a given location, in order, for example, to adjust the directional aerial for satellite communication if automatic positioning has been interrupted.

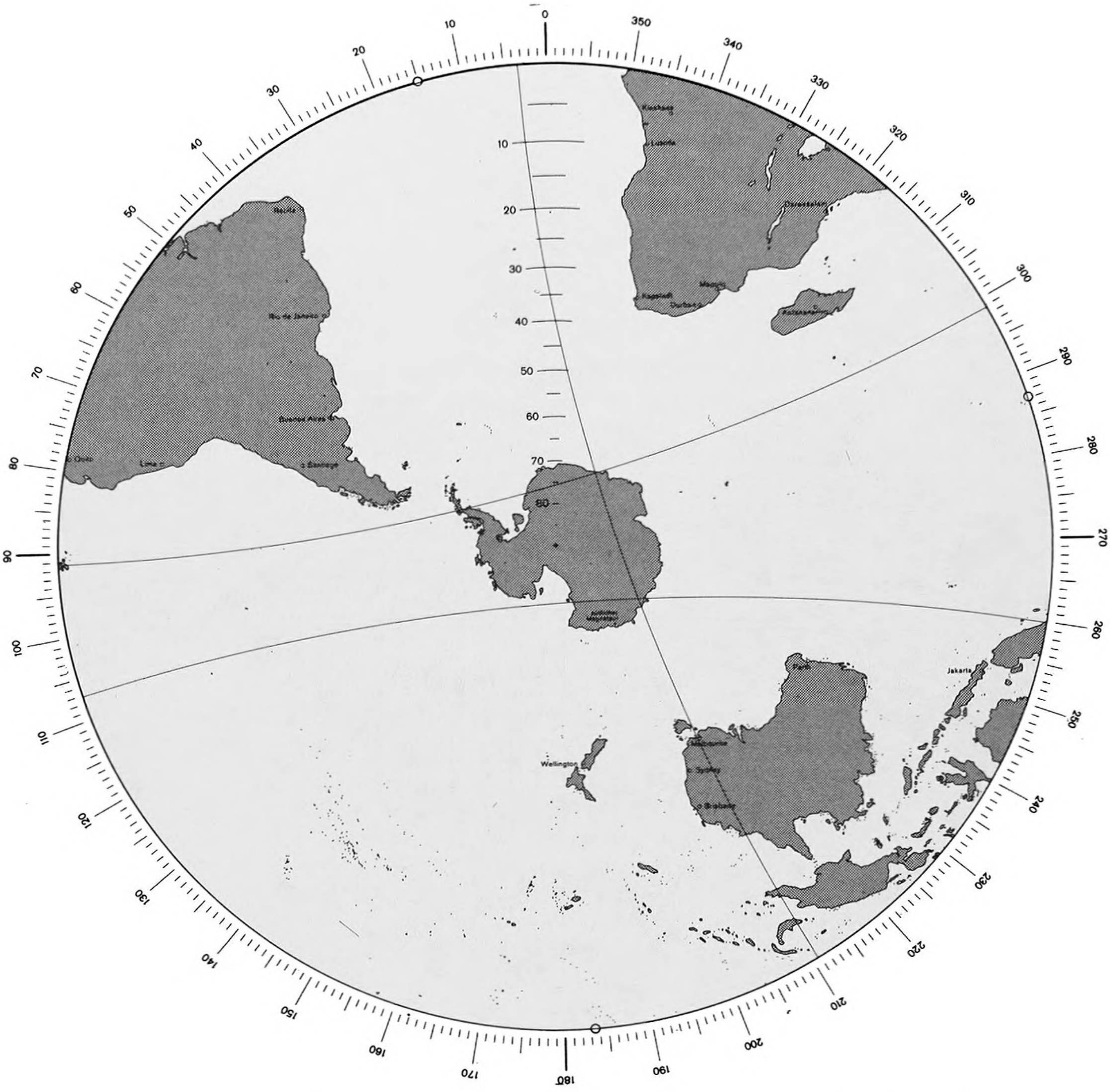

FIG. 4b. – Coverage area of the three geostationary MARISAT satellites (h = 5°) : Southern hemisphere.

4.4. Orbiting satellites

When – as is generally the case – a satellite orbits in relation to the Earth's surface, its subsatellite point pulls with it the system of satellite-bound circles of altitude, and it would be possible to draw only a "snapshot". A "stationary picture" can, however, be drawn for a definite observation point if the height of the satellite's orbit remains constant (circular orbit). If the satellite S (compare fig. 2) is "seen" by an observer B at a definite altitude h, the observer is, according to equation 3, at a certain spherical distance ρ from the subsatellite point S', so that, on the other hand, S' must lie on a circle with a spherical

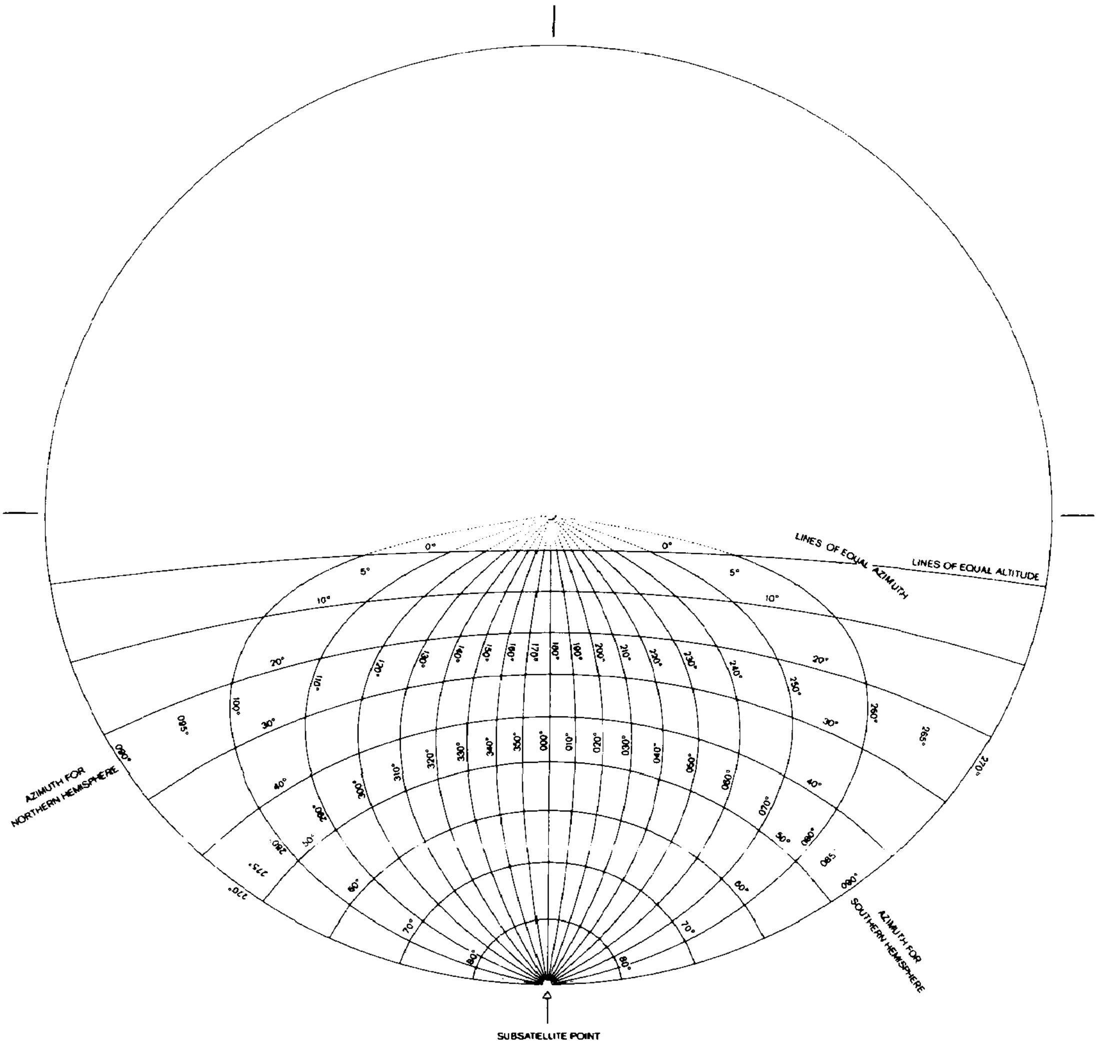

FIG. 5. – Lines of equal altitude and equal azimuth for geostationary satellites.

radius ρ around B. Thus one once again obtains circles of equal altitude, although these are in this case described around the stationary observer B and refer to the variable subsatellite point S'.

Figure 6 shows circles of equal altitude of this kind with reference to Hamburg and the satellite AMSAT-OSCAR 8 (AO 8) for the amateur radio service (inclination of orbit 98.98°; period 103.23 min).

Each azimuth of the satellite is identical with the initial great circle course leading from the observer to the subsatellite point. These circles of equal azimuth are also shown in figure 6. They always intersect the circles of equal altitude at right angles, as in nature.

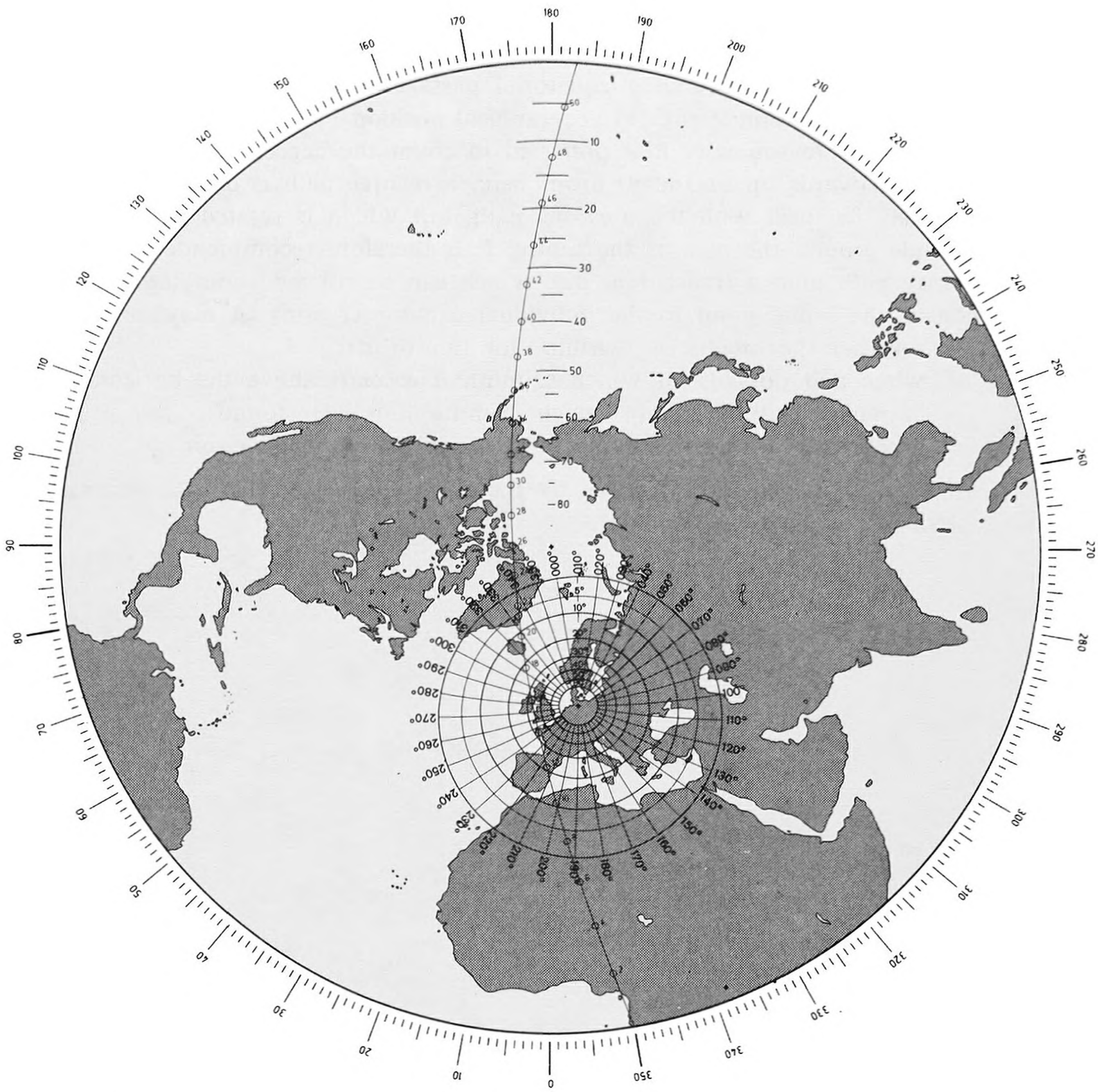

FIG. 6. – Circles of equal altitude and equal azimuth for the amateur radio satellite AMSAT-OSCAR 8, with reference to an observer in Hamburg, and the relative path of this satellite with time markings in minutes from the ascending equatorial passage (longitude of equatorial passage chosen at random).

Both altitude and azimuth can thus be seen from the diagram if the position of S′ for a definite time is known. The course of the relative path of the subsatellite point – with time markings from the equator passage onwards – can be determined from the data of the period and the inclination of the orbit towards the equator. The relative path is a great circle, on which the rotation of the earth about its axis and the rotation of the plane of orbit about the same axis (precession) are superimposed.

Figure 6 shows such a relative path for AO 8. The time markings show how many minutes have elapsed since the ascending equatorial passage.

On account of the superimposed rotations the descending equatorial passage is not exactly opposite the ascending equatorial passage in the case of orbiting satellites, and the next ascending equatorial passage shows in comparison with the previous one a definite shift of geographical position in a westward direction. (This is also the reason why it is preferred to count the degrees of geographical longitude westwards up to 360° !) From here, a relative path is described which is completely identical with the previous path, but which is rotated by the shift of longitude around the pole as the centre. It is therefore recommended to trace the relative path onto a transparent disc which can be rotated about the pole. If one adjusts the initial point to the individual equator crossing, it may be seen :

- whether the satellite is "visible" for this orbit;
- when and possibly in which azimuth it ascends above the horizon;
- at which altitude and in which azimuth it is later found;
- when and in which azimuth it descends below the horizon.

Such an arrangement is known by the name OSCALATOR. The pertinent sheets can be supplied by the DHI.