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AUTOMATIC POSITIONING BY REDUNDANT MEASUREMENTS

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ABSTRACT

The paper investigates a short-range positioning system, e.g. for surveying of rivers, using electromagnetic phase measurements. The system consists of a transmitter on a ship and three shore based transponders. For computing the initial position for a calibration, differences of ranges to four successive transmitter positions are measured. The differences yield an overdetermined hyperbolic equations system for the unknown coordinates of the transmitter positions. The redundant measurements are used to get a unique solution of the equations system in almost all situations. The equations system is well adapted for an iterative, numerical solution by means of the secant method. The algorithm of an automatic positioning system is developed which calls the radio link for suitable measurements, computes the above indicated hyperbolic method for a calibration, uses only two transponders and computes the classic range-range method if the last position is known, checks periodically the result of the range-range method by means of two redundant measurements from the third transponder, restarts the hyperbolic method in the case of an error, and informs the operator when the course should be altered. Some results of numerical experiments and some properties of the algorithm are reported which show that the proposed hyperbolic method can be advantageous in application.

INTRODUCTION

Positioning systems for application to hydrography primarily use electromagnetic distance measuring [3]. We are interested in very short range systems, especially for surveying of rivers, harbors, etc., which use shore-based transpon-

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ders and a transmitter, for example, on a ship. Exact positioning is required for precise coordination between the depth value measured and the ship position (e.g. for suitable paper transport of a survey sounder or for a bottom chart recorder).

Robert C. MUNSON presented in 1977 a report [5] on positioning systems of potential application to hydrography. He investigated two types of electromagnetic distance measuring systems, short range systems using microwaves from a mobile transmitter to a pair of fixed transponders, and medium range systems using three or more shore-based transmitters at about 2 MHz and passive receivers. The former systems fall into three classes: pulse systems, CW systems using phase measurements on modulation frequencies, and systems using long coded pulses on a CW signal. He concluded that the phase measuring systems in general have lower accuracy specifications than the pulse systems, and also carry a higher price tag. However, this is not correct for very short range systems which are of interest in this paper.

Our investigations are related to experiments with the distance measuring system ATLAS RALOG 22 of Krupp Atlas-Elektronik [4]. It is a range-range system with a mobile transmitter and two fixed shore stations not applying the pulse mode of operation like the Mini-Ranger III of Motorola [5] which directly obtains the ranges to the stations, but using phase measurements to get the difference of ranges from a station to pairs of transmitter positions. In other words, the system measures ranges that have an unknown fixed bias, for each station, but range differences between successive observation points are correct. Knowing, for example, the initial position of the ship, the following positions can be computed by the differences from both stations using the known simple formulas.

The ATLAS RALOG 22 was developed especially for river and harbour surveying and the chosen frequency of 34 MHz (about 8 m wavelength) is a good compromise, keeping position errors due to reflections and phase anomalies low, while maintaining a sufficient range of more than 10 km. It could be demonstrated that at this wavelength the transmitted signal "jumps" over smaller obstacles rather than being totally shadowed, and stable operation could be achieved, even in unfavourable areas like harbours, across forest-covered islands, or in the vicinity of iron bridges. While all microwave systems are bothered by Fresnel zone interference, at least at distances below 1 km the 34 MHz propagation is stable in this respect and the minimum distance between transmitters can be less than 10 m. The RALOG system is coupled to intelligent data-processing equipment called SUSY, which is well able to run the mathematical procedure described later. Although the following investigation was made with the RALOG 22 system in mind, the general method seems to be applicable to other short range systems which cannot be replaced by satellite systems, and can be used for both electromagnetic and acoustic systems.

The RALOG 22 has a high accuracy; there are, however, two disadvantages. First, operator intervention may be needed to suppress ambiguities, and second, more important, the system has to be zero set. Inshore position fixing is done by means of a laser. The differences have to be continuously monitored. A signal loss results in a loss of differences, and a new calibration is necessary.

The problem was to avoid both the system's zero set by optical means and the possible ambiguities when only phase measurements with a similar system are possible. The idea for a solution is summarized as follows. A third land station is introduced and redundant measurements to successive transmitter positions are executed. Measurements to three stations supply one redundant range difference, so at least three ships' positions are required to solve for three fixed biases. We take four positions and get redundancy which is used to avoid ambiguities in solving a set of hyperbolic equations. In this way, which we call the hyperbolic method, a calibration without optical means is achieved.

After calibration, the following positions can be computed as in the range-range system without the third land station. For quality control, however, it is recommended to check the computed positions periodically by use of additional measurements from the third station. If there is disagreement, one has to calibrate with the hyperbolic method once again. Since the hyperbolic method does not work in the case of a straight course, the helmsman must know that a restart is being effected and should alter the course. Based on these ideas, an automatic computer-controlled positioning system was designed. The system works automatically in that all necessary activities of the radio system are called, for example "switch on the third station" and "difference measurements from three land stations"; in that it calls for such measurements until the hyperbolic method is successful; and in that it checks the last computed position as explained above.

The next chapter discusses the geometry of the hyperbolic positioning problem in the plane. A suitable system of linear and non-linear equations for the unknown coordinates of the successive positions of the ship is constructed. The use of the differences from three stations and four positions yields more independent equations than unknown coordinates. An iterative solution of the complete system avoids the generation of ghost positions, i.e. computes the unique solution. The following chapter describes an algorithm for the automatic positioning system which executes the iterations for the hyperbolic and the calculations for the range-range method and all the things already mentioned. Finally, some remarks on numerical experiments, on properties of the algorithm, and on possibilities for handling noise problems and continuous quality control are added.

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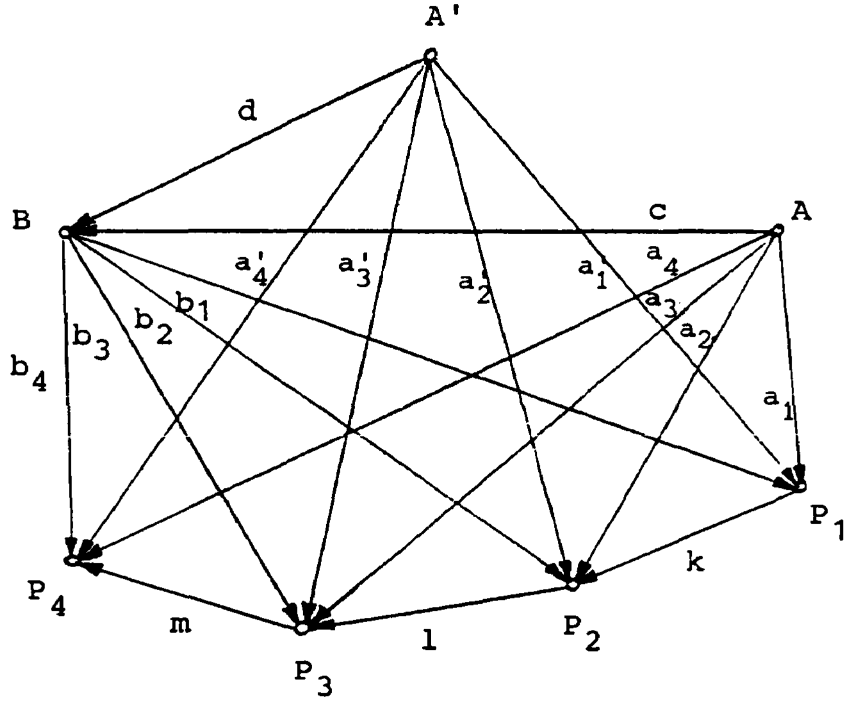
I gratefully acknowledge the fact that the Editor of this Journal called the author's attention to Mr. RIEMERSMA'S paper [7] and inspired the detailed comments of Mr. SLUITER. I thank Mr. SLUITER for the critical review of this paper, especially for his statements on problems in connection with disturbed measurements.

GEOMETRY OF THE POSITIONING PROBLEM

Let us assume that we have two land stations at locations A and B and a third at A' which is not on the straight line matching A and B. Four successive positions of the transmitter on the ship are called P_1, P_2, P_3, P_4. The following sketch illustrates the geometry and defines the vectors of interest.



We are given the points A, B and A′ and then the vectors c and d. These vectors define the base of the coordinate system. The other vectors are assumed to be unknown. The problem is to find the inner products of these vectors with c and d, for example lc and ld. As known, the vector l can be expressed by :

(1) $$l = \frac{d^2\,lc - cd\;ld}{E}\,c + \frac{c^2\,ld - cd\;lc}{E}\,d$$

where $d^2 = dd$, etc., and

(2) $$E = c^2 d^2 - (cd)^2$$

is a constant greater than zero since c and d are linearly independent.

The phase measurements by the radio link give us the following differences of ranges

(3) $$|b_{i+1}| - |b_i| = \beta_i, |a_{i+1}| - |a_i| = \alpha_i, |a'_{i+1}| - |a'_i| = \gamma_i \text{ for } i = 1,\ 2,\ 3.$$

Herein, the range is the length of a vector, e.g. $|b_i| = (b_i b_i)^{1/2}$.

We find some elementary vector identities from the sketch. To simplify matters, we omit the subscript 2 of b_2.

(4) $$\begin{cases} b_1 = b - k,\ b_2 = b,\ b_3 = b + l,\ b_4 = b + l + m, \\ a_1 = c + b - k,\ a_2 = c + b,\ a_3 = c + b + l,\ a_4 = c + b + l + m, \\ a'_1 = d + b - k,\ a'_2 = d + b,\ a'_3 = d + b + l,\ a'_4 = d + b + l + m. \end{cases}$$

Thus, the unknown vectors are b, k, l, m, and the variables bc, bd, kc, kd, lc, ld, mc, md have to be determined.

From (3), we have nine independent, nonlinear equations for eight variables, i.e. the equations system is overdetermined.

If we did not use measurements to the fourth position of the transmitter, we would have six corresponding equations for six variables. Since the equations could be stated as polynomial equations, cf. the arguments below, a large number of solutions would exist in general. The overdetermination reduces this number to one in almost all situations. We remark that working with only two land stations gives too few equations: the use of n positions results in $2n - 2$ equations for $2n$ variables.

The next problem is the construction of an equations system equivalent to (3) which is well suited for an iterative numerical solution. When the equations of (3) are written $|b_{i+1}| = |b_i| + \beta_i$, etc., they can be squared without loss of information. Suitable differences of the squared equations and substitutions result in

1) $kc = (\beta_1^2 - \alpha_1^2 + 2\alpha_1 |b + c| - 2\beta_1 |b|)/2$
2) $kd = (\beta_1^2 - \gamma_1^2 + 2\gamma_1 |b + d| - 2\beta_1 |b|)/2$
3) $lc = (\alpha_2^2 - \beta_2^2 + 2\alpha_2 |b + c| - 2\beta_2 |b|)/2$
4) $ld = (\gamma_2^2 - \beta_2^2 + 2\gamma_2 |b + d| - 2\beta_2 |b|)/2$
5) $mc = (\alpha_3^2 - \beta_3^2 + 2\alpha_3(\alpha_2 + |b + c|) - 2\beta_3(\beta_2 + |b|))/2$
6) $md = (\gamma_3^2 - \beta_3^2 + 2\gamma_3(\gamma_2 + |b + d|) - 2\beta_3(\beta_2 + |b|))/2$.

From (1) and (2) we obtain :

7) $k^2 = (d^2(kc)^2 + c^2(kd)^2 - 2cd\ kc\ kd)/E$
8) $l^2 = (d^2(lc)^2 + c^2(ld)^2 - 2cd\ lc\ ld)/E$
9) $m^2 = (d^2(mc)^2 + c^2(md)^2 - 2cd\ mc\ md)/E$

and the inner product

10) $lm = (d^2 lc\ mc + c^2 ld\ md - cd\ (lc\ md + ld\ mc))/E$.

Two of the squared equations of (3) give

11) $bl = (\beta_2^2 - l^2 + 2\beta_2 |b|)/2$
12a) $bk = (k^2 - \beta_1^2 + 2\beta_1 |b|)/2$.

Like 10), we can write

12) $bk = (d^2 bc\ kc + c^2 bd\ kd - cd\ (bc\ kd + bd\ kc))/E$.

The vectors k and l are linearly independent if, and only if,

(5) $$F = kc\ ld - kd\ lc$$

is not equal to zero. Presuming the latter, we can write bk and bl as 10), solve for bc and bd, and obtain equations 13a) and 14a). If we interchange the role of k, l and F with that of l, m and

(6) $$G = lc\ md - ld\ mc$$

respectively, we get 13b) and 14b).

13a) $bc = (c^2\ (bk\ ld - bl\ kd) - cd\ (bk\ lc - bl\ kc))/F$
13b) $bc = (c^2\ (bl\ md - bm\ ld) - cd\ (bl\ mc - bm\ lc))/G$
14a) $bd = (d^2\ (bl\ kc - bk\ lc) - cd\ (bl\ kd - bk\ ld))/F$
14b) $bd = (d^2\ (bm\ lc - bl\ mc) - cd\ (bm\ ld - bl\ md))/G$.

We have, similar to 7),

15) $|b| = ((d^2\ (bc)^2 + c^2\ (bd)^2 - 2cd\ bc\ bd)/E)^{1/2}$

The overdetermination gives additional equations for $|b|$. First, expressions for bm are found as 10) or from squared equations of (3),

16a) $bm = (d^2bc\ mc + c^2\ bd\ md - cd\ (bc\ md + mc\ bd))/E$
16b) $bm = (\beta_3^2 + 2\beta_3\ (\beta_2 + |b|) - m^2 - 2lm)/2.$

The last equation re-arranged yields

17a) $|b| = (m^2 + 2bm + 2lm - \beta_3^2 - 2\beta_2\beta_3)/(2\beta_3)$, if $\beta_3 \neq 0$.

We have from 12a)

17b) $|b| = (\beta_1^2 - k^2 + 2bk)/(2\beta_1)$, if $\beta_1 \neq 0$.

Finally, we write

19) $|b + c| = (b^2 + c^2 + 2bc)^{1/2}$
20) $|b + c| = (b^2 + d^2 + 2bd)^{1/2}$

We can assume the constants c^2, d^2, cd and E and the measured values α_i, β_i, γ_i $(i = 1, 2, 3)$ are known. An equations system for the unknowns $|b|$, $|b + c|$ and $|b + d|$ with known coefficients was constructed since all other variables of interest can be expressed as functions of only these variables by means of substitutions. For a numerical solution of the system, suitable subsets of equations have to be chosen. Three cases are discriminated. If $|F|$ is not small against c^2d^2, i.e. k and l are not parallel, then we use the equations 1) to 20) except for those additionally marked by b). If $|F|$ is small and $|G|$ not small against c^2d^2, i.e. l and m are not parallel, we exclude the equations marked by a). If both, $|F|$ and $|G|$ are small, i.e. the ship is on a straight course, we cannot solve in the above manner. The overdetermination for $|b|$ is handled by mixing the right hand sides of 15) and 17) with weights λ and $1 - \lambda$, respectively, where $0 < \lambda \leqq 1$ and $\lambda = 1$, if β_3 or β_1 are relatively small. The mixture which we call 18) makes sense since $|b|$ is greater than zero. If we call the input values for $|b|$, $|b + c|$ and $|b + d|$ in 1), 2), etc., by u_1, u_2 and u_3, respectively, and the output values of 18), 19) and 20) by v_1, v_2 and v_3, we have the desired equations system of the hyperbolic method:

$$(5) \qquad f_i\ (u_1, u_2, u_3) = v_i - u_i = 0 \quad (i = 1, 2, 3).$$

For completeness, we note the equations systems for the range-range method if the initial position is known, for example the vector b_1. We first assume that there are differences from land stations A and B. We find from (3) that

21) $|b_2| = |b_1| + \beta_1$
22) $|b_2 + c| = |b_1 + c| + \alpha_1$

and like 19) and 7)
25a) $b_2c = ((b_2 + c)^2 - c^2)/2$
$b_2^2 = (d^2\ (b_2c)^2 + c^2\ (b_2d^2) - 2cd\ b_2c\ b_2d)/E.$

Solving for b_2d results in

26a) $b_2d = (cd\ b_2c \pm (E(c^2b_2^2 - (b_2c)^2))^{1/2})/c^2.$

Because of $(b_2 + d)^2 = d^2 + b_2^2 + 2b_2d$, the plus sign is correct if A′ and P_2 are on opposite sides of the vector c and, if not, the minus sign. The sign cannot be concluded from the measurements. This is in contrast to the case in which we have additional measurements from the third station A′. We would have 21), 22)

23) $|b_1 + d| = (b_1^2 + d^2 + 2b_1d)^{1/2}$
24) $|b_2 + d| = |b_1 + d| + \gamma_1$

and from $b_2 = b_1 + k$, 1), and 2) the unique assignments

25b) $b_2c = b_1c + (\beta_1^2 - \alpha_1^2 + 2\alpha_1|b_2 + c| - 2\beta_1|b_2|)$
26b) $b_2d = b_1d + (\beta_1^2 - \gamma_1^2 + 2\gamma_1|b_2 + d| - 2\beta_1|b_2|)$.

Storing the values $|b_2|$, $|b_2 + c|$ and in case b_2c and b_2d, the start values for the computation of a succeeding position are prepared.

ALGORITHM FOR AN AUTOMATIC POSITIONING SYSTEM

The problem of the preceding chapter was the statement of a suitable hyperbolic equations system. We now indicate a method for a numerical solution in connection with a description of an automatic positioning system. We formulate an algorithm in an ALGOL-like programming language, explain a running example of the positioning system, and describe the subprograms which do the computations.

The programming language we use is PIDGIN ALGOL which was introduced in [1]. PIDGIN ALGOL is a high-level language and is unlike any conventional programming language in that it allows one to use any type of mathematical statement as long as its meaning is clear and the translation into a machine language is evident. The language does not need data declaration if the meaning of the variables is obvious. The language uses expressions, conditions, statements and procedures as ALGOL. The variables are universal and can be used in all procedures. The procedures we apply usually contain, in the parameter list, only the variables which describe the results of a computation. If necessary, we add comments in the program to explain the activities called or the next steps. Statements not of interest or realizations of formulas of the last chapter are only indicated by a verbal description.

Algorithm of the positioning system

```
        begin;
        compute constants c², d², cd, ... parameters n₀, ε, ...;
        FINIS := false;
        comment: FINIS is boolian and is set « true » if the operator wants to
              stop the computations;
        ONA';
        comment: ONA' switches on station A'; it is assumed that A and B
              are already on;
start : MABA'4 (α₁, α₂, α₃, β₁, β₂, β₃, γ₁, γ₂, γ₃);
        comment: measures differences from stations A, B, A' to four succes-
              sive positions for the hyperbolic method called by SECANT;
```

```
hyperb : SECANT (bc, bd, kc, kd, lc, ld, mc, md, u1, u2, u3, i);
         comment : if i = 0, solution with u1 = |b|, u2 = |b + c|, u3 = |b + d|; if
              i = 1, restart with new random start values, otherwise restart
              with more recent measurements;
         if i = 0 then goto « run »;
         if FINIS then goto  stop;
         si i = 1 then goto  hyperb;
         FLAG ("alter the course - restart");
         for n := 2, 3 do begin
         α(n-1) : = α(n); β(n-1) := β(n); γ(n-1) = γ(n) end;
         MABA'1 (α3, β3, γ3);
         comment : measures differences from stations A, B, A' between the
              latest and one new position;
         goto hyperb;
run :    comment : now the (c, d)-coordinates of the last four successive posi-
              tions are put out to the link of interest;
         OUTPUT (bc - kc, bd - kd); OUTPUT (bc, bd);
         OUTPUT (bc + lc, bd + ld);
         b4c := bc + lc + mc; b4d := bd + ld + md;
         OUTPUT (b4c, b4d);
         r1 := u1 + β2 + β3; r2 := u2 + α2 + α3;
         comment : r1 = |b4|, r2 = |b4 + c|, now id := 1 if A' and the course are
              on opposite sides of c, and id := - 1 otherwise, must be put in if
              c and b4 are parallel, e.g. ε = 10^-8 c²d²;
         q := c²r1² - (b4c)²;
         if q < ε then INPUT (id) else
         id := sgn (c²b4d - cd b4c);
         comment : now n0-times range-range method with stations A, B, posi-
              tion P4 is interpreted as P1;
range : OFFA';
         if FINIS then goto stop;
         for n := 1, step 1 until n0 do begin;
         MAB 1 (α1, β1; TWOP (b1c, b1d, r1, r2);
         comment : differences from stations A and B are measured between
              the latest and one new position, TWOP computes the coordinates
              and the ranges of the new position which then is interpreted as
              P1;
         OUTPUT (b1c, b1d); if FINIS then goto stop end;
         comment : now for checking two additional measurements from A' are
              required;
         ONA'
         MABA'1 (α1, β1, γ1);
         comment : γ1 may not be used at this time;
         TWOP (b1c, b1d, r1, r2); OUTPUT (b1c, b1d);
         MABA'1 (α1, β1, γ1); THREEP (b1c, b1d, r1, r2, ic);
         comment : if ic ≠ 0, error;
         if ic = 0 then begin OUTPUT (b1c, b1d); goto range end;
         FLAG ("error - restart"); goto start;
stop : end
```

Let us assume the operator starts the program. The program first determines the constants and parameters where the vectors c and d must be put in. The land station A′ is switched on by the procedure ONA'. The call of MABA'4 has the effect that, corresponding to a fixed program, difference measurements from the stations A, B, A′ to four successive positions of the transmitter are executed by the radio link. The succeeding differences are assigned to the variables α_n, β_n, γ_n (n = 1, 2, 3). Then, the procedure SECANT is called. This program tries to solve the hyperbolic equations (5) by means of the secant method which is described below. After returning, the main program branches depending on the index i. If i is equal to zero, the procedure has been successful and the program jumps to run. In the other case, FINIS is checked to stop the program when the operator likes it. If i equals 1, the random start values of the secant method have been unsuitable and must be generated again. The program goes back to the label hyperb. For other values of i, SECANT could not find a solution since, for example, it is suspected that the ship was on a straight course. FLAG indicates to the operator that the course should be altered. The oldest differences are cleared, and new differences from stations A, B, A′ between the latest position and one new position are measured by calling of MABA'1. Having now more recent differences, the method is tried once again by jumping back to hyperb. We now assume i = 0 and the program continues at run. The (c, d)-coordinates of the positions P_1, P_2, P_3, and P_4 are put out to a bottom chart recorder for example. The coordinates b_4c and b_4d of P_4 and the ranges $r_1 = |b_4|$ and $r_2 = |b_4 + c|$ are stored. We need them e.g. for checking if b_4 and c are parallel by means of q. If they are approximately parallel, the operator has to put in whether the next position of interest is on the same side of c as station A′ or not by setting the indicator i_d equal to -1 or 1 respectively. In the other case, 26a) implies the formula for i_d. The value of i_d is required in the range-range method with only stations A and B. Station A′ is switched off. If not interrupted by the operator, the range-range method is applied n_0-times. By calling of MAB1, differences from A and B between the latest position now called P_1 and one new position are measured. TWOP uses the old ranges r_1 and r_2, realizes 21) and 22) by $r_1 := r_1 + \beta_1$ and $r_2 := r_2 + \alpha_1$ to get the new ranges as well as 25a) and 26a), where « + » is used if $i_d = +1$ and « − » otherwise, for the coordinates of the new position which are now b_1c and b_1d. These coordinates are put out. In the next steps, the results are checked. Station A′ is switched on, and one measurement from three stations is executed. Since the difference γ_1 from A′ is not correct at this time, we once more proceed as with two stations. The second measurement from A′ gives a correct difference. THREEP first computes the new coordinates as TWOP, then the new coordinates by 23), 24), 25b) and 26b) and compares the results. If there is a significant deviation, THREEP assigns $i_c := 1$, and $i_c := 0$ otherwise. In the latter case, the coordinates of the new position are put out, and the program continues with the range-range method at label range. If i_c equals 1, the operator is informed that there are errors and the program jumps back to a new start with the hyperbolic method.

The next point of interest is how the SECANT procedure realizes the hyperbolic method. We have to solve numerically the equations system (5) and we do it iteratively by means of the secant method [6]. First, a procedure is required which computes the differences $\delta_j = f_j(u_1, u_2, u_3) = v_j - u_j$ (j = 1, 2, 3) for any input values u_1, u_2, u_3:

procedure FCT (u_1, u_2, u_3, δ_1, δ_2, δ_3, i);

begin

if $0 \leqq u_1$ *and* $|u_1 - |c|| \leqq u_2 \leqq u_1 + |c|$ *and* $|u_1 - |d|| \leqq u_3 \leqq u_1 + |d|$

then i: = 0 *else begin* i: = 1 ; *goto* out *end*;

comment: input values should satisfy triangular equations; now the eqs. 1) to 20) are used as assignments in a suitable sequence, e.g. $\varepsilon_1 = 10^{-8}c^2d^2$, $\varepsilon_2 = 10^{-3}|c|$;

compute 1) to 11) and (5) where $|b|$, $|b + c|$, $|b + d|$ are replaced by u_1, u_2, u_3;

if $|F| < \varepsilon_1$ *then goto* change;

compute 12a), 13a), 14a), 16a), assign right hand side of 15) to v_1;

if $|\beta_3| > \varepsilon_2$ *then* assign mixture of v_1 and the right hand side of 17a) to v_1;

goto finish;

change: compute (6);

if $|G| < \varepsilon_1$ *then begin* i : = 2 ; *goto* out *end*;

compute 16b), 13b), 14b), assign right hand side of 15) to v_1;

if $|\beta_1| > \varepsilon_2$ *then* compute 12b), assign the mixture of v_1 and the right hand side of 17b) to v_1;

finish: $\delta_1 := v_1 - u_1$;

$\delta_2 := (v_1^2 + c^2 + 2bc)^{1/2} - u_2$;

$\delta_3 := (v_1^2 + d^2 + 2bc)^{1/2} - u_3$;

out : *end*.

The idea of the secant method is to approximate the functions f_j in (5) by linear functions λ_j so that $f_j = \lambda_j$ (j = 1, 2, 3) for four points in general position. The solution of $\lambda_1 = \lambda_2 = \lambda_3 = 0$ is used as an approximate solution of $f_1 = f_2 = f_3 = 0$, which is called secant approximation. Iterative computations of such solutions are done by the SECANT procedure.

procedure SECANT (bc, bd, kc, kd, lc, ld, mc, md, u_1, u_2, u_3, i);

begin

RANDOM ((u_{nj}));

comment: generates non-negative random numbers in the 4×3 matrix (u_{nj});

for n : = 1, 2, 3, 4 *do begin*

FCT (u_{n1}, u_{n2}, u_{n3}, δ_{n1}, δ_{n2}, δ_{n3}, i); *if* i $\neq 0$ *then goto* return;

$s_n := \delta_{n1}^2 + \delta_{n2}^2 + \delta_{n3}^2$ *end*;

comment: the elements of (u_{nj}) are the start values of the secant method in the Wolfe formulation, now the rows of (u_{nj}) are iteratively corrected until a sum s_n of squared differences δ_{nj} is less than a parameter;

for p : = 1, *step* 1 *until* 100 *do begin*

solve the linear equations system :

$$\sum_{n=1}^{4} t_n = 0, \quad \sum_{n=1}^{4} t_n\delta_{nj} = 0 \ (j = 1, 2, 3) \text{ for } t_1, t_2, t_3, t_4;$$

if insolvable *then begin* i : = 3 ; *goto* return *end*;

for $j := 1, 2, 3$ *do* $u_j := \sum_{n=1}^{4} t_n u_{nj}$;

comment : u_1, u_2, u_3 are the secant approximation;
FCT $(u_1, u_2, u_3, \delta_1, \delta_2, \delta_3, i)$; *if* $i \neq 0$ *then goto* return;
$s := \delta_1^2 + \delta_2^2 + \delta_3^2$;
if $s > \max(s_1, s_2, s_3, s_4)$ *then begin* $i := 4$; *goto* return *end*;
assign u_1, u_2, u_3, δ_1, δ_2, δ_3, s to those u_{n1}, u_{n2}, u_{n3}, δ_{n1}, δ_{n2}, δ_{n3}, s_n for which s_n is maximum;
comment : if s is small enough, u_1, u_2, u_3 are solution of (5); e.g. $\varepsilon_3 = 10^{-8} |c|\ |d|$;
if $s < \varepsilon_3$ *then goto* return; *end*;
$i := 5$;
comment : error messages $i = 0$, solution, $i = 1$, wrong start values, $i = 2$, probable straight course; $i = 3$, linear equations system insolvable; $i = 4, 5$, no convergence;

return : *end*

REMARKS

Some numerical experiments were carried out with a simplified version of the program and with artificial data generated from typical transponder locations and randomly chosen courses of the ship. SECANT computed a solution with four ship positions in almost all situations. In every case of a solution, it was correct. No ghost positions were generated, in contrast to experiments where we used measurements from only three ship positions.

In connection with the algorithmic description of the solution method for the equations (5), some remarks about the numerical behavior apply. The critical points are, of course, the divisions in 13), 14), and 17). In the algorithm, we branch if the dividents are relatively small and we therefore expect some numerical stability. The secant method itself has good properties; for example, computing in floating point gives high accuracy to the solution. However, the speed of convergence could be faster by use of other methods [6].

A simple check of the algorithm and the observed fast convergence of the calibration method show that implementation of the automatic positioning system either by one of the more powerful table computers having an interrupt system or by one of the inexpensive process computers with a floating point processor would suffice for the real-time requirements in the surveying problems concerned.

In his review, Mr. SLUITER pointed out the problem of the distribution of the four ship positions for a calibration which is indeed of importance if the measurements are disturbed. In this paper, we assumed good conditions for measurements with a low noise level except for occasional, short distortions resulting in a signal loss or similar things. For a calibration, we have assumed weak measurement noise at most. Then, the hyperbolic method works if the last four ship positions are not on a straight course and there is a sufficient distance between succeeding ship positions. The latter means that the differences α_i, β_i, γ_i are not all small in magnitude against the distances between the land stations.

Finally, let us touch on the problem of disturbed measurements caused by instrumentation noise, propagation conditions, etc. Continuous quality control is required for a positioning method as stated in [7] and [2]. A careful investigation in this direction has still to be accomplished. Statistical modelling of noise can help to find statistical methods for estimating the positions. In the application of this paper a quality control after a calibration could be effected as follows. The third transponder (and maybe a fourth if there are no problems with frequency licensing and prices) is continuously used, and a linear least square estimation [2] is substituted for the simple range-range method. More difficult is the calibration with disturbed measurements. As stated by Sluiter, working with three transponders and four measurements with moderate noise can result in alternative hypotheses for the ship positions. There are some possibilities of addressing this problem with redundant measurements from additional ship positions. Since the hyperbolic method is comparatively fast, we use it, for example, recursively. Beginning with four measurements, the method computes four points. One new measurement and the last three measurements are taken, and the hyperbolic method is executed again. The latter is done in a loop. In the sequence, we obtain, for each position, three points regarded as rough data. By means of a tracking algorithm similar to those applied by radar people, the ship positions are estimated. From a statistical viewpoint, it is better to use the differences directly as input data for an estimation procedure. Then, we have to solve a non-linear stochastic equations system. However, we do not anticipate future developments.

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