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ANALYSIS OF THE GEOMETRY OF TRANSIT OF A NAVIGATION SATELLITE

by Mark SZYMOŃSKI(*)

INTRODUCTION

The basic parameters characterising the conditions in which measurements to determine ship's position by means of Artificial Satellites are made, are defined and analysed in this paper. These are the size of the service area, the period of satellites' transit through the service area, the elevation angle in the culmination, and the coverage of the globe with the Satellite Navigation System.

THE GEOMETRY OF PASSING OF A NAVIGATION SATELLITE

The users of Satellite Navigation Systems should be aware of the geometry of the movement of the Artificial Satellites in the ship's radiocommunication area (service area) and the resulting conditions of measuring and receiving the data necessary for accurate determination of the ship's position in order to maximize the use of radionavigation receivers of Satellite Systems.

The geometry is dependent directly upon the orbit parameters of the navigation satellites and also the ship's position related to the orbit surface. Both the orbit parameters of artificial satellites and the method of measurement influence the exploitation of the radionavigation Satellite Systems. Taking the above into consideration, the systems can be divided as shown in Table I.

Table I

Characteristics of Navigation Satellites
Altitude Ranges

Altitude Range	Altitude (km)	Period
Low	900 - 2,700	100 - 150 min
Medium	13,000 - 20,000	8 - 12 h
Synchronous	22,000 - 48,000	24 h

(*) Institute of Marine Navigation, Gdynia, Poland

The American System TRANSIT/NNSS belongs to the system with satellites in low altitude orbits. This system is in general used by merchant ships all over the world. The technique used by TRANSIT/NNSS measures Doppler shift (shift in frequency due to satellite motion relative to the navigator's receiver) of signals emanating from the satellite. The Artificial Earth Satellites are placed in polar orbits of about 1,100 km altitude and of eccentricity $e = 0.002 - 0.02$. The orbital movement period of an Artificial Earth Satellite amounts to about 107 min.

The Soviet System CYKADA, started with the satellite KOSMOS-1000 on 31 March 1978, should be mentioned here also (Pravda, April 2, 1978 – TASS agency). It, too, utilizes the doppler technique ("Sputnik as Navigator" by A. Zelezonov, Leningrad Pravda, April 4, 1978).

The system NAVSTAR/GPS now being developed in the USA uses synchronised clocks in the satellite and ship equipment to find the range from satellite to navigator. It belongs to the system with satellites in medium altitude orbits. The navigation satellites will be placed in their circular orbits at an angle 63° to the equator, at an altitude of about 20,000 km.

The Systems MARISAT and INMARSAT are systems with geostationary satellites in high altitude synchronous orbits. Although they are being used exclusively for sea radiocommunications, the possibility of the use of geostationary satellites for navigation purposes also cannot be excluded.

The value of Satellite Navigation Systems lies in the fact that they provide global coverage with great accuracy. The geometry of the relationship between a ship and a navigation satellite determines the conditions of receiving radio signals emanating from the satellite and, therefore, the accuracy of position determination. The conditions assuring the radiocommunication of a satellite with a ship are characterised *inter alia* by such parameters as the size of the service area, the time of satellite passing through the service area, the elevation angle in the culmination, and coverage of the globe with the Satellite Navigation System.

To analyse the above parameters it is convenient to use a simplified model of Artificial Earth Satellite movements, and assume that satellites move around circular orbits in the centre of which the globe is situated.

Figure 1 presents the service area limited by a circle with radius l, and geocentric angle 2 l. The point S refers to the position of a satellite on the orbit at the moment of its rise. Out of triangle OP_0S is found the formula :

$$l_{max} = \arccos\left(\frac{R_e}{R_e + D_s}\right) \tag{1}$$

where l_{max} = the half of the geocentric angle SOS, limiting the service area,
R_e = the Earth radius,
D_s = the orbit altitude.

Time t of satellite passing in the service area is found from the equation :

$$t = \frac{T_s \left[\arccos\left(\frac{R_e}{R_e + D_s}\right)\right]}{180^\circ} \tag{2}$$

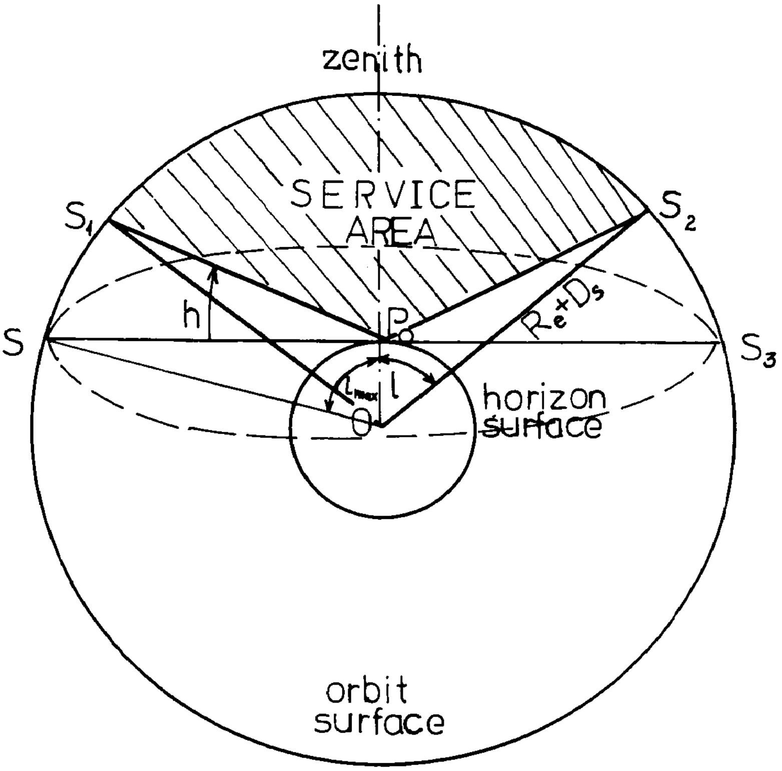

FIG. 1. – The service area for the ship in the orbit surface.

where : T_s = period of Earth passing by satellite,

$$T_s = 2\pi \sqrt{\frac{(R_e + D_s)^3}{\mu}}$$

μ = coefficient of the Earth gravitation field.

The service area, however, under the actual conditions of radio reception, is a little limited. This limitation is due to the influence of tropospheric refraction, interrupting the reception of satellite signals on elevations smaller than about 5°. As a result of the above the angle l, as it is illustrated by Fig. 1, will be limited and will be found from the formula :

$$l = \left[\arccos\left(\frac{R_e}{R_e + D_s} \cdot \cos h\right)\right] - h \qquad (3)$$

where h = elevation angle limiting the non-interrupted tropospheric refraction receipt of radio signals.

The geometry of a satellite's transit presented in Fig. 1 refers, however, to a particular case where the observer is in the orbit surface.

Figure 2 illustrates a more general case of navigation satellite observation from a ship far from the orbit surface at an angle Φ. For $\Phi = 0°$ the satellite's track reflected on the Earth surface is the arc A'L'B' of a great circle. When the ship moves away from the orbit surface the track becomes the arc of a small circle (analogous with parallels). From Fig. 2 :

$$\cos l = \frac{OL}{R_e + D_s} \tag{4}$$

where : OL = a segment on the orbit surface formed from the cutting of the horizon surface with the line joining the Earth center with the point S_o of satellite culmination.

The following equations result from this :

- for the service area :

$$l = \arccos\left[\frac{R_e \cos h}{(R_e + D_s)\cos\phi}\right] - h \tag{5}$$

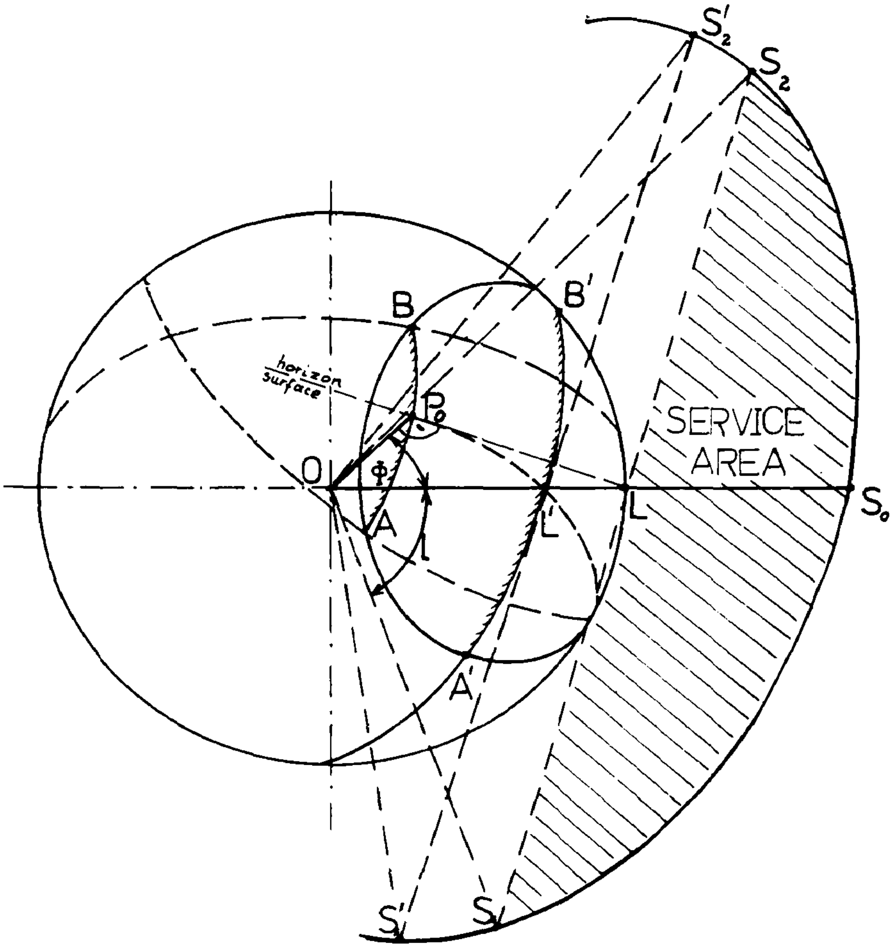

FIG. 2. – The service area for the ship far from the orbit surface at angle Φ.

– for the passing time of the Artificial Earth Satellite in the service area :

$$t = \frac{T_s \left\{ \arccos\left[\dfrac{R_e \cos h}{(R_e + D_s)\cos \phi}\right] - h \right\}}{180^\circ} \tag{6}$$

The following parameter characterising the geometry of the relationship between a ship and a satellite is the elevation angle of the satellite in the culmination h_{max}. From Fig. 3, the elevation h_{max} is :

$$h_{max} = \arctan\left(\frac{z_s - R_2}{x_s}\right) \tag{7}$$

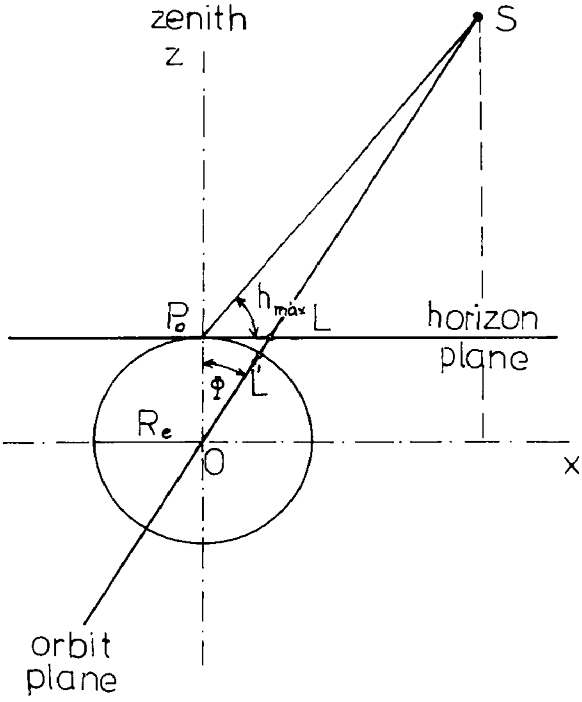

FIG. 3. – The geometry of the relation – a ship (P_0) and navigation satellite (S).

where : x_s, z_s = the Cartesian coordinates of the satellite position.

The curves of l, t, h_{max} from the angle Φ for both the low and the medium orbital systems are presented in Fig. 4.

The parameters of the TRANSIT system, as an example of solving the System with satellites in low altitude orbits, and of NAVSTAR, as an example of the System using satellites in medium altitude orbits, were used for computations. The corresponding data are presented in Table II.

In the use of the Systems mentioned in Table II, the limitations of the service area of the navigation satellites under the two different methods of determination of ship's position ought to be examined separately.

In the first case of satellites in low altitude orbits (doppler technique) the service area is limited both due to the influence of tropospheric refraction and to the effect of cross-track error. The accuracy of the determination of a ship's

Table II

The data for computing the formulas (5), (6), (7)

System	R_e	D_s	h	T_s
TRANSIT . .	6,370 km	1,100 km	0°	107 min
NAVSTAR .	6,370 km	20,240 km	5°	12 h

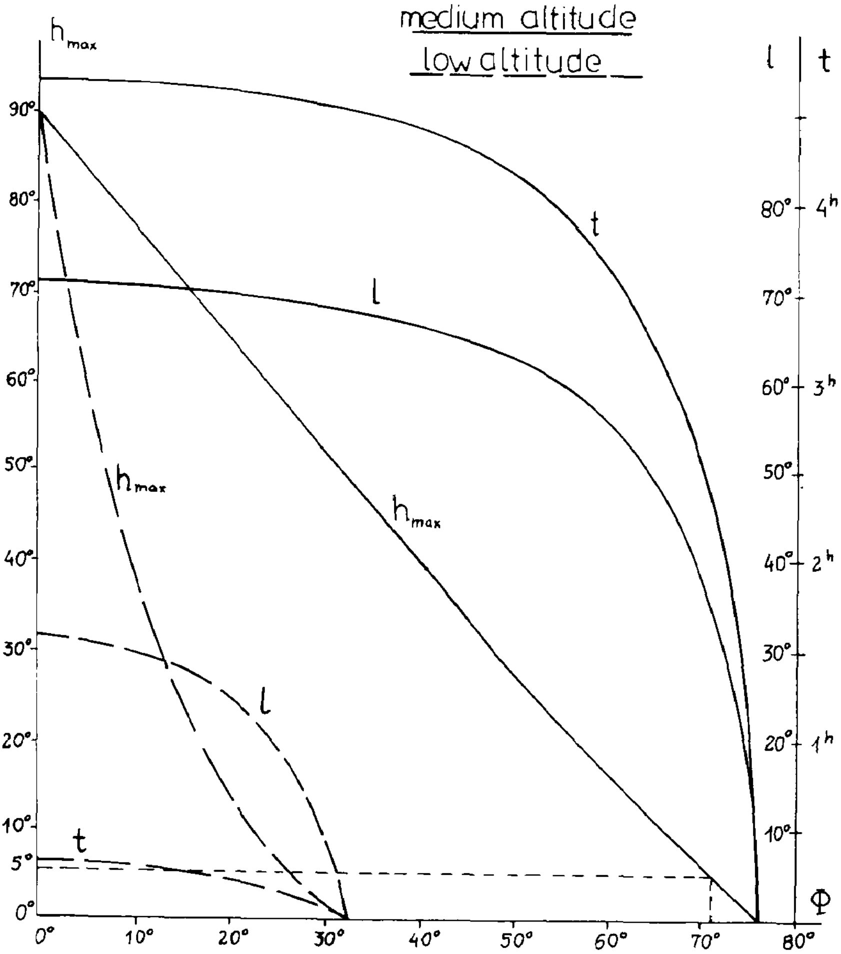

FIG. 4. – The curves l, t, h_{max} from the angle Φ.

position depends primarily on the pass elevation angle. Satellite passes at an elevation angle lower than 10° and higher than 80° at closest approach to the receiver do not afford a satisfactory accuracy of the positions.

But the measurements are continuously made during the whole transit of the satellite on the observer's side. If the elevation angle at the point of closest approach to the receiver is higher than 10° a great number of measurements (in relation to the number required to determine the ship's position) are accomplished, mostly uninterrupted by tropospheric refraction as the Artificial Earth Satellite remains longer on the elevation angle $h \geqslant 5^\circ$ than very low over the horizon.

It should be added that the problems of the service area limitation due to the interruption of the signal reception and those due to the minimizing of the error of the ship's position determination are different. The coordinates of the ship's position may be obtained if the signal is uninterrupted. But the users should evaluate the usefulness of the observations from the geometry of transit of a satellite.

In the second case, when simultaneous measurements are made against several (3 - 4) Artificial Earth Satellites, there is a limitation on the minimum elevation of the satellites being observed. For minimizing the error of position determination in the mean-squared error sense, the criteria GDOP = minimum should be taken into consideration, too.

In the case when the ship is in the surface of the orbit ($\Phi = 0^\circ$), in the System TRANSIT the service area is $2\,l = 64^\circ$, i.e. it covers a surface of the globe limited by a circle with a radius equal to about 1,900 nm. It is not large when compared with the service area for the satellite of NAVSTAR system, amounting to $2\,l = 144.8^\circ$ which corresponds to the surface on a globe limited by a circle with a radius of 4,300 nm.

It should be stressed that the satellite in a low altitude orbit is in the service area for about 19 minutes, while the satellite in medium altitude orbit may be observed for about 5 hours.

It should be noted that the geostationary satellite placed in the altitude of 25,000 km covers an area with a radius of about 4,700 nm, i.e. from the Equator to latitude ± 78°.

But while the satellite in the low altitude orbit, at a given moment, covers a much smaller service area than the stationary satellite, its fast revolution around the earth in 24 hours permits navigation on practically the entire global surface.

The service area also passes according to the movement of the Artificial Earth Satellite. The passing is found from :

$$\Delta l\,(T_s) = \arcsin\left[\sin\left(\frac{360^\circ}{24}\cdot T_s\cos\varphi\right)\cdot\sin i\right] \qquad (8)$$

where φ = geographical latitude,
i = incline of orbit plane.

On the Equator, during one period of the navigation satellite's passing around the globe the service area in the low altitude orbit will change by 26° (T_s = 107 min., $i = 90^\circ$), but for a satellite in medium altitude orbit Δl amounts to 180° (T_s = 12 h, $i = 63^\circ$).

The coverage of the globe by Radionavigation Satellite System is found from

$$\Delta = k \cdot \frac{2l - \frac{1}{N} \cdot \Delta l}{2l} \tag{9}$$

where k = the number of satellites on one orbit,
N = the number of orbits.

The curves of dependence Δl and Δ from geographical latitude are presented in fig. 5.

It appears that on the Equator one satellite of the TRANSIT System gives the coverage in 58%, while in the case of the NAVSTAR System the same coverage will be possible for 3 satellites placed one by one in 3 orbit surfaces. In both cases the coverage is similar, but in the first case the satellite appears periodically over a chosen point of the globe, and in the second case it can be seen at a given point of the globe and then at least one Artificial Earth Satellite is visible continuously.

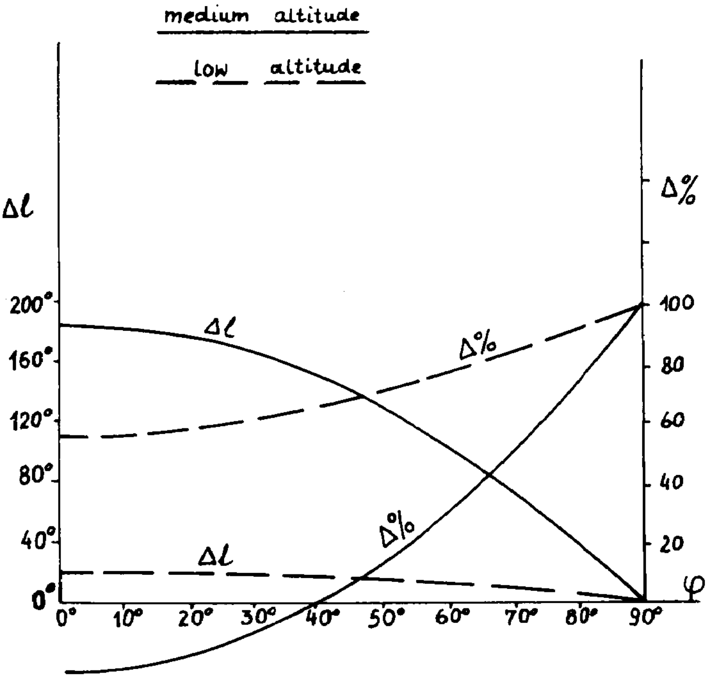

FIG. 5. – The curves Δl, Δ% from the latitude φ.

CONCLUSIONS

The above considerations point out the clear relationship between the choice of parameters of navigation satellite orbits and the measurement method used in a given system.

One of the basic requirements to select a method of measurement is to know the orbit parameters and the number of satellites in the system.

That is why Satellite Systems can be divided in two essential groups :

- systems with only periodical observation of Artificial Earth Satellites,
- systems with continuous observation of one or several satellites simultaneously.

In using satellites in low altitude orbits quick changes of measured parameters (doppler frequency shift or range) are achieved. The use of the measurement of doppler frequency shift is very useful here. The transmitting radio station on the navigation satellite and receiving equipment on a ship are rather simple, and what is important is that the requirement concerning the stability of the reference oscillators in the satellites as well as in the receivers is met by classical crystal oscillators.

The main limitations in the use of the low orbit Navigation Satellite Systems are :

- relatively great discretion or care in observations which would be unnecessary if the number of satellites were increased;
- the dependence on ship's movement in the accuracy of ship's position determined by satellite navigation.

The system of navigation satellites in medium altitude orbits seems to have the greatest potential because of the possibility of practically continuous observation and because results are independent of ship's movement. The simultaneous observation of several satellites in a given region of the globe is assured with the establishment of a reasonable quantity of Artificial Earth Satellites. For example, in the System NAVSTAR/GPS, 18 satellites moving subsynchronously six by three around orbit planes assure the observation of 3 simultaneous measurements from at least 3 satellites which are in the service areas ($\Delta = 3.47$ for $\varphi = 0^\circ$), and 24 satellites in this system would assure the simultaneous observation of at least 5 navigation satellites ($\Delta = 4.63$ for $\varphi = 0^\circ$).

It seems to be most useful to use the range method in this case because the measurements of doppler frequency shift are limited by a small (in comparison with low altitude orbit) relative velocity between the satellite and the receiver, and a long period of the satellite's passing through the service area.

A system with geostationary satellites, in turn, does not assure the coverage of subpolar regions, and near the equator the accuracy of position determination deteriorates. Thus, the system of geostationary satellites can only be used for sea navigation purposes in conjunction with satellites in orbits of a different type.

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