A NUMERICAL METHOD
FOR COMPUTING THE CRAFT'S POSITION
DIRECTLY FROM OBSERVATIONS OF TWO CELESTIAL BODIES OR SIMULATION METHOD

by Petar CUMBELIĆ, B.Sc., Master Mariner[*]

FOREWORD

The modern concept of determining a craft's position by observation of celestial bodies, now almost universally accepted, is the altitude method, often called intercept method, which was first proposed by Commander Marcq Saint-Hilaire, of the French Navy, who later became an admiral. The craft's position, i.e. latitude and longitude, is thus computed indirectly, as the solution of the problem is to determine lines of position.

Several methods for computing the position directly have been proposed so far. All those proposals begin with simultaneous or quasi-simultaneous observations of two celestial bodies, or observations of one or two bodies within a short interval of time.

Spherical trigonometry formulae are used to compute different parameters and eventually to determine latitude and longitude.

Nowadays, with the achievements made in the field of electronics and computer technology, those proposals have become attractive.

Mathematical solutions which seemed inconvenient and cumbersome and involved work forms which were long and complicated, are now quite acceptable, as programmable calculators do all the work, giving us the final results after checking.

Today navigators can choose between different programmed and programmable calculators which solve navigational problems fast and with great accuracy, thus substituting for nautical tables and in some cases even almanacs.

(*) c/o Dabinovic S.A.M., 57, rue Grimaldi, MC 98000 Monaco.

Nautical tables and almanacs were, until recently, indispensable for solving navigational problems, in particular those of celestial navigation, and great circle, composite, plane and middle latitude sailing. These books have to be included, even now, in a craft's library, but for routine work calculators can substitute for them successfully.

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This article will try to demonstrate a new method and a new approach to the perennial problem of determining a craft's position by observation of celestial bodies. This method has been called the "Simulation Method", as in our calculations the latitude is simulated until the actual one is determined. The simulated latitude can differ considerably from a craft's actual latitude without any adverse effect on the calculation and final result. We need not simulate longitude to determine the craft's position; however, by simulating it, too, we could shorten our program considerably. Once we have computed latitude, longitude is easily found.

The Simulation Method is based upon simultaneous or quasi-simultaneous observations of two celestial bodies, or observations of one or two bodies within a short interval of time. In the latter case, the altitude of the first body must be corrected for the craft's run between the two observations, thus enabling the navigator to consider both observations as having been made in the same place. Better results will be obtained if three celestial bodies, symmetrically situated around the horizon, are observed. In this latter case, instead of a fix we have a triangle of position. The centre of the circle inscribed in the triangle is the craft's position, or we can develop formulae for systematic (constant) error for observed altitudes.

By observing three celestial bodies with a difference in azimuth of about 120°, we eliminate constant errors in the observed altitudes, which are predominant in normal circumstances.

To compute latitude, first we enter the almanac to find the Greenwich hour angles, declinations and corrections for altitudes of the observed celestial bodies. (If we have a calculator with sufficient memory, we can program almanac data and corrections for observed altitudes). Then we have to compute local hour angles with simulated latitude, which in practice is dead reckoning or assumed latitude.

The differences between the Greenwich hour angles for two celestial bodies observed simultaneously and the local hour angles for the same two observed bodies are equal. However, when local hour angles are calculated with a simulated latitude, which is not actual latitude, these differences are not equal. We have to try another latitude, and perhaps yet another, until the differences actually become equal. This can be written mathematically as follows :

$$(GHA2 - GHA1) - (LHA2 - LHA1) = 0 \qquad (1)$$

In order to approach the zero in formula (1) as soon as possible we should not change the simulated latitude haphazardly; instead, we should determine the rate of change of one variable with respect to another by means of differentiation. The independent variable in this case is the hour angle and the dependent variable is the latitude. The rate of change of the independent variable will be the difference of the above mentioned differences, i.e.,

$$dt = (GHA2 - GHA1) - (LHA2 - LHA1) \qquad (2)$$

As it will be demonstrated later in the calculations, the rate of change of the dependent variable is :

$$dL = \pm \ \frac{\cos \ aL}{\cot Z_2 \pm \cot Z_1} \ \times dt \tag{11}$$

In normal navigational practice, and when observed celestial bodies are not close to the meridian, results will be obtained after the first trial. The first latitude obtained by correcting simulated latitude for dL will be named Approximate Latitude One (L1). By repeating the calculation with L1 instead of aL the Approximate Latitude Two (L2) is obtained. The procedure should be repeated until Ln satisfies the precision criterion established earlier. However, L1 will very often satisfy this criterion.

DEVELOPMENT OF THE FORMULAE

The rate of change of the independent variable has already been defined by formula (2). If we substitute local hour angles (LHA) with meridian angles (t) we get the following formula :

$$dt = (GHA2 - GHA1) \pm (t2 - t1) \tag{2a}$$

In computing meridian angles the law of cosine for sides for spherical triangles is used :

$$t_1 = \pm \ arc \ cos \ \frac{\sin \ h_1 - \sin \ aL \ \sin \ d_1}{\cos \ aL \ \cos \ d_1} \tag{3}$$

$$t_2 = \pm \ arc \ cos \ \frac{\sin \ h_2 - \sin \ aL \ \sin \ d_2}{\cos \ aL \ \cos \ d_2} \tag{4}$$

Subtracting equation (3) from equation (4), taking the absolute value and substituting $t_2 \sim t_1$ with Δt we have the following equation :

$$\Delta t = arc \ cos \ \frac{\sin \ h_2 - \sin \ aL \ \sin \ d_2}{\cos \ aL \ \cos \ d_2} \ \pm \ arc \ cos \ \frac{\sin \ h_1 - \sin \ aL \ \sin \ d_1}{\cos \ aL \ \cos \ d_1} \tag{5}$$

In equation (5) the variables are Δt et aL. After differentiation of equation (5) and transformation, we have equation :

$$dL = \pm \ \frac{dt}{\dfrac{\sin \ h_2 \ \sin \ aL - \sin \ d_2}{\cos^2 \ aL \ \cos \ d_2 \ \sin \ t_2} \ \pm \ \dfrac{\sin \ h_1 \ \sin \ aL - \sin \ d_1}{\cos^2 \ aL \ \cos \ d_1 \ \sin \ t_1}} \tag{6}$$

If it undergoes the following substitution :

$$\sin \ h_{1/2} \ \sin \ aL - \sin \ d_{1/2} = a_{1/2}$$
$$\cos^2 \ aL \ \cos \ d_{1/2} \ \sin \ t_{1/2} = b_{1/2}$$

equation (6) can be written :

$$dL = \pm \ \frac{b_1 b_2}{a_2 b_1 \pm a_1 b_2} \ dt \tag{7}$$

Now approximate latitude one (L_1) can be stated as :

$$L_1 = aL + dL \tag{8}$$

and actual latitude :

$$L = aL + dL_1 + dL_2 + dL_3 + ... + dL_n \tag{9}$$

dL_1, dL_2, dL_3, etc., are computed by formula (6), so that we enter it with aL, L_1, L_2, etc., respectively.

For practical purposes in celestial navigation the first two summands on the right-hand side of equation (9) will suffice. In less convenient cases we should take three or four summands.

If formula (6) undergoes the following substitutions :

$$\frac{\sin h_{1/2} \sin aL - \sin d_{1/2}}{\cos^2 aL \cos d_{1/2} \sin t_{1/2}} = -\frac{\cos Z_{1/2}}{\sin Z_{1/2}} \times \frac{1}{\cos aL} = -\frac{\cot Z_{1/2}}{\cos aL}$$

we have formula :

$$dL = \pm \frac{\cos aL}{\cot Z_2 \pm \cot Z_1} \times dt \tag{11}$$

Azimuth angles ($Z_{1/2}$) we compute by formula :

$$Z_{1/2} = \pm \arccos \frac{\sin d_{1/2} - \sin aL \sin h_{1/2}}{\cos aL \cos h_{1/2}} \tag{18}$$

As can be seen from the above, this method uses quite complex formulae, not particularly suitable for step-by-step calculation by means of logarithms and tables. However, now that compact, pocket-size, sophisticated and programmable calculators are at our disposal, the Simulation Method, in combination with a programmable calculator, I believe, has a significant advantage over other methods as will be seen from the examples in this article.

To compute longitude we use equation :

$$\lambda = GHA_1 \pm \arccos \frac{\sin h_1 - \sin L \sin d_1}{\cos L \cos d_1} \tag{10}$$

or equation :

$$\lambda = GHA_2 \pm \arccos \frac{\sin h_2 - \sin L \sin d_2}{\cos L \cos d_2} \tag{10a}$$

CRITERION OF ACCURACY

Now we shall discuss the afore-mentioned Criterion of Accuracy, and also analyze the case when the calculator computes L_1, L_2, L_3, etc.

By definition we have the derivate of y with respect to x when dx becomes infinitesimal. In practice it means that both dt and dL must be small. This is normal nowadays except when one of the bodies comes close to the celestial meridian. In the latter case dt can be considerable though dL may be small. The consequence is that we must calculate L_2 or L_3 to satisfy the accuracy criterion.

As far as latitude and longitude are concerned, accuracy within half a mile will suffice for practical purposes; however, we calculate longitude with computed latitude and when the celestial body is close to the meridian a small error in latitude can produce a considerable error in longitude.

In order to analyze the magnitude of an error in longitude caused by an error in latitude, we must find the derivative of t with respect to L of the following equation :

$$\cos t = \frac{\sin h - \sin L \sin d}{\cos L \cos d}$$

$$dt = \frac{\sin h \sin L - \sin d}{\cos^2 L \cos d \sin t} dL$$

$$dt = dL \cot Z \sec L \tag{12}$$

From equation (12), we can see that the change of t (dt) due to the change of L (dL) increases when the azimuth decreases and latitude increases. When the azimuth approaches 0° (360°) or 180° and the latitude 90°, dt approaches an infinite number.

In order to make our calculations precise, we shall use 0.001° (3.6″) or about 0.05 nautical mile for our accuracy criterion. The program is written so that the calculator calculates the latitudes (L_1, L_2, etc.) as long as the above mentioned criterion is met, and after that it starts to compute the longitude. The calculator also displays "L" for latitude and "long" for longitude, as well as the minus sign for south latitude and west longitude.

OBSERVATIONS FOR A RUNNING FIX

When observations are taken at two different times, the correction for any run made in between must be applied. The first observed altitude must be corrected for the course and distance covered in between the two observations. To find the rate of change of altitude due to the rate of change of latitude we must calculate the derivative of h (altitude) with respect to L (latitude) from the equation :

$$\sin h = \sin L \sin d + \cos L \cos d \cos t$$

$$dh = \frac{\cos L \sin d - \sin L \cos d \cos t}{\cos h} dL$$

$$dh_L = dL \cos Z \tag{13}$$

To find the rate of change of altitude due to the rate of change of longitude (meridian angle) we must calculate the derivative of h with respect to t from the same equation :

$$\sin h = \sin L \sin d + \cos L \cos d \cos t$$

$$dh_t = - \frac{\cos L \cos d \sin t}{\cos h} dt$$

$$dh_t = dt \cos L \sin Z \tag{14}$$

(the minus sign has been dropped because dh_t will be applied to west longitude)

From mid-latitude sailing we have the following formulae :

$$dL = 1 = D \cos C; \quad dt = DLo = D \sin C \sec L$$

Now we can write formulae (13) and (14) as :

$$dh_L = D \cos Z \cos C \tag{15}$$

$$dh_t = D \sin Z \sin C \tag{16}$$

If we add formulae (15) and (16) we have formulae :

$$dh = D \; (\cos Z \cos C + \sin Z \sin C) \qquad (17/1)$$

$$\text{and} \quad dh = D \cos (Z - C) = D \cos RZ \qquad (17/2)$$

The azimuth angle is calculated by formula

$$Z = \pm \arccos \frac{\sin d_1 - \sin aL \sin h_1}{\cos aL \cos h_1} \qquad (18)$$

SOLUTION BY CALCULATOR

For computing the craft's position i.e., latitude and longitude, by this method we can use any programmable calculator, or even a simple, non-programmable one, but in the latter case we have to work out our problem step by step. If we use a sophisticated calculator with sufficient memory we can incorporate data which we would normally obtain from an almanac.

For this work, the Japanese made "CASIO FX-602P" programmable calculator has been used. It is an inexpensive, compact, liquid-crystal, pocket-size calculator (see Fig. 1) which, apart from four basic mathematical operations, incorporates logarithmic, trigonometric, hyperbolic and other functions. It has also a fixed program for calculation of standard deviation, alphabetic mode, as well as ten programmes. As its brochure notes "This unit has a variable range of the input capacity from 32 program steps with 88 independent memories to 512 steps with 22 memories. These combinations can be selected optionally to execute an effective programming. The FA-1 adaptor, an optional accessory, allows the connection of a cassette tape recorder to the calculator for the storage of programmes and memory content on cassette tapes for later re-loading". The Casio Calculator, as we can see, has limited capacity, therefore the data from an almanac cannot be stored in the memory. The first step would be to extract from an almanac GHA_1, GHA_2, d_1, d_2 and corrections for the observed altitudes. These data should be put into the calculator memory and after that the calculator will do the rest, and after an interval of time it will display "L" and "long" on the screen.

The interval of time taken for Approximate Latitude One (L_1) is about 9.5 seconds, for L_2 about 19 seconds, and for every next Approximate Latitude about 9.5 seconds.

The question : "Where are we ?" no longer arises nowadays. Instead we ask ourselves : "Is our position there where other navigational systems indicate ?" Dead reckoning or assumed position is not far away from the real one. That means the result will appear on the screen after about 19 seconds for most problems.

To bring this analysis to a conclusion we may state : The navigator who uses this method, this calculator and this program will have the craft's position after about 19 seconds in most cases. The result is always accurate.

If by any chance we select a celestial body very close to the celestial meridian the result is not obtainable, as the cotangent of zero is infinite. In the latter case we may disregard that body or calculate latitude by meridian transit instead.
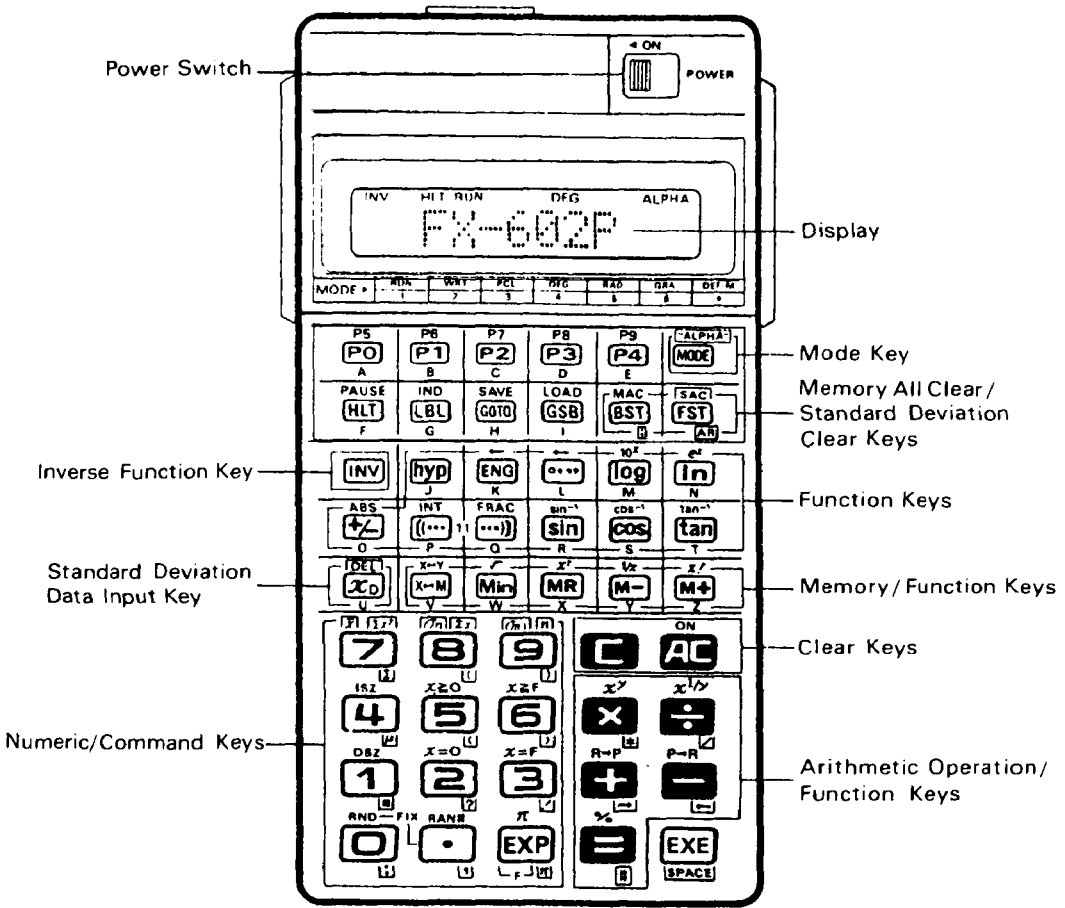
FIG. 1. – The calculator CASIO FX-602 P.

The same program is used for simultaneous observations as well as for observations in an interval of time. A subordinate program corrects the first altitude for the run made in between, thus enabling the navigator to consider both observations as having been made in the same place.

Step-by-Step Instructions for the Use of the Calculator "CASIO FX-602P"

1. Switch on the power and select program PO. "h1?" displays on the screen.
2. Write in the first observed and corrected altitude [*].

    Minutes and parts of a minute write in the calculator as if they were parts of a degree, for example 25°56' write in as 25.56. A subordinate program converts it to 25.9333.

    Press the execute key. "h2?" displays on the screen.

(*) When stars are observed, we feed the calculator with sextant altitude and a subordinate program corrects it for refraction and depression.

PROGRAMMES FOR CALCULATOR "CASIO FX-602P" FOR COMPUTING LATITUDE AND LONGITUDE

Program P0   DEG "h1?" HLT GSBP7 Min01 "h2?" HLT GSBP7 Min02 (GSBP9)
             "d1?" HLT GSBP7 Min03 "d2?" HLT GSBP7 Min04
             "G1?" HLT GSBP7 +/- GSBP8 Min05 "G2?" HLT GSBP7 +/- GSBP8
             Min06 "aL?" HLT GSBP7 Min00 "aλ?" HLT GSBP7 Min10
             MR05 - MR10 = GSBP8 Min07 MR06 - MR10 = GSBP8 Min08 (GSBP4)
             (GSBP3) LBL3 90 - MR01 = Min09 90 - MR02 - Min11 MR03 + MR09
             = Min12 MR03 - MR09 = Min13 MR04 + MR11 = Min14 MR04 - MR11
             = Min15 MR12 MinF MR00 X≥F GOTO0 MR14 MinF MR00 X≥F GOTO0
             MR00 MinF MR13 X≥F GOTO1 MR15 X≥F GOTO1 GOTO2
             LBL0 MR00 - .05 = Min00 GOTO3 LBL1 MR00 + .05 = Min00 GOTO3
             LBL2 MR03 Min19 MR01 GSBP5 MR07 GSBP6 Min07
                  MR04 Min19 MR02 GSBP5 MR08 GSBP6 Min08
                  MR01 Min19 MR03 GSBP5 MR07 GSBP6 Min09
                  MR02 Min19 MR04 GSBP5 MR08 GSBP6 Min11
             MR06 - MR05 = - (MR08 - MR07) = Min12 MR11 TAN 1/X - MR09
             TAN 1/X = : MR00 COS X↔Y = x MR12 = Min13 + MR00 = Min00
             .001 MinF MR13 ABS X≥F GOTO3 MR05 - MR07 = GSBP8 Min10
             MR00 GSBP1 "L = ╫ " HLT MR10 GSBP1 "λ=╫ "

Program P1   Min19 INT + (MR19 FRAC x .6) = FIX3

Program P2   + .7 = TAN 1/X+MR19 = : 60= +/- +

Program P3   MR07 COS x MR03 COS x MR00 COS + MR03 SIN x MR00 SIN = SIN^{-1}
             Min19 - MR01 X↔Y = x 60 = FIX1 "a = ╫ " HLT MR03 GSBP5 MR07
             GSBP6 X≥0 GOTO0 + 360 = LBL0 FIX1 "Zn = ╫ " HLT MR02 Min01
             MR04 Min03 MR08 Min07 GSBP3

Program P4   1 MinF MR03 SIN - MR00 SIN x MR01 SIN = : MR00 COS : MR01
             COS = Min19 ABS X≥F GOTO0 MR19 GOTO1 LBL0 MR19 1/X LBL1
             COS^{-1} Min19 MR07 GSBP6 Min09 0 "C?" HLT Min12 "v?" HLT :
             60 = Min13 "T1?" HLT GSBP7 Min14 "T2?" HLT GSBP7 - MR14 =
             Min14 x MR13 x (MR12 - MR09) COS = MR01 = Min01

Program P5   SIN - MR00 SIN - MR19 SIN = : MR00 COS : MR19 COS = COS^{-1} Min19

Program P6   X≥0 GOTO0 MR19 +/- GOTO1 LBL0 MR19 LBL1

Program P7   Min19 INT + (MR19 FRAC x 2 10^X) : 60 =

Program P8   Min19 180 MinF MR19 ABS X≥F GOTO0 MR19 GOTO1 LBL0 360
             + MR19 = Min19 ABS X≥F GOTO0 MR19 LBL1

Program P9   "he?" HLT√ x 1.77 = Min19
             MR01 GSBP2 MR01 = Min01 MR02 GSBP2 MR02 = Min02

FIG. 2

3. Write in the second observed and corrected altitude [*]. Press the execute key. "he?" (height of eye) displays on the screen.
4. Write in the height of eye. Press the execute key. "d1?" displays on the screen.
5. Write in the declination of the first body (South declination is written with the minus sign). Press the execute key. "d2?" displays on the screen.

[*] When stars are observed, we feed the calculator with sextant altitude and a subordinate program corrects it for refraction and depression.

6. Write in the declination of the second body. Press the execute key. "G1?" displays on the screen.

7. Write in the Greenwich hour angle of the first body. Press the execute key. "G2?" displays on the screen.

8. Write in the Greenwich hour angle of the second body. Press the execute key. "aL?" displays on the screen.

9. Write in the simulated (DR or assumed) latitude (South latitude is written with the minus sign.) Press the execute key. "al?" displays on the screen.

When a celestial body is close to the meridian, the simulated latitude may not fall within the required limits i.e., declination plus/minus zenith distance. In the latter case the program is compiled so that the latitude will be changed until it is brought within its limits.

10. Write in the simulated longitude (an integral degree). Press the execute key. For simultaneous or quasi-simultaneous observations after pressing the execute key, the calculator starts computing and after about 19 seconds actual latitude will be displayed on the screen.

11. Press the execute key. Longitude displays on the screen.

When observations are made in an interval of time step 10 is as follows :

10. After pressing the execute key, "C?" displays on the screen.

11. Write in the true course. Press the execute key. "v?" displays on the screen.

12. Write in the craft's speed in knots. Press the execute key. "T1?" displays on the screen.

13. Write in the time of first observation. It is written in a similar way as other data i.e., minutes and parts of a minute are entered as if they were parts of an hour, for example 1^h30^m is written 1.3. The same subordinate program converts it to 1.5.

After about 19 seconds latitude displays on the screen, for example L = 32.076 (that is lat. = 32°07.6'N).

14. Press the execute key. Longitude displays on the screen, for example l = 30.241 (that is long. = 30°24.1'E).

This is the end of the program.

Example 1. – Two stars observed quasi-simultaneously

During the evening twilight on May 15th, 1979, the 19.12 DR position of the craft is lat. 30°06.5'N; long. 44°45'W. At C = $22^h10^m37^s$ the navigator observes Capella with a marine sextant having no IC, from the height of eye of 10 metres, hs = 25°56'. At C = $22^h12^m05^s$ the navigator observes Sirius, hs = 15°16.5'.

Required. – The observed position at 19.12 hours.

Solution. – In table 1.

This is a typical problem from actual navigating practice which the author of this article worked out on a ship of which he was the Master.

If we wish to check this calculation by the intercept method, we can do it with this same calculator with a program stored in it, and if we use the latitude and longitude we calculated above, it will be found that $a_1 = 0$, $Zn_1 = 310°$, $a_2 = 0$ and $Zn_2 = 240°$. For this we do not need to feed any additional entries into the calculator as it already has all the data required and so all we need to do is to press a program key.

TABLE 1

STEP	DATA INPUT OPERATION			READ-OUT	REMARKS
1			PO	hs1	Sextant altitude
2	(hs1)	25.56	EXE	hs2	
3	(hs2)	15.165	EXE	he ?	Height of eye
4	(he)	10	EXE	d1 ?	
5	(d1)	45.586	EXE	d2 ?	
6	(d2)	− 16.415	EXE	G1 ?	
7	(G1)	126.547	EXE	G2 ?	
8	(G2)	105.004	EXE	aL ?	Simulated latitude
9	(aL)	30	EXE	al ?	Simulated longitude
10	(a)	− 45	EXE	L = 29.584	lat. = 29°58.4 'N
11			EXE	l = − 44.104	long. = 44°10.4 'W

Example 2. − Two stars in an interval of time of 7^m30^s
(Data are given in table 2)

Required. − The observed position.

Solution. − In table 2.

TABLE 2

STEP	DATA INPUT OPERATION			READ-OUT	REMARKS
1			PO	hs1	Sextant altitude
2	(hs1)	75.48	EXE	hs2	
3	(hs2)	37.225	EXE	he ?	Height of eye
4	(he)	10	EXE	d1 ?	
5	(d1)	8.491	EXE	d2 ?	
6	(d2)	− 26.231	EXE	G1 ?	
7	(G1)	291.526	EXE	G2 ?	
8	(G2)	344.097	EXE	aL ?	Simulated latitude
9	(aL)	11.20	EXE	al ?	Simulated longitude
10	(al)	54	EXE	C ?	Course
11	(C)	288	EXE	v ?	Speed
12	(v)	10	EXE	T1 ?	Time, first observation
13	(T1)	18.15	EXE	T2 ?	Time, second observation
14	(T2)	18.225	EXE	L = 11.184	lat. = 11°18.4 'N
15			EXE	l = 53.48	long. = 53°48 'E

If we wish to check this calculation by the intercept method we continue :

16		P3	a = 0.0	a_1	= 0.0
17		EXE	Zn = 98.7	Zn_1	= 98.7°
18		EXE	a = 0.0	a_2	= 0.0
19		EXE	Zn = 223.8	Zn_2	= 223.8°

Example 3. − The Sun in an interval of time of 1^h30^m
(Data are given in table 1)

Required. − The observed position.

Solution. − In table 3.

TABLE 3

STEP	DATA INPUT OPERATION			READ-OUT	REMARKS
1			PO	h1 ?	
2	(h1)	57.102	EXE	h2 ?	
3	(h2)	72.416	EXE	d1 ?	
4	(d1)	17.052	EXE	d2 ?	
5	(d2)	17.042	EXE	G1 ?	
6	(G1)	297.328	EXE	G2 ?	
7	(G2)	320.101	EXE	aL ?	Simulated latitude
8	(aL)	32.1	EXE	al ?	Simulated longitude
9	(al)	30	EXE	C ?	Course
10	(C)	81	EXE	v ?	Speed
11	(v)	10	EXE	T1 ?	Time, first observation
12	(T1)	10	EXE	T2 ?	Time, second observation
13	(T2)	11.3	EXE	L = 32.077	lat. = 32°07.7'N
14			EXE	l = 30.241	long. = 30°24.1'E

If we wish to check this calculation by the intercept method we continue :

15		P3	a = 0.0	a_1	= 0.0
16		EXE	Zn = 109.7	Zn_1	= 109.7°
17		EXE	a = 0.0	a_2	= 0.0
18		EXE	Zn = 148.2	Zn_2	= 148.2°

Position by three stars symmetrically situated around the horizon

As mentioned before, when three stars symmetrically situated around the horizon are observed, a triangle of position is obtained, as each pair of stars gives a position (a vertex of a triangle).

In the centre of the circle inscribed in the triangle is the craft's position, free of constant error, or, in other words, the best position one can obtain from celestial observations.

I have been determining my ship's position, for more than twenty years spent at sea, by observing three stars symmetrically situated around the horizon and on most occasions the position was accurate within one mile.

Now, using the Simulation Method and a Casio FX-602P calculator, I can have the three stars' position about 58 seconds after all the data have been fed into the calculator. It computes three positions (vertices of triangle of position) and, eventually, the best position free of constant error. It displays only the best position, but, if we wished, we could take out from the memory the first three positions. Also, we can check the solution by the Intercept Method, merely by pressing a program key and without feeding the calculator additional data.

The additional feature here is a subordinate program which corrects the sextant altitudes we feed into the calculator.

Example 4. – Three stars observed in an interval of 7ᵐ

During the evening twilight on December 23rd, 1982, the 18.41 satellite position of the vessel is lat. 36°N, long. 6°33.5′W. At C = 17ʰ34ᵐ23ˢ the navigator observes Fomalhaut with a marine sextant having no IC, from the height of eye of 16 metres, hs = 24°16′. At C = 17ʰ36ᵐ11ˢ the navigator observes Capella, hs = 25°29′, and at C = 17ʰ41ᵐ26ˢ, Vega hs = 34°39.2′. Course is : C = 112°, speed is : v = 10.5 knots.

Required. – The observed position at 18.41 hours.

Solution. – In table 4.

TABLE 4

STEP	DATA INPUT OPERATION			READ-OUT	REMARKS
1			PO	hs1 ?	Sextant altitude
2	(hs1)	24.16	EXE	hs2 ?	
3	(hs2)	25.29	EXE	hs3 ?	
4	(hs3)	34.392	EXE	he ?	Height of eye
5	(he)	16	EXE	d1 ?	
6	(d1)	– 29.43	EXE	d2 ?	
7	(d2)	45.589	EXE	d3 ?	
8	(d3)	38.461	EXE	G1?	GHA
9	(G1)	371.213	EXE	G2 ?	
10	(G2)	637.071	EXE	G3 ?	
11	(G3)	438.128	EXE	aL ?	Simulated latitude
12	(aL)	36	EXE	al ?	Simulated longitude
13	(al)	– 6	EXE	C ?	Course
14	(C)	112	EXE	v ?	Speed
15	(v)	10.5	EXE	T1 ?	Time, first observation
16	(T1)	17.344	EXE	T2 ?	Time, 2nd observation
17	(T2)	17.362	EXE	T3 ?	Time, 3rd observation
18	(T3)	17.414	EXE	L = 36.002	lat. = 36°00.2′N
19			EXE	l = – 6.343	long. = 6°34.3′W

The above position is free from constant error and if we wish the other three positions (vertices of triangle) we continue :

	EXE	L = 36.007	Fomalhaut-Capella	
	EXE	l = – 6.358		
	EXE	L = 36.006	Fomalhaut - Vega	
	EXE	l = – 6.331		
	EXE	L = 35.592	Capella - Vega	
	EXE	l = – 6.340		

If we wish to solve the problem by the intercept method, where aL = 35°59.2′N and al = 6°34′W, that is the position from Capella - Vega, we continue :

	P3	a = – 1.4	a₁
	EXE	Zn = 184.6	Zn₁
	EXE	a = 0.0	a₂
	EXE	Zn = 50.2	Zn₂
	EXE	a = 0.0	a₃
	EXE	Zn = 296.1	Zn₃

If we work out positions with the above parameters (a and Zn) we shall obtain about the same results as those above worked out directly by the Simulation Method.

Theoretically speaking, the results obtained by the Simulation Method are more accurate than those obtained by the Intercept Method especially for high altitudes.

CONCLUSION

Calculators can be used by applying this method of programming for solving most navigational problems, such as position by intercept method, identification of stars, great circle, composite, plane and mid-latitude sailing. If we have a "Casio FX-602 P" calculator, once all the memory has been used up and the steps have been completed, we can feed our programmes into a cassette and reload them when needed.

ANNEXE

ABBREVIATIONS

a,	difference of altitudes
aL,	assumed latitude
C,	chronometer time, course
d,	declination
D,	distance
DLo,	difference of longitude
DR,	dead reckoning
dh,	differential of h (altitude)
dh_L,	differential of h with respect to L
dh_t,	differential of h with respect to t
dt,	differential of t (meridian angle)
Δt,	difference of t
G,	Greenwich hour angle
GHA,	Greenwich hour angle
h,	altitude
hs,	sextant altitude
L,	latitude
l,	difference of latitude, longitude
LHA,	local hour angle
RZ,	relative azimuth angle
T,	time, interval of time
t,	meridian angle
Z,	azimuth angle
Zn,	azimuth
λ,	longitude
v,	speed
he,	height of eye