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ANALYSING CLIPPED SEA-LEVEL RECORDS FOR HARMONIC TIDAL CONSTITUENTS

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Coastal sea-level measurements must sometimes be made at sites where the gauge dries out at low levels. By progressively removing the lower part of a tidal record, and analysing the remainder with the conventional least-squares criteria, we have obtained stable values of the principal constants until only half of the original range remained. This stability has implications for the definitions of tidal constants and of mean sea level in regions of very shallow water and drying banks.

INTRODUCTION

As part of a recent survey of tidal conditions in the Severn Estuary and Bristol Channel (ALCOCK and PUGH) [1], it was necessary to install a sea-level recorder on the island of Flat Holm, with a zero level 2.8 m above Lowest Astronomical Tide. The gauge therefore dried out for the lowest part of the large spring range (10.5 m) which occurs in the area. In order to compare the data from Flat Holm with those from contemporary bottom pressure records at Lavernock Point and Steep Holme, 4.0 km northwest and 5.0 km south of Flat Holm respectively, harmonic tidal analyses were made of the data at all three sites.

ANALYSES

Following the usual practice, harmonic constituents were determined by fitting the function :

$$T(t) = Z_0 + \Sigma_N H_n f_n \cos(\sigma_n t - g_n + V_n + u_n)$$

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where Z_0 is mean sea level or pressure,

H_n and g_n are the amplitude and phase of the constituent,

σ_n is the constituent speed,

and

V_n, u_n and f_n are astronomical arguments (See, for example, MURRAY) [5], [6].

The parameters were fitted subject to the conditions that $\Sigma R^2(t)$ is a minimum, where

$$R(t) = 0(t) - T(t)$$

and 0(t) is the observed level or pressure. 29 days of simultaneous data were analysed for Z_0, 27 independent constituents and 8 related constituents, the relationships being based on analyses of a year of data at six sites in the Bristol Channel.

For Lavernock Point and Steep Holme the function was fitted to the complete record, and the pressure constituent amplitudes were converted to levels using the appropriate values of gravitational acceleration and water density. At Flat Holm the function was fitted only to those data obtained when the level was above the gauge zero, 85.4% of the total period. The results of these three analyses are summarised for the major tidal constituents in Table 1. Differences in the amplitudes and phases among the three analyses are satisfyingly small, and in most cases do not exceed the errors associated with the individual analyses. However, for the principal constituent, M_2, the differences, though small, are significant, and vary systematically with geographical position. It is apparent that our different method of measurement and analysis for Flat Holm is commensurate with that used at the other two sites.

NUMERICAL EXPERIMENT

To confirm the validity of our analysis of the incomplete Flat Holm data set, we performed a numerical experiment with the initially complete 29 days of hourly Steep Holme data. The Harmonic analysis described was applied to a series of progressively reduced data sets; the lower part of the record was removed in 1.0 m steps. In electronics this treatment of a signal is called "clipping". Lower levels were removed in 1.0 m steps, but smaller increments were used as the results became unstable. Figure 1 shows the percentage of the data remaining for analysis at each level of truncation. The values of amplitude and phase for the principal diurnal (O_1 and K_1), semi-diurnal (M_2 and S_2) and quarterdiurnal (M_4) constituents are plotted in Figures 2 and 3, against the level of clipping. Figure 4 shows the corresponding values of mean sea level, Z_0. Figure 5 shows the standard deviation of the analysed record about the mean value (not Z_0) of that record, and the standard deviation of the residuals, R(t), at each level of truncation.

For the diurnal constituents, the amplitude of O_1 and K_1 behave in a similar manner. They remain stable until the clipping level reaches mean sea level (msl). Both amplitudes then decay rapidly, giving meaningless results at a clipping level 3 m above msl. The phases remain stable to a clipping level 2 m below msl, and then decrease progressively, becoming meaningless 2 m above msl.

For the large semidiurnal constituents, the amplitudes of both M_2 and S_2 decay at a clipping level 1 m below msl, and become meaningless at 1 m above msl. Stable phases are obtained up to msl, but meaningless values are obtained at 2 m above msl.

Table 1
Comparison of principal harmonic constants.
Severn Estuary

		Steep Holme	Flat Holm	Lavernock Point
O_1	H (cm)	8.5	9.2	8.3
	G°	359.0	359.3	359.3
K_1	H (cm)	7.5	7.4	7.4
	G°	124.5	120.5	127.0
M_2	H (cm)	387.5	389.3	393.1
	G°	186.1	188.8	189.5
S_2	H (cm)	137.4	137.2	138.8
	G°	239.7	243.1	243.8
N_2	H (cm)	60.7	60.0	61.4
	G°	176.0	179.6	180.0
M_4	H (cm)	8.1	12.7	12.3
	G°	30.0	32.1	13.1
M_6	H (cm)	5.6	4.1	6.4
	G°	224.2	225.4	231.7

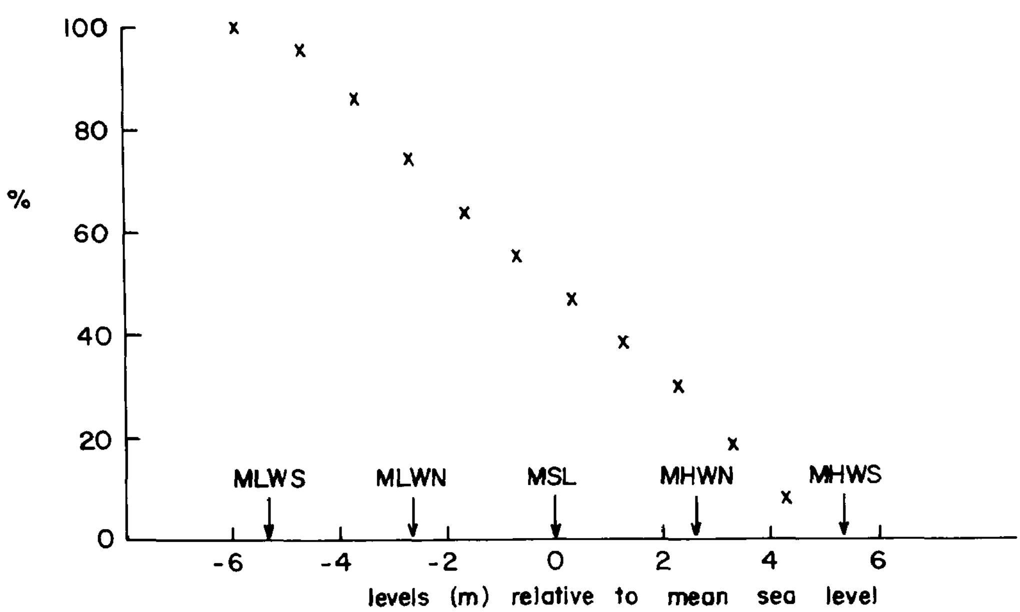

FIG. 1. – Showing the percentage of the complete 29 days of Steep Holme data available for analysis after truncation at different levels, relative to mean sea level. Also shown are the levels of mean high and low water for spring and neap tides.

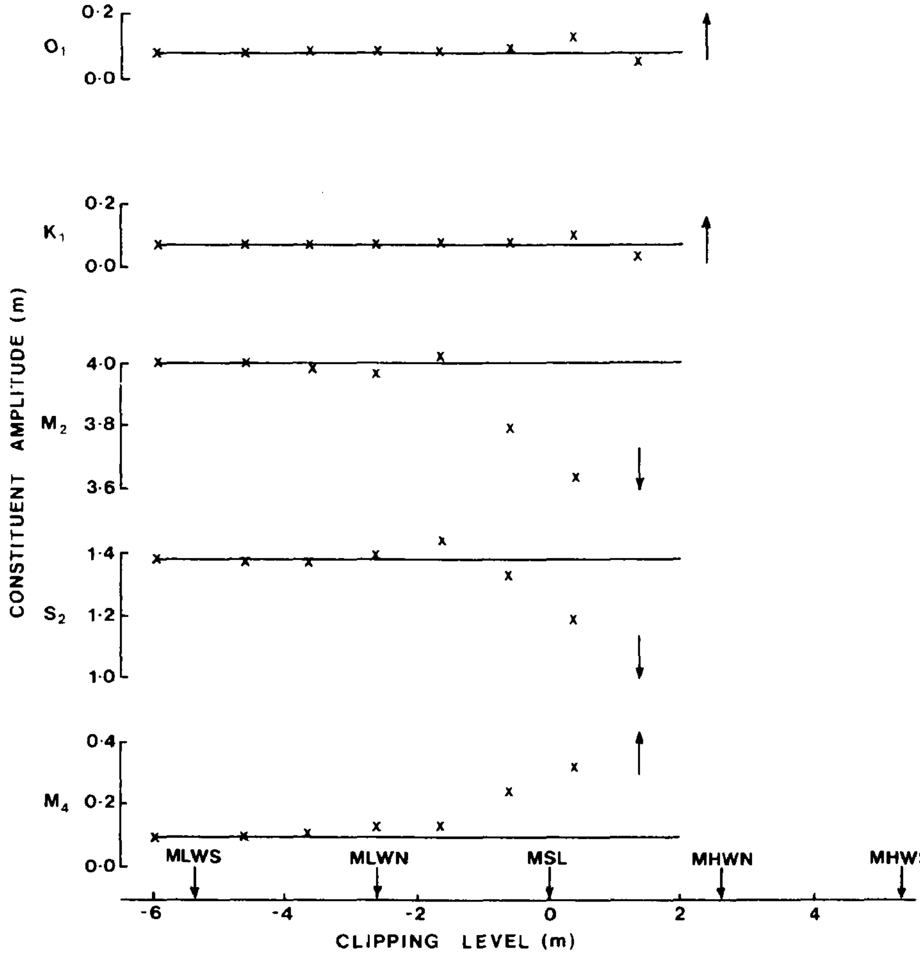

FIG. 2. – Values of constituent amplitudes obtained as a function of the level of clipping of the total record, for Steep Holme data. The continuous lines show the values obtained with a complete record. The arrows show the direction in which the first off-scale value lies.

The quarterdiurnal constituent M_4 has an amplitude which remains stable to 1 m below msl and becomes meaningless at 1 m above msl. The phase is slightly more variable than for the other constituents, but remains meaningful beyond a clipping level 2 m above msl.

The value of mean sea level obtained remains stable to within 0.02 m of the true value for clipping levels up to 2 m below msl. Even at 1.5 m above msl, when only 47% of the record is analysed, the value obtained is only 0.25 m above the true value.

DISCUSSION

Clearly, the numerical experiment with the Steep Holme data confirms the validity of our method of analysing the available 29 days of Flat Holm data. We were surprised to find that useful results were obtained for mean sea level and for all the major harmonic constituents for clipping levels up to mean low water on neap tides, and in many cases, for levels up to mean sea level. The purpose of this brief report is to draw attention to the possibility of analysing a month of necessarily truncated records in this way. Although our empirical approach precludes any general statements about the maximum clipping or drying level at other sites, the fact that

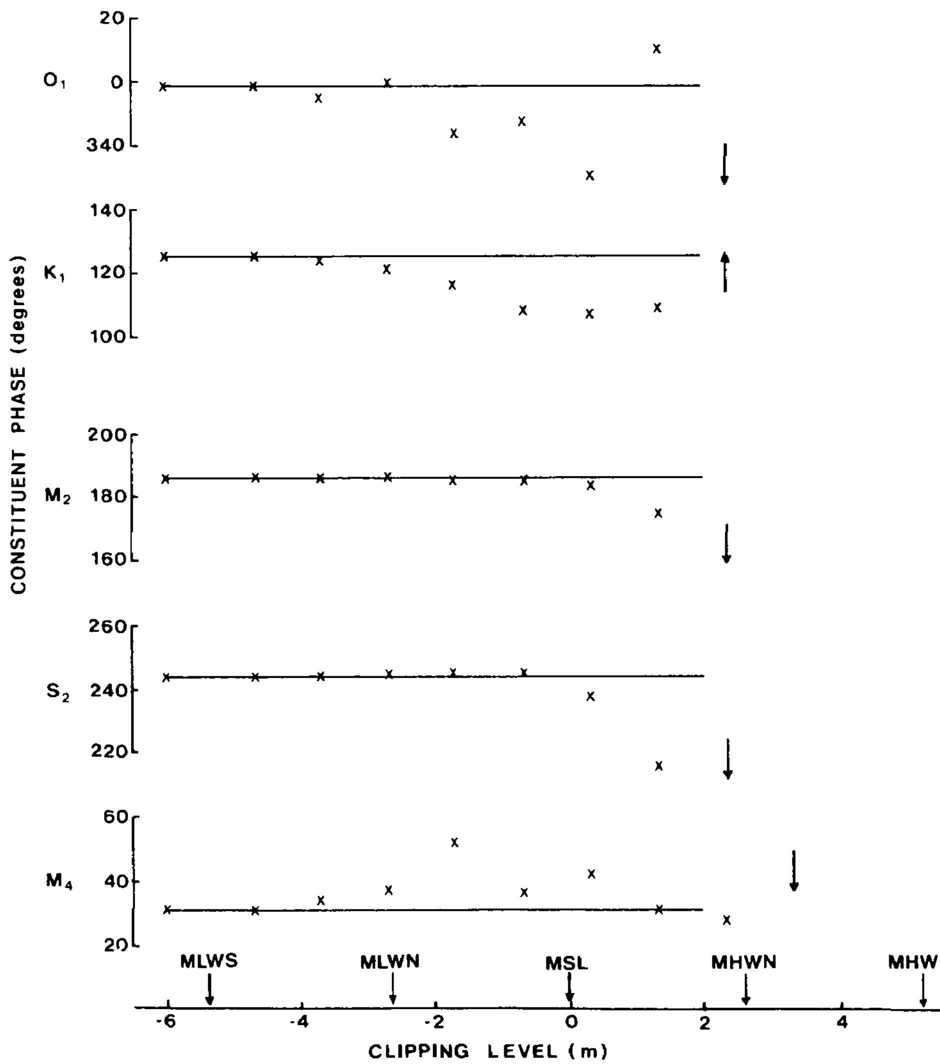

FIG. 3. – As for Figure 2, showing constituent phases.

the Severn Estuary is a region of extensive shallow water, with very large tidal ranges, suggests that results can also be obtained elsewhere, where more normal tidal conditions prevail. Even higher levels of clipping may be tolerable where longer records are obtained. MERRIMAN [4] reports similar success with the Admiralty semi-graphic method for records which dry on all but three tides in a 30-day period.

In addition to using this technique for analysing data from gauges which dry out, it may also be applied to measurements in rivers which are tidal for only the upper part of the range. Such measurements are sometimes analysed as a complete record, including the flat lower part, giving tidal amplitudes which decrease rapidly up-river. This can be seen from Table 2, where the "all values" column means fitting to the constant clipped level as if it were a genuine sea level, even though the sea level is below this. It can also be seen in the Admiralty Tide Tables (1981), giving decreasing M_2 amplitude for locations upstream from Flat Holm. Such an analysis also yields enhanced higher harmonic terms.

To illustrate this point, consider a level given by :

$$\left.\begin{array}{lll} Z_0 + H \sin \sigma t & \text{if} & \sin \sigma t + \text{ve} \\ Z_0 & \text{if} & \sin \sigma t - \text{ve} \end{array}\right\} \quad (1)$$

Harmonic analysis of this would give several terms :

$$T(t) = Z_0 + \frac{H}{\pi} + \frac{H}{2} \sin \sigma t - \frac{2H}{3\pi} \cos 2\sigma t - \frac{2H}{15\pi} \cos 4\sigma t + \ldots$$

Table 2

Comparison of M_2 amplitudes obtained at Steep Holme for three clipping levels

Clipping level	Fitting to :			
	a) values above clip level		b) all values	
	H	g	H	g
Msl − 3.75 m. .	4.00 m	185.8°	3.82 m	185.6°
Msl − 1.75 m. .	4.02 m	185.3°	2.99 m	185.6°
Msl + 0.25 m. .	3.64 m	184.1°	1.82 m	186.3°

However, our results suggest that a good fit would be obtained to the original function (1) using only the tidal part of the data, and omitting the periods of constant level. A further refinement would be to vary Z_0 both over a spring-neap period to allow for the increase in level due to a forced fortnightly harmonic (LEBLOND) [3], and to allow for variable river discharge. This method would have the advantage of modelling the levels in a more physically meaningful way, in terms of two distinct hydrodynamic regimes.

Acknowledgements

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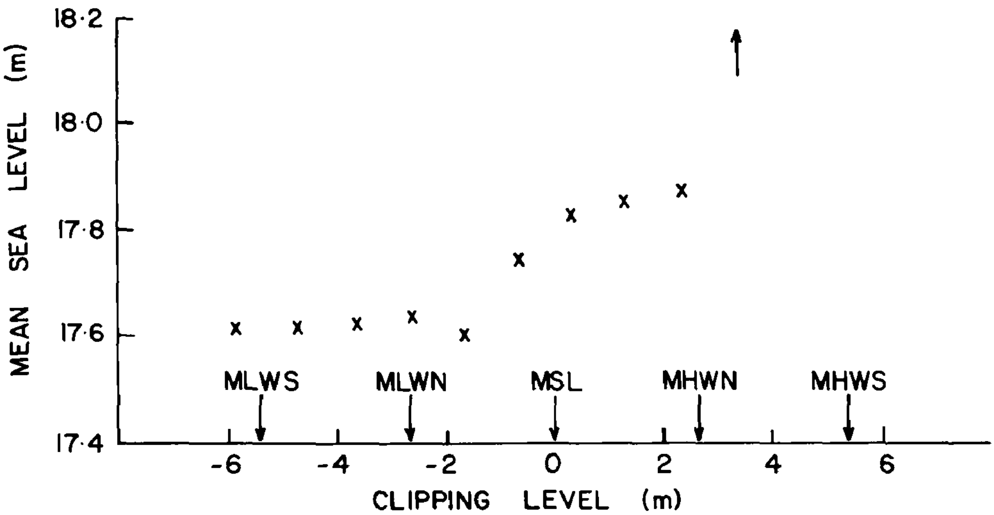

FIG. 4. – Variations in the mean sea level value (Z_0) as a function of clipping level.

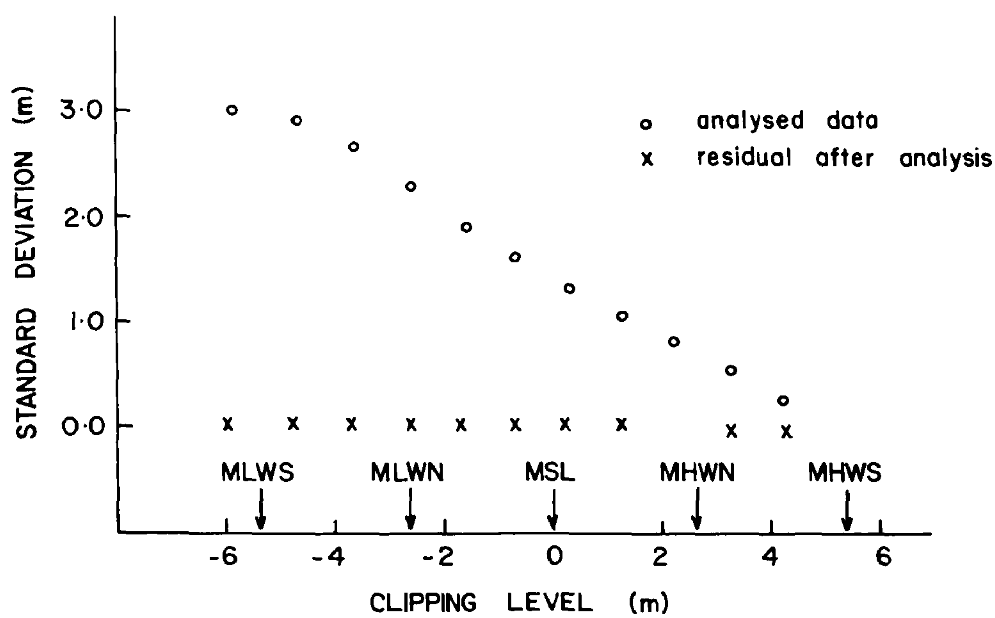

FIG. 5. – Standard deviations of analysed data about a mean value and of residuals, R(t), after analysis, as a function of clipping level.

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