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THE SPECIES CONCORDANCE METHOD OF TIDE PREDICTION IN ESTUARIES

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ABSTRACT

The harmonic method of tide prediction is developed to its fullest extent, so that it can deal with tide curves which are so distorted that they have gradient discontinuities. It is better, however, to use a new two-step method known as the method of species concordance. This method is applied to the Gironde, and then to the Loire, where the variation in fresh-water level is important.

1. PROBLEMS OF TIDE-PREDICTION IN ESTUARIES

Two major problems arise in predicting water levels in an estuary :
(a) the distortion of the tide-curve by shallow water;
(b) the effect of the variation in fluvial discharge.

These two problems are different in character : (a) is fundamentally linked to the astronomical tide, which is predictable in the long term (deterministic); whereas the variation of discharge which causes (b) is largely unpredictable in the long term (stochastic). Furthermore, while (b) becomes more and more important with increasing distance from the mouth, (a) at first increases and later decreases. (See Fig. 1).

Studies of the tide in the Gironde (Fig. 2) were concerned more with (a) than with (b); in the Loire (Fig. 3) which is more affected by variations of fresh-water discharge than the Gironde, the emphasis was on (b).

We now consider the problems separately.

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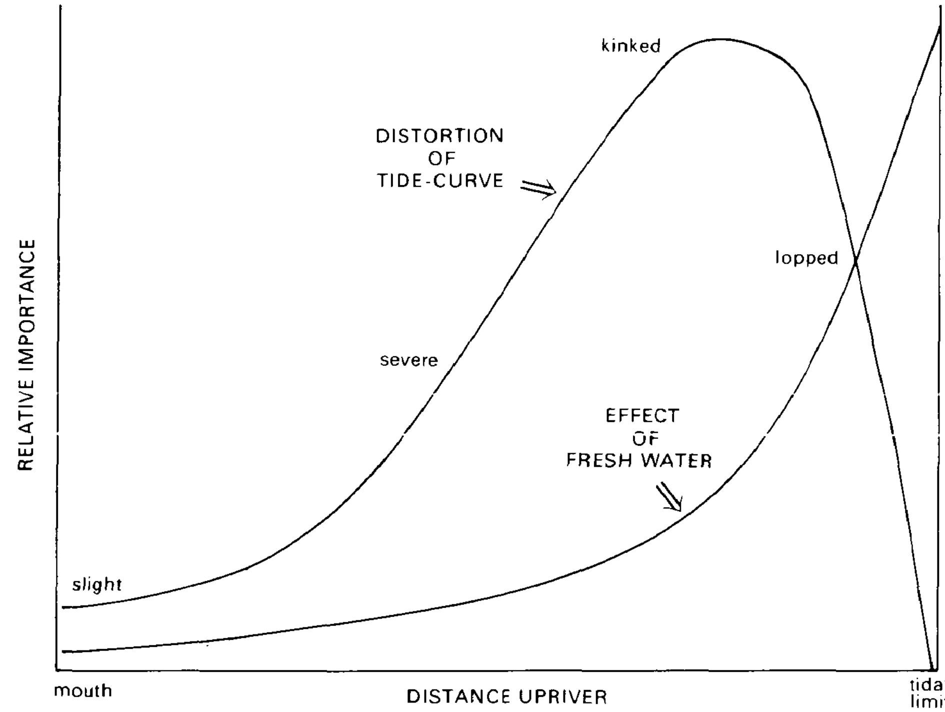

FIG. 1. — Relative importance of tide-prediction problems in an estuary.

2. THE DISTORTION OF THE TIDE-CURVE

2.1. Introduction

The tide-curve, which at the mouth may approximate a sine curve (over a single tidal cycle), becomes distorted as one moves up-river, principally owing to bottom friction. Low water is retarded more than high water, and as a result the curve becomes asymmetric; the tide rises quicker than it falls. The distortion may become so great that the curve is no longer smooth; a discontinuity in the gradient (kink) develops at commencement of rise. In extreme cases, a bore may form. Beyond the point of maximum distortion, the tide becomes increasingly drowned by the river flow, which smoothes the curve, but renders the tide almost unpredictable in the long term.

2.2. Conventional Harmonic Methods

In order to take the asymmetry of the curve into account, the harmonic method uses a large number of shallow-water constituents, whose frequencies are combinations of those of the basic astronomical constituents.

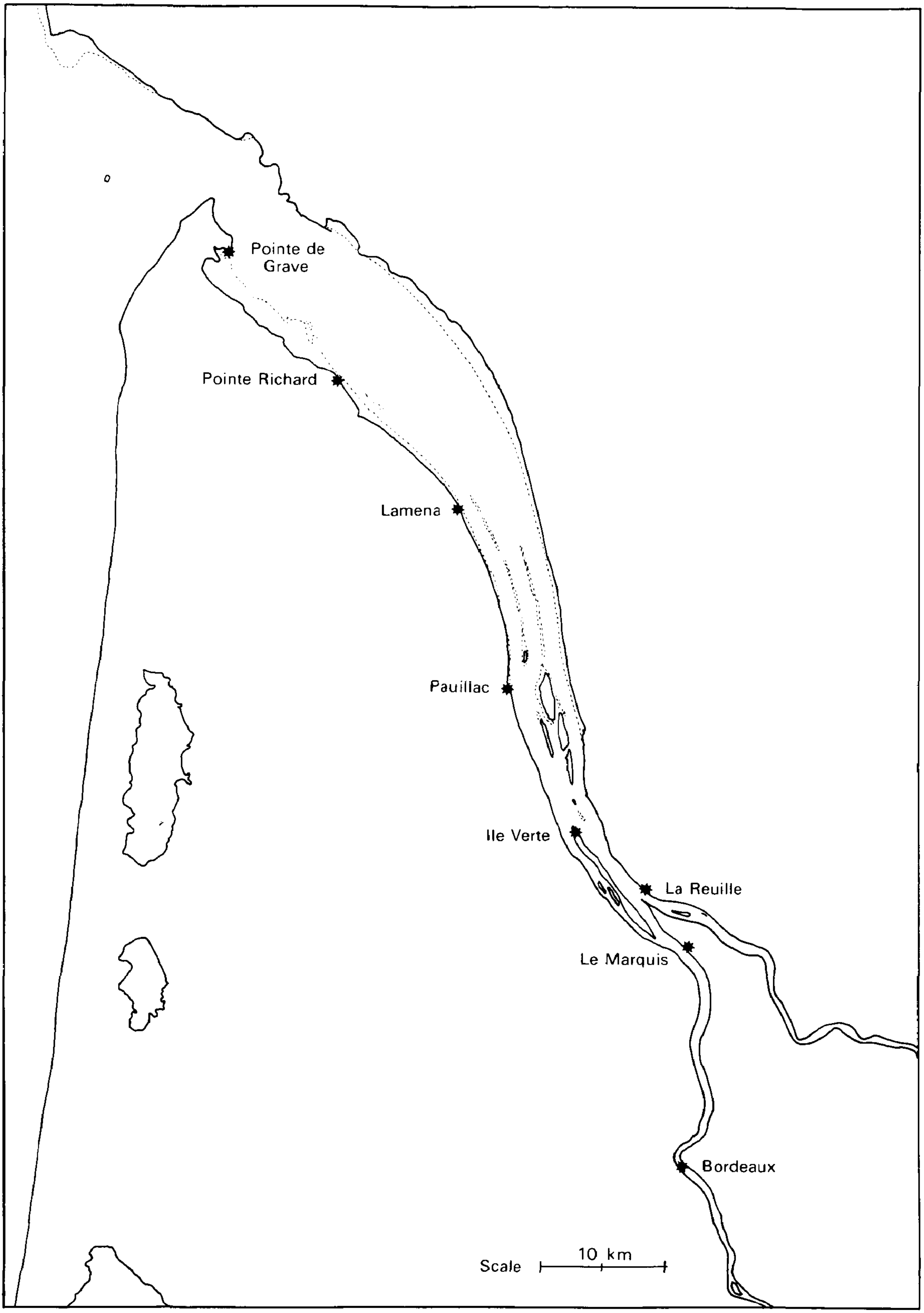

FIG. 2. — **Map of the Gironde estuary.**

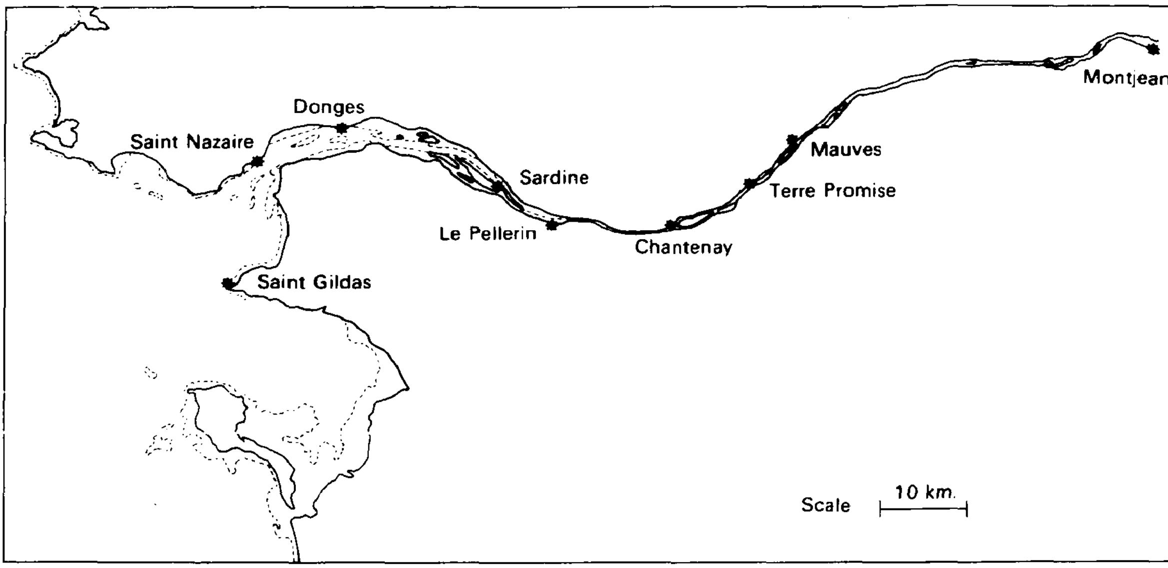

FIG. 3. — Map of the Loire estuary.

The form of the harmonic method introduced by DOODSON in 1921 (henceforth called the Standard Harmonic Method — S.H.M.), used a standard list of 60 constituents, in species 0, 1, 2, 3, 4 and 6. This was about the maximum number that could be accommodated, either in a tide-predicting machine or by hand, and the method worked well for tides of any regime, provided that the curve was not too distorted.

Although it was realized that the S.H.M. was inadequate to represent a severely distorted tide, until the mid-1960s it was generally believed that the proliferation of shallow-water constituents required would be too great for practical prediction. Two methods, those of DOODSON (1957) and HORN (1960) had been devised to predict the turning-points of shallow-water tide-curves, but the "vexatious problem of continuous tide prediction" remained unsolved. Then ROSSITER and LENNON (1967) showed that the tide in the Thames, at Tower Pier, could be represented adequately by a list of 113 constituents, and predicted by computer. Subsequently they modified the list of constituents, adding 40 shallow-water terms to Doodson's 60 to make the Extended Harmonic Method — E.H.M. This list of 100 constituents is designed for a semi-diurnal tidal regime, and extends as far as species 12. It is used for routine predictions for the following ports in Great Britain (and also some ports elsewhere) :

(i) certain estuarine ports, notably in the River Thames;
(ii) ports in the southern North Sea, an area particularly affected by shallow water distortion and also by storm surges;
(iii) Newlyn in Cornwall, which is the basis of the land levelling system.

Other ports use the S.H.M.

The formula used for these routine predictions is well known :

$$\zeta(t) = \zeta_0 + \sum_{i=1}^{M} f_i H_i \cos(\omega_i t + E_i + u_i - g_i)$$

where $\zeta(t)$ is the height of the tide above chart datum at a time t;
ζ_0 is the height of the mean sea level above the chart datum;
M is the number of constituents used (N = 60 for the S.H.M., 100 for the E.H.M.);
E_i is the equilibrium phase-lag of the i^{th} constituent;
H_i and g_i are the amplitude and phase-lag of the i^{th} constituent (the harmonic constants);
f_i and u_i are the nodal factors, introduced to allow for the fact that observations are made over a period of one year rather than 19 years;
ω_i is the frequency of the i^{th} constituent.

The selection of harmonic constants given in Fig. 4 illustrates the increasing effect of fresh-water discharge as one proceeds away from the mouth of an estuary. The mean level A_0 increases monotonically, which reflects the increase in level of the river bed. The amplitude of the fortnightly luni-solar constituent MSf increases some twelvefold, for the following reason : low waters in the upper reaches fall to the same level on each tide (provided that the F.W.L. remains constant), whereas the level of high waters, which is not so dependent on the F.W.L., varies between springs and neaps.

HARMONIC / PORT	A_0 H (m)	MSf H (m)	M_2 H (m)	M_2 g (°)	M_2 Δg (h m)
St. Gildas	3.161	0,029	1.698	99.5	0 00
St. Nazaire	3.182	0.021	1.748	105.6	0 13
Donges	3.263	0.018	1.788	110.7	0 23
Sardine	3.692	0.133	1.603	126.7	0 56
Le Pellerin	3.770	0.165	1.468	134.5	1 12
Chantenay	4.256	0.227	1.260	153.2	1 51
Terre Promise	4.637	0.244	1.034	163.2	2 12
Mauves	5.881	0.144	0.450	188.1	3 03

FIG. 4. — Harmonic constants for the Loire : A_0, MSf, M_2 (from an analysis of tidal records for 1977).

The amplitude of the principal lunar semi-diurnal constituent M_2 at first slightly increases (as far as Donges), and subsequently decreases; near the tidal limit (at Mauves) it is quite small (< 0.5 m). Its phase increases progressively; the phase difference Δg relative to the mouth (St. Gildas), expressed in time units, indicates the mean delay of the tide. This value is used in the method of species concordance described in para. 2.7.

2.3. Shallow water distortion in the Gironde and the Loire

Fig. 5 shows complete tidal spectra for the Gironde at low resolution. It represents the results of a Fourier analysis of 30 days of observations. Fig. 6 is a similar ensemble of spectra for the Loire, representing the mean of analyses by Fast Fourier Transform (F.F.T.) of heights for eleven consecutive periods of 32 Lunar days.

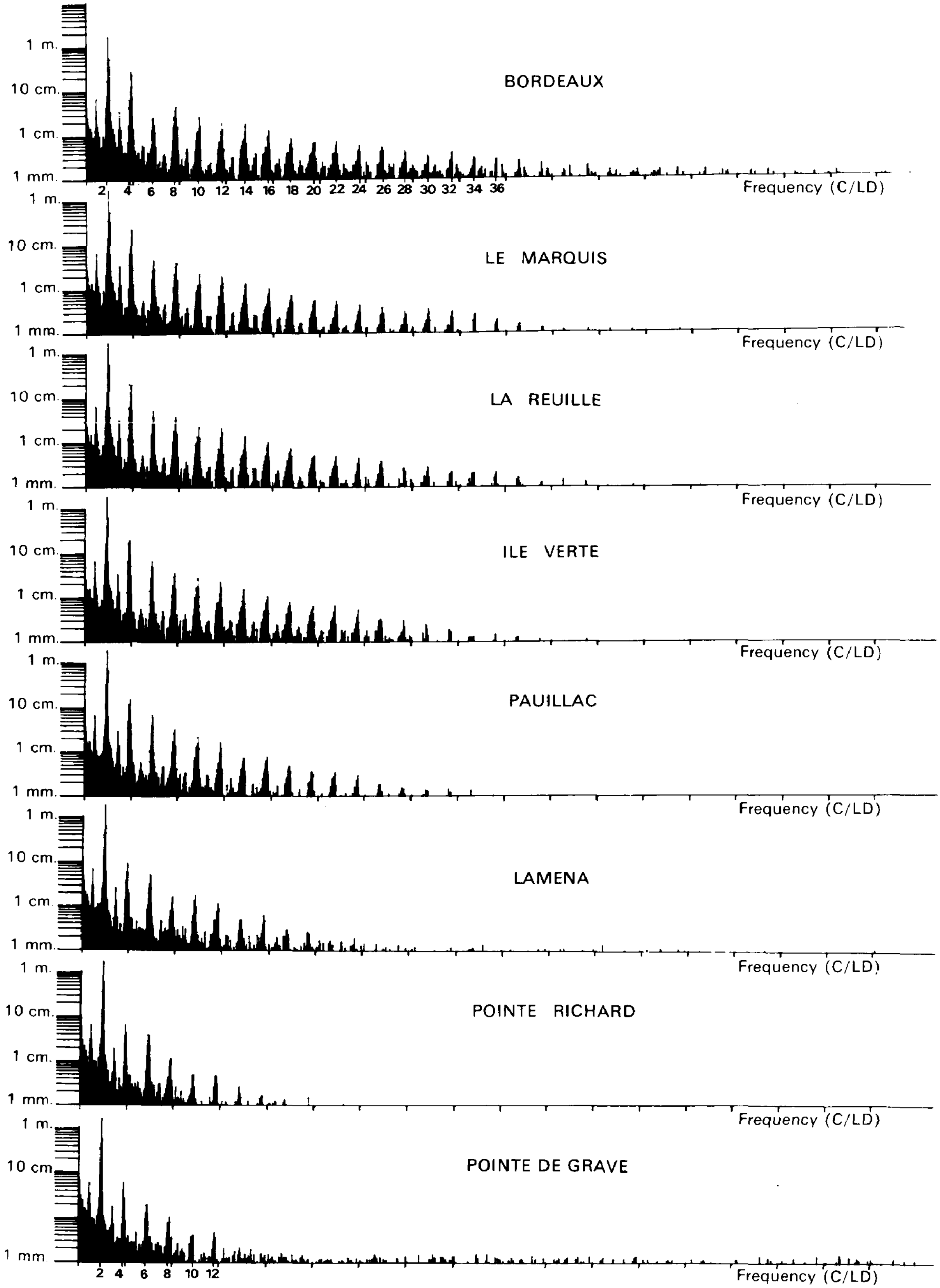

FIG. 5. — Low-resolution spectra for the Gironde.

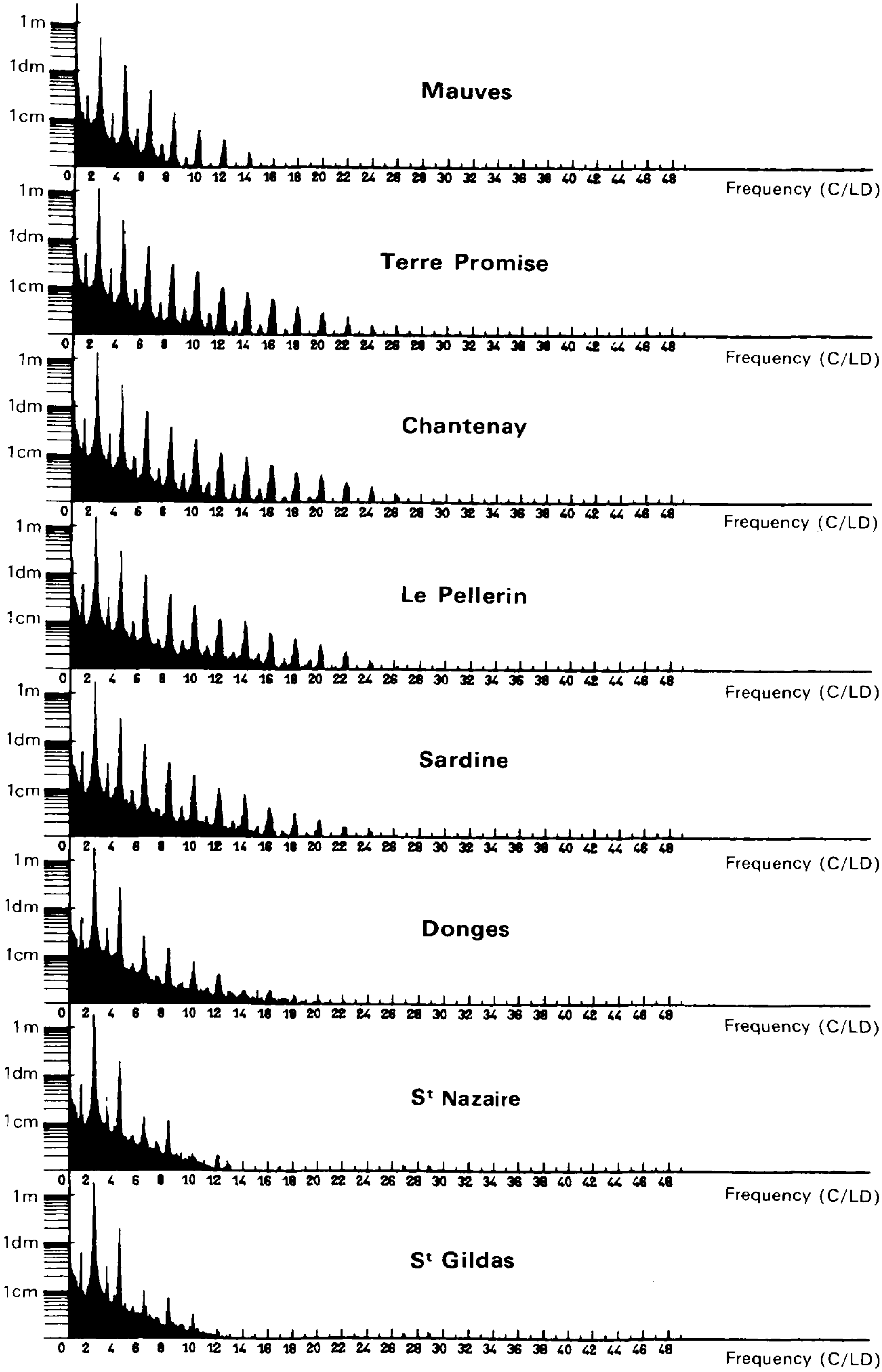

FIG. 6. — Low-resolution spectra for the Loire.

As expected in a semi-diurnal regime, every spectrum in both sets shows a series of peaks of energy at frequencies in the neighbourhood of 1, 2, 3,... cycles per lunar day, and especially in the even species. The striking feature of many of these spectra is the considerable amount of energy at high frequencies above 12 c/d, suggesting that the E.H.M. would be inadequate.

In the Loire, the shallow-water distortion reaches a maximum at Chantenay, where the spectral lines are identifiable up to species 28. Further up-river, there is less energy at high frequency.

The tide in the Gironde is markedly more distorted than that in the Loire. The four ports above Pauillac all have tides with a degree of distortion greater than the maximum observed in the Loire. At Bordeaux, lines can be identified in species 40; the spring tide curve there (illustrated in Fig. 7) has a discontinuity of gradient or "kink" at commencement of rise, which necessitates treating the falling and rising limbs separately when interpolating heights. Fig. 7 does not show a reduced distortion as the tidal limit is approached, but no doubt it exists above Bordeaux.

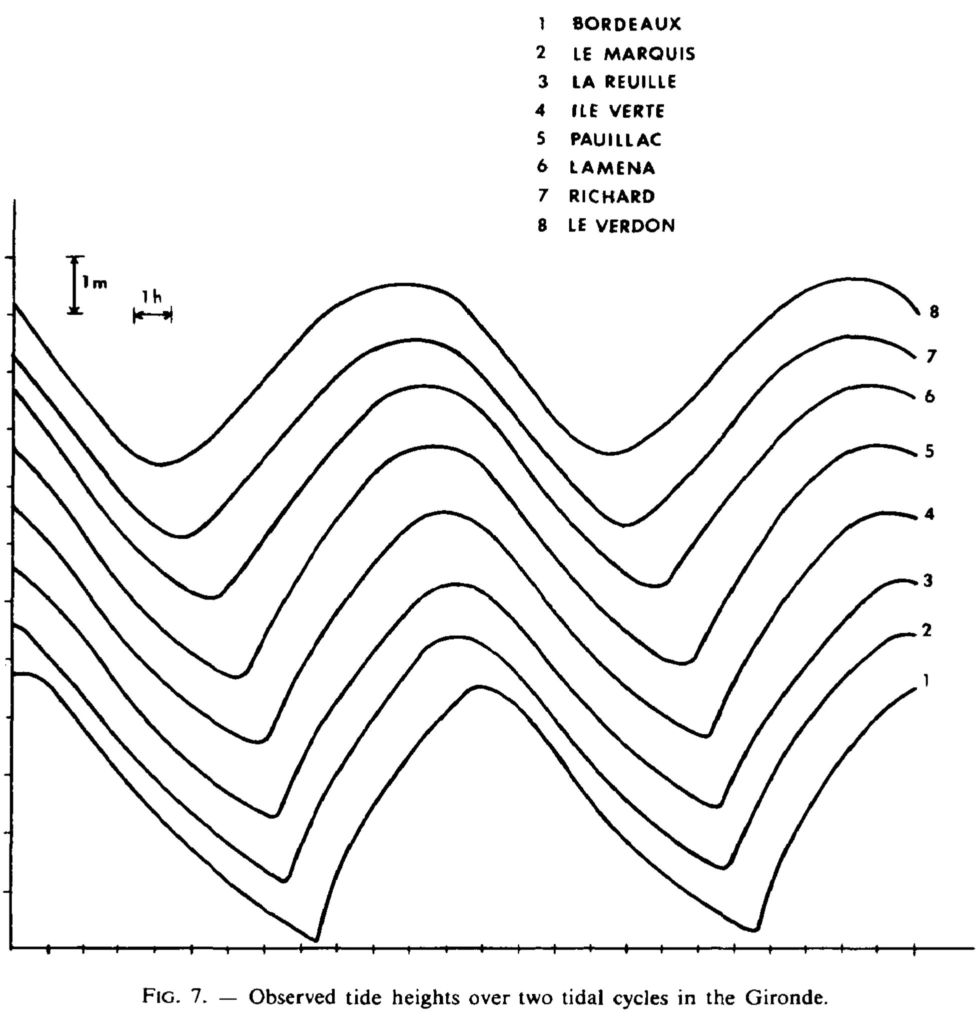

FIG. 7. — Observed tide heights over two tidal cycles in the Gironde.

In order to obtain a sufficiently accurate representation of the distorted tide-curves, the tide height has to be specified every ten minutes in the Gironde, and every fifteen minutes in the Loire.

2.4. Significant constituents in shallow-water tides

A difficulty associated with the E.H.M. is that, in order to evaluate the nodal factors f_i and u_i, one ought to be able to identify all the high-frequency constituents. This becomes impracticable in the higher species, because several different combinations of basic astronomical frequencies can combine to give the same (or very nearly the same) high frequency. For example, in species 6, MNK_6 and $4MN_6$ are very close; so are $2MU_6$ and $2MNKS_6$: it would need more than a year of observation to separate them.

When this problem arises in species with frequencies as low as 6 c/d, one can imagine how difficult it would be to extend the conventional technique of the harmonic method to species 28 or even further. A new approach was clearly required; because it takes the harmonic method as far as it can possibly go, it is called the Full Harmonic Method (F.H.M.). It was introduced at E.P.S.H.O.M. by DEMERLIAC (1973) and developed by SIMON (1981).

We first need to know the frequencies of the constituents in the high-frequency species. Fig. 8 shows the spectra of the even species 4 to 28 at Bordeaux, each to the same frequency scale, and arranged so that the central frequency of each is a multiple of that of M_2 (denoted henceforth by ω_m). Lines in similar positions relative to the central frequency recur in all species, enabling a standard list of 30 lines in species 2 to be drawn up. Thus the frequency of the i^{th} line in species s (s = 0, 2, 4,... 28) is given by

$$\omega_{si} = \omega_{2i} + \left(\frac{s-2}{2}\right)\omega_m \qquad i = 1, 2, \ldots 30$$

The same reasoning applies to the odd species, though of course in a semi-diurnal regime there is no need to go to quite such high frequencies, and only 22 lines are significant. The frequency of the i^{th} line in species s (s = 1, 3, 5, 7,...) is given by

$$\omega_{si} = \omega_{1i} + \left(\frac{s-1}{2}\right)\omega_m \qquad i = 1, 2, 3, \ldots 22$$

2.5. Formulation of the Full Harmonic Method (F.H.M.)

We start from the well-known formula quoted in para. 2.2, viz.

$$\zeta(t) = \zeta_0 + \sum_{i=1}^{M} f_i h_i \cos(\omega_i t + E_i + u_i - g_i) \qquad (1)$$

Let $A_i = f_i h_i$ (2)

and $\varepsilon_i = g_i - (E_i + u_i)$ (3)

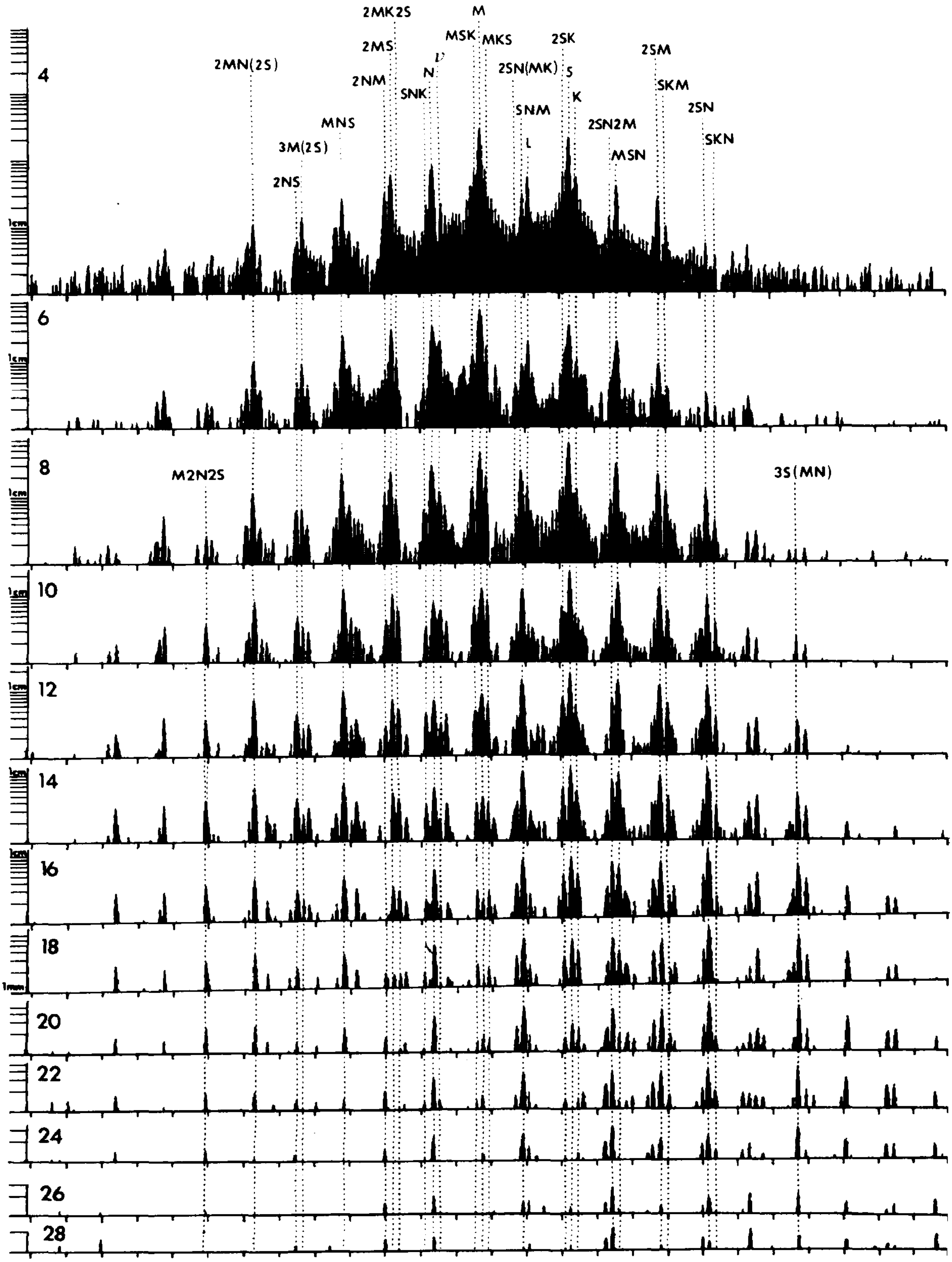

FIG. 8. — High-resolution spectra of even species at Bordeaux.

the formula simplifies to :

$$\zeta(t) = \zeta_0 + \sum_{i=1}^{M} A_i \cos(\omega_i t - \varepsilon_i) \qquad (4)$$

Here M = 60 for the S.H.M., 100 for the E.M.H., and several hundred for the F.H.M.

We group the constituents into species, and includes ζ_o in species 0, giving :

$$\zeta(t) = \sum_s \sum_i A_{si} \cos(\omega_{si} t - \varepsilon_{si}) \qquad (5)$$

where, for even species, i = 1, 2, 3,... 30, and for odd species, i = 1, 2, 3,... 22.

Note that we already have formulae for ω_{si} (see para. 2.4); those for E_{si} and u_{si} are similar. Those for f_{si} are :

$$f_{si} = f_{2i} \times f_m^{(s-2)/2} \text{ for even species} \qquad (6)$$

$$f_{si} = f_{1i} \times f_m^{(s-1)/2} \text{ for odd species} \qquad (7)$$

We introduce complex numbers by writing :

$$\zeta(t) = \frac{1}{2} \sum_s \sum_i A_{si} [\exp\{j(\omega_{si} t - \varepsilon_{si})\} + \exp\{-j(\omega_{si} t - \varepsilon_{si})\}] \qquad (8)$$

Let a differential frequency ν_{si} be defined by $\omega_{si} = S\omega_1 + \nu_{si}$ (9)
where $\omega_1 = 2\pi$ radians per lunar day. On making this substitution, we have :

$$\zeta(t) = \frac{1}{2} [\sum_s \exp(js\omega_1 t) \sum_i A_{si} \exp\{j(\nu_{si} t - \varepsilon_{si})\}$$
$$+ \sum_s \exp(-js\omega_1 t) \sum_i A_{si} \exp\{-j(\nu_{si} t - \varepsilon_{si})\}] \qquad (10)$$

This may be simplified by defining a set of complex numbers :

$$C_s(t) = \sum_i A_{si} \exp\{j(\nu_{si} t - \varepsilon_{si})\} \qquad (11)$$

so that :

$$\zeta(t) = \frac{1}{2} \sum_s \{C_s(t) \exp(js\omega_1 t) + C^*_s(t) \exp(-js\omega t)\} \qquad (12)$$

$C_s(t)$ is the complex amplitude of the tide in species s, and is known as the reduced vector in species s. It varies but slowly, since ν_{si} is a relatively small quantity. We suppose that it is specified at intervals of T = 1 lunar day.

Let $\eta(t_1, t_0)$ be the approximate height of the tide obtained by replacing the slowly-varying $C_s(t)$ by its fixed value at a specified epoch t_0. (This is known as "the height of the tide at time t reduced to the epoch t_0").

$$\text{i.e. } \eta(t_1, t_0) = \frac{1}{2} \sum_s \{C_s(t_0) \exp(js\omega_1 t) + C^*_s(t_0) \exp(-js\omega_1 t)\} \qquad (13)$$

The tide height $\zeta(t)$ may be approximated thus :

$$\text{by } \eta(t, t_0 - T) \text{ in the interval } t_0 - \frac{3T}{2} \leqslant t < t_0 - \frac{T}{2}$$

$$\text{by } \eta(t, t_0) \text{ in the interval } t_0 - \frac{T}{2} \leqslant t < t_0 + \frac{T}{2}$$

$$\text{by } \eta(t, t_0 + T) \text{ in the interval } t_0 + \frac{T}{2} \leqslant t < t_0 + \frac{3T}{2}$$

To obtain the true height $\zeta(t)$ in the interval $t_0 - T/2 \leqslant t < t_0 + T/2$ quadratic interpolation may be used; we suppose that :

$$\zeta(t) = a_0 + a_1(t - t_0) + a_2(t - t_0)^2 \qquad (14)$$

By putting $t = t_0 - T$, t_0 and $t_0 + T$, a_0, a_1 and a_2 are easily evaluated, and we obtain the following expression for $\zeta(t)$:

$$\zeta(t) = \eta(t, t_0) + \frac{t - t_0}{2T} \eta(t, t_0 + T) - \eta(t, t_0 - T)$$

$$+ \frac{(t - t_0)^2}{2T^2} \eta(t, t_0 - T) - 2\eta(t, t_0) + \eta(t, t_0 + T) \qquad (15)$$

To summarize, prediction is carried out in the following steps (Fig. 9) :

(i) Computation of the reduced vectors C_s at successive intervals of 1 lunar day, using equations (2), (3), (9) and (11).

(ii) Computation of the reduced heights $\eta(t, t_0)$ using equation (13); this is performed using the Fast Fourier Transform.

(iii) Computation of the real heights $\zeta(t)$ in lunar time, at intervals of 1/128 lunar day (i.e. every 11.6443 minutes).

(iv) A further interpolation to express the heights in solar time.

The use of the F.F.T., and the need to compute the reduced vectors only once every lunar day, makes the computing time manageable, even though the number of constituents (about 500 for a kinked tide) is much greater than that used in the E.H.M.

2.6. Results of the Full Harmonic Method in the Gironde

Bordeaux was used as the test port in the Gironde. Most of the comparisons between the observed and the predicted tide were deliberately made for periods when the stage in the rivers Garonne and Dordogne was close to zero. The results using the F.H.M. were a considerable improvement on those using the previously used routine method (the conventional concordance method). Nevertheless, certain discrepancies remained at low tide, as shown by the examples in Fig. 10. It was therefore decided to introduce a new method, which may be called the species concordance method.

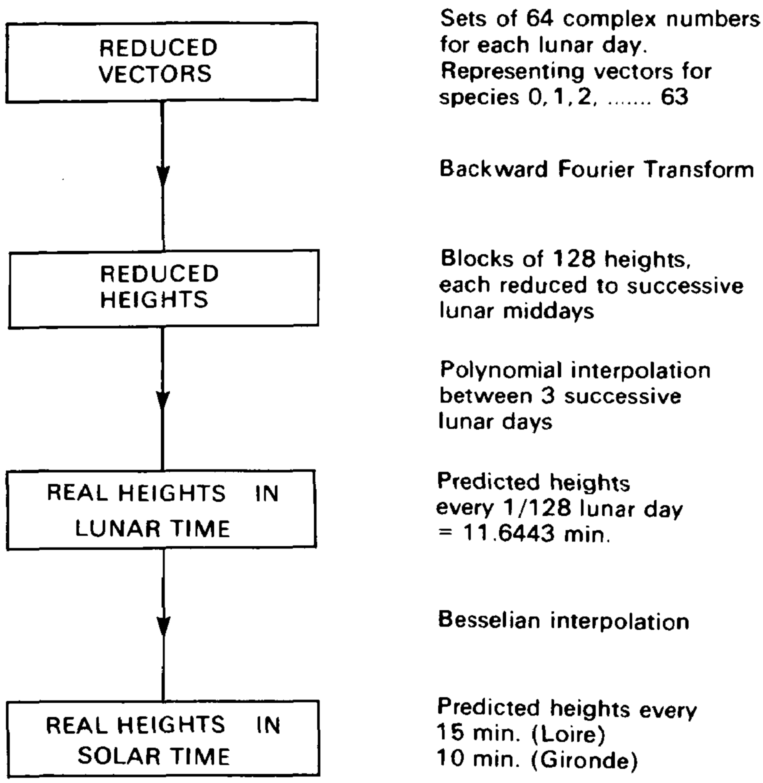

FIG. 9. — Prediction by the Full Harmonic Method.

2.7. The method of species concordance

The harmonic method, in whatever guise, is a one-port method, in that it goes from the harmonic constants for a port to the predicted heights for the same port. The best way to avoid the proliferation in the number of terms is to use a two-port method. The harmonic method (e.g. in its standard form) is used to predict the relatively undistorted tide at the mouth of the estuary, and a secondary technique to predict therefrom the tides in the estuary. After all, most of the shallow-water distortion in the higher species originates within the estuary.

The same prediction sequence is used as in the full harmonic method, and the relationship between the tide at the standard port and that at the secondary port is expressed in terms of the reduced vectors (Fig. 11).

The reduced vectors C_s for the standard port at the mouth are computed as indicated in para. 2.5. Because the tide there is relatively less distorted, only a few species need be taken into account, (e.g. $s = 0, 1, 2, 3, 4, 6, 8$, for the Gironde). We suppose that the reduced vectors D_s for a secondary port are given by :

$D_s(t + \tau) = M_s(t)\, C_2^{s/2}(t)$ for even species

and $D_s(t + \tau) = M_s(t)\, C_s(t)$ for odd species (only 1 and 3 were considered in Gironde)

Here τ represents the time taken for the tide to propagate from the standard port to the secondary port. It is given by the difference between phase of M_2 at the

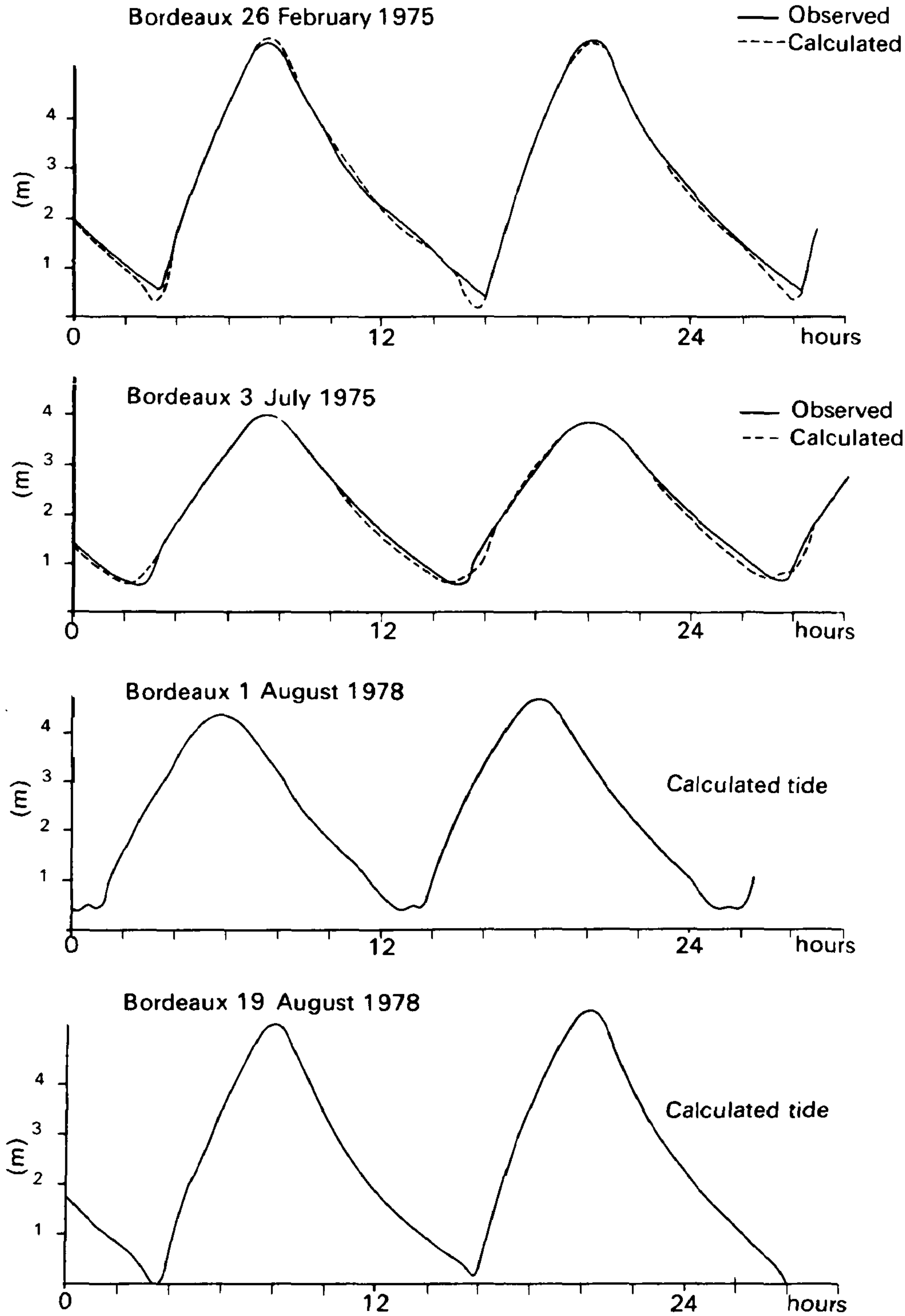

FIG. 10. — Observed and predicted tides at Bordeaux.

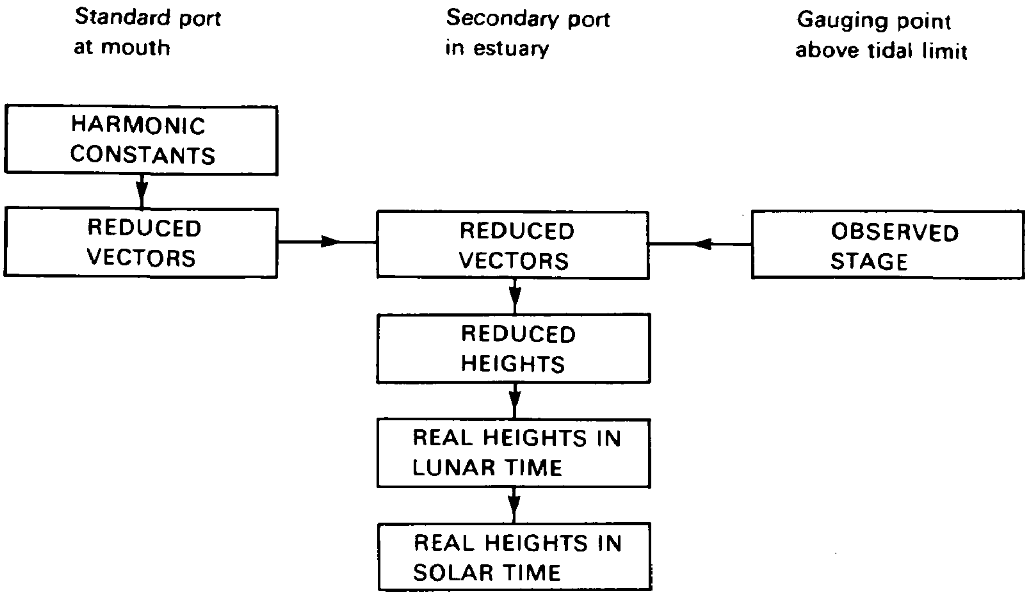

FIG. 11. — Prediction by the method of Species Concordance.

two ports, expressed in time. For example, at the mouth of the Gironde, g of M_2 = 139°.3 and at Bordeaux, g of M_2 = 217°.5. In this case, putting ω of M_2 = 28.9841 °/h, we obtain $\tau = 2^h\ 42^m$. The method is insensitive to the precise value of τ, which can be found by a pilot harmonic analysis.

M_s may be regarded as the (complex) magnification of species s; it relates species s in the tide at the secondary port to that at the standard port. It is another slowly-varying quantity, and is a function of :

(a) the mean level at the mouth, A_0, which is affected by the weather (particularly wind and pressure) over the local shelf sea;
(b) The amplitude of the semi-diurnal tide at the mouth, A_2;
(c) the fresh-water level, S;
i.e. $M = f(A_0, A_2, S)$.

It remains to find a suitable function f. After several attempts the following was found to give satisfactory results for the Gironde :

$$M = k_1 + k_2A_2 + k_3A_2^2 + k_4A_2^3 + k_5A_0 + k_6S_G + k_7S_D \qquad \text{(F1)}$$

where S_G and S_D are the F.W.L. in the Garonne and Dordogne respectively, and k_1, k_2,... k_7 are (complex) constants, which are found by least squares analysis.

3. THE EFFECTS OF VARIABLE FLUVIAL DISCHARGE

3.1. Introduction

It is worth remarking that there has been relatively little attention paid to hydrological perturbations of the tide. The uppermost reaches of estuaries are usually shallow; there is little maritime traffic, and the masters of the vessels which do navigate there are well imbued with local knowledge. It is natural that the port authority is more interested in the lower reaches of the estuary. Too often it follows that tide-gauges near the tidal limit do not exist, or are badly maintained. Fortunately it was possible to use the results of two gauges in the uppermost part of the Loire, those at Mauves and Terre Promise. (During the period studied, the gauge at Thouaré was malfunctioning).

The two questions to be answered are :

(i) How should the effects of variable F.W.L. be treated in long-term predictions ? (Section 3.2)

(ii) How should the long-term predictions be modified in the short-term for the observed F.W.L. ? (Section 3.4)

3.2. The treatment of fresh-water level in long-term predictions

The variations of F.W.L. (and consequently its effects on the tide) are mostly stochastic (unpredictable in the long term), but they do have deterministic elements in the annual variation of sea-level, represented by the constituents Sa and Ssa; and in the annual modulations of the constituent M_2 i.e. the side-bands $M(SK)_2$ and $M(KS)_2$.

The values of harmonic constants for these constituents in the Loire are given in Fig. 12. The amplitude of the annual variation of sea level (Sa) increases twelvefold from mouth to head, and its phase increases by more than 70°. Although this constituent figures in the tide-generating potential, in reality most of its energy has a geophysical rather than an astronomical origin. Here in the Loire, we see a progressive change from a maritime regime at St. Gildas, with maximum sea level on October 20, to the fluvial regime at Mauves, with maximum river level on January 1.

The side-bands $M(SK)_2$ and $M(KS)_2$, sometimes known as MA_2 and MB_2, are not included in DOODSON's standard list. They were introduced by HORN in the 1940s. If their amplitude is large, it often indicates a fault in the tide-gauge, but in the Loire above Sardine, this is not the case. There they are important because of the modulation effect of the annual variation in F.W.L.

If the river has a regime which varies but little from year to year, then the amplitude and phase constituents will also vary but little, and they will contain a substantial part of the variance associated with the changes of F.W.L. Such is the case in rivers draining a permeable catchment, e.g. the Test in southern England.

PORT \ HARMONIC	Sa H (m)	Sa g (°)	M (SK)2 H (m)	M (KS)2 H (m)
St. Gildas	0.087	285.1	0.036	0.039
St. Nazaire	0.117	284.8	0.025	0.028
Donges	0.115	302.2	0.044	0.014
Sardine	0.251	340.6	0.104	0.082
Le Pellerin	0.326	324.7	0.126	0.087
Chantenay	0.593	347.1	0.164	0.111
Terre Promise	0.650	349.1	0.162	0.114
Mauves	1.053	358.6	0.137	0.124

FIG. 12. — Harmonic constants for the Loire : Sa, $M(SK)_2$, $M(KS)_2$ (from an analysis of tidal records for 1977).

More usually, however, there are great differences between one year and the next, and therefore between any individual year and the mean of a number of years. In Fig. 13, the recorded daily F.W.L. in the Loire during 1977 (top full curve) may be compared with the annual variation averaged over several years. During the summer the river was higher than normal for the time of year, and during the last three months it was lower than normal. There was a remarkable freshet, over 6 m high, during the last week of February, and another freshet of note during the first week of June. It is clear that in the Loire there is little value in using the mean annual variation to predict heights for an individual year. Furthermore in order to obtain a good estimate of the amplitude and phase of the constituents Sa, Ssa, $M(SK)_2$ (and also A_0), one ought to observe for several years and take the mean.

The middle curve on Fig. 13 represents the level of observed L.W. at Chantenay. The fluctuations of this level closely follow those of the F.W.L. It was only in October and November, when there was little water in the river, that the fortnightly tidal pulse was able to make itself felt.

In the method of species concordance, the F.W.L. may be explicitly introduced by the tidal forecaster. This is a great improvement if short-term predictions are to be made, say one day in advance, using the recently observed F.W.L. For the more usual long-term predictions, however, which are made years in advance, a level has to be chosen by the forecaster. Possible choices are :

(i) a constant level equal to the *mean annual F.W.L.*; this has little meaning because of the extreme asymmetry (positive skew) of the frequency distribution of river levels.

(ii) a level which varies annually according to the known *mean annual variation*, as in the conventional harmonic analysis.

(iii) a constant level equal to the *zero stage*; this is the system which is actually used in published predictions in the Gironde and the Loire.

If there are to be errors in tide prediction, then for navigational purposes it is better to err on the low side. Thus a navigator who uses predictions made for zero stage will almost always have more water than predicted. Such is not always the case with conventional predictions which include the annual variation; for example, during the period 1977 Oct. 10 to Nov. 20, the F.W.L. of the Loire was constantly more than 0.5 m below its average level for the time of year.

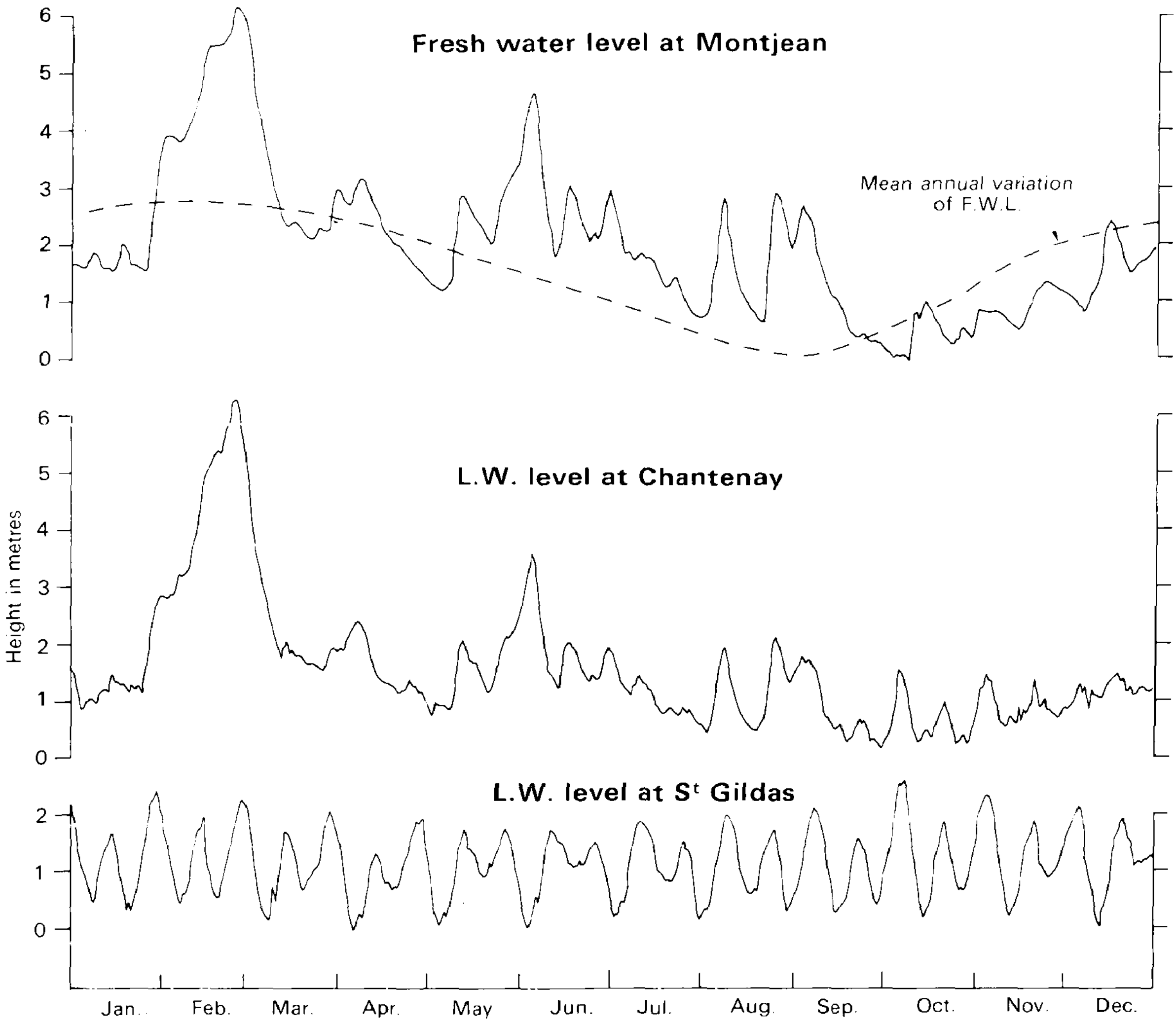

FIG. 13. — Mean annual variation of F.W.L. for 1977.

3.3. Improvement of the species concordance method

In applying the method of species concordance to the Loire, the formula (F1) of section 2.7 was improved as follows.

In the middle and upper reaches, the influence of the mean level at the mouth (A_0) is much less than that of the F.W.L. (S), and can be neglected. The complex magnification M therefore becomes a function of the variable A_2 and S. It follows that the semi-diurnal amplitude at a secondary port (B_2) is also a function of A_2 and S. The variation of B_2 in the (A_2, S) plane is shown in Fig. 14. After taking into account the nature of the B_2 surface, the formula (F1) was replaced by a cubic polynomial in two dimensions, viz.

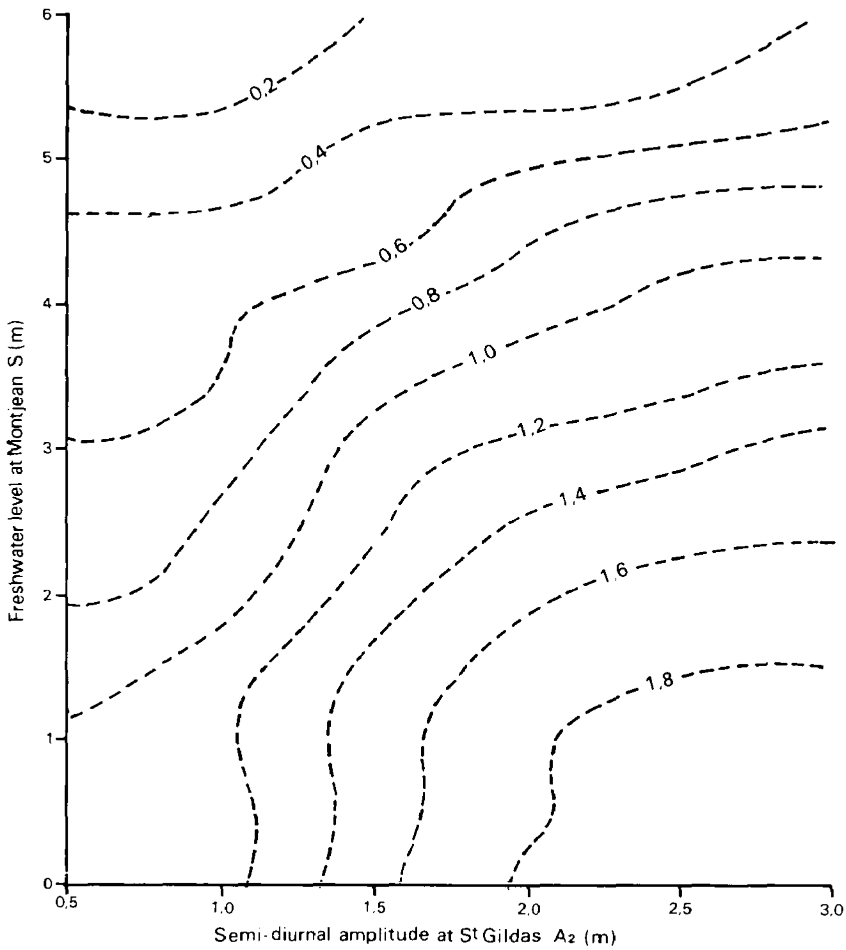

FIG. 14. — Semi-diurnal amplitude at Chantenay (B_2).

$$M = C_{00} + C_{10}A_2 + C_{01}S + C_{20}A_2^2 + C_{02}S^2 + C_{11}A_2S + C_{30}A_2^3 + C_{03}S^3 + C_{12}A_2S^2 + C_{21}A_2^2S \quad \text{(F2)}$$

where the Cs are constants to be determined, again by least squares.

The analyses for the Loire having been carried out for the year 1977, a different period was chosen for comparing observations and predictions. This was a span of 59 solar days (= 1.9979 synodic months) centred on the epoch 1976 Nov 29^d00^h U.T. It comprised 114 tides, and the F.W.L. varied between 0.41 m and 4.47 m.

Chantenay was chosen as the principal test port. The tide there is not only the most distorted of all the ports studied, but is severely perturbed by the variations of river level. It therefore constitutes a difficult test for the methods of prediction. Moreover, since fairly large vessels regularly use the port, the study of the tide there has more practical significance than for ports further upriver.

Species	Observations	Predictions by formula F1		Predictions by formula F2	
			% in each species		% in each species
0	1 667,02 cm²	1 291,31 cm² 12,37 %	77,46	1 203,28 cm² 11,52 %	72,18
1	59,54 cm² 0,57 %	31,81 cm² 0,30 %	53,42	32,11 cm² 0,31 %	53,91
2	7 393,21 70,80	7 222,98 69,17	97,70	7 287,39 69,78	98,57
4	611,88 5,86	546,80 5,24	89,36	570,72 5,47	93,27
6	67,63 0,65	55,67 0,53	82,32	59,31 0,57	87,69
8	15,67 0,15	9,47 0,09	60,47	13,09 0,13	83,57
10	7,10 0,07	4,05 0,04	57,05	5,13 0,05	72,34
12	2,21 0,02	0,96 0,01	43,32	1,40 0,01	63,35
14	1,20 0,01	0,34 0,00	28,23	0,74 0,01	62,11
16	0,71 0,01	0,30 0,00	42,21	0,41 0,00	58,40
Other even species	0,89 0,01	0,22 0,00	24,72	0,42 0,00	47,19
Other odd species	23,82 0,23	1,03 0,01	4,32	1,45 0,01	6,09
In the tidal wavebands	9 842,96 94,25	9 164,92 87,76	93,11	9 175,44 87,86 %	93,22
Between the tidal wavebands	600,12 5,75	nil		nil	
GRAND TOTAL	10 443,08 cm² 100,00 %	9 164,92 cm² 87,76 %		9 175,44 cm² 87,86 %	

FIG. 15. — Comparison of predictable variance at Chantenay.

Fig. 15 shows the results of the species concordance method; the figures given for each species are the predictable variance, defined as :

variance of observations — variance of residuals, if this is + ve, zero otherwise.

Apart from species 0, the predictable variance using formula (F2) is greater in every even species than that using formula (F1). The superiority of (F2) is particularly marked in even species higher than the twelfth-diurnal. The relatively poor performance of the method in the odd species is attributed to the abnormally strong presence of M_3, M_5, etc.

The accuracy of (F2) is also evident in the figures for the high and low waters :

	LOW WATER				HIGH WATER			
	Mean error (h)	S.D. (h)	Mean error (m)	S.D. (m)	Mean error (h)	S.D. (h)	Mean error (m)	S.D. (m)
Formula (F2)	−0.08	0.28	0.00	0.25	−0.07	0.52	0.06	0.28
Formula (F1)	0.09	0.30	−0.15	0.35	−0.20	0.57	−0.10	0.30

3.4. Short-term modification of predictions for variable F.W.L.

Before considering how best to modify long-term predictions in the short-term for the observed F.W.L., it is perhaps worth asking whether they need to be modified at all. The answer is for estuaries such as the Loire, where the difference between the highest and lowest F.W.L. (> 6 m) exceeds the extreme tidal range, is an emphatic "yes". This is clear from Fig. 16, which shows the absurdity of trying to predict the tide at Chantenay using a F.W.L. of zero on a day when the actual F.W.L. was 4.4. m.

During a major freshet, the effect of the F.W.L. is striking. Fig. 17 shows that of 1977 February 23 to 26. The fresh water influence on the level of L.W. extended as far seaward as Sardine. The level of H.W. was very seriously affected. At Mauves, the tide completely disappeared for three days, being "drowned" by the exceptional discharge.

In principle, short-term modifications can be made anywhere in the chain of prediction. For the harmonic method, one could in theory have several sets of "constants", each corresponding to a different F.W.L., and use the set corresponding to the F.W.L. of the day. In the method of species concordance, the observed F.W.L. is introduced at the stage of calculating the reduced vectors.

The resulting predictions are remarkably good, and better than those by any other method tested. Three examples are given; in Figs. 18 and 19 for Chantenay and in Fig. 20 for Mauves. We believe the only serious competitor of the species concordance method for day-to-day prediction would be a one-dimensional numerical model of the estuary, like those which were developed in the early 1960s, e.g. by Rossiter and Lennon (1965).

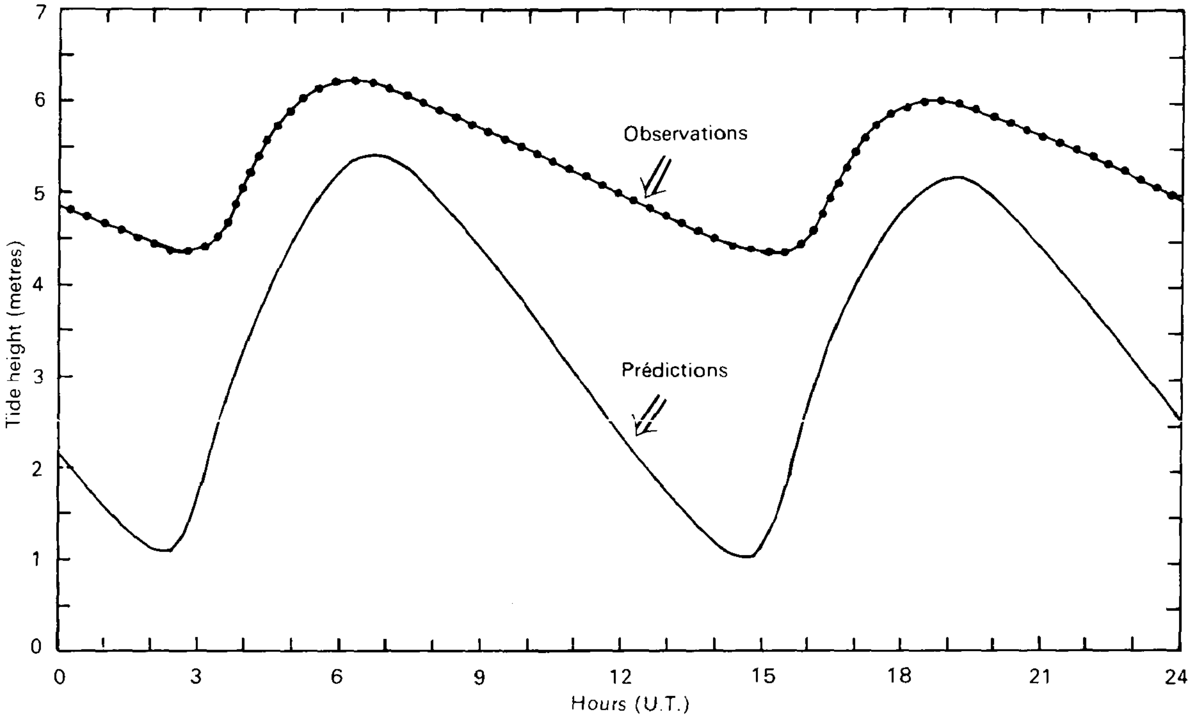

FIG. 16. — Chantenay, 10/12/76, coefficient 76, S = 4.4 m.
Predictions for Chantenay using zero stage during a freshet. The predictions are absurdly low, because they are for a F.W.L. of zero.

3.5. Corrections for variable F.W.L. in published tide tables.

It is worth bearing in mind that the average user of an estuary does not have access to a computer, and probably relies on a pocket set of tide tables. If the predictions have been made for zero stage using the species concordance method, as is recommended in this paper, then an extensive set of auxiliary tables is required in order to correct for the F.W.L. of the day. These would give the height of the tide at a secondary port as a function of the tide at the mouth and the F.W.L. at the tidal limit. It is suggested that these take a form similar to that of the "Table permanente des hauteurs d'eau", published by E.P.S.H.O.M. A separate table would be needed for a number of different fresh water levels.

Clearly, it should be possible for the average user to ascertain the current F.W.L. in the river. For the Loire, Garonne and Dordogne, the previous day's levels are published in the local daily press. This level will suffice in these cases; the catchment areas are so large that the F.W.L. changes but slowly in these rivers. Elsewhere, the local hydrological authority should be able to provide the information.

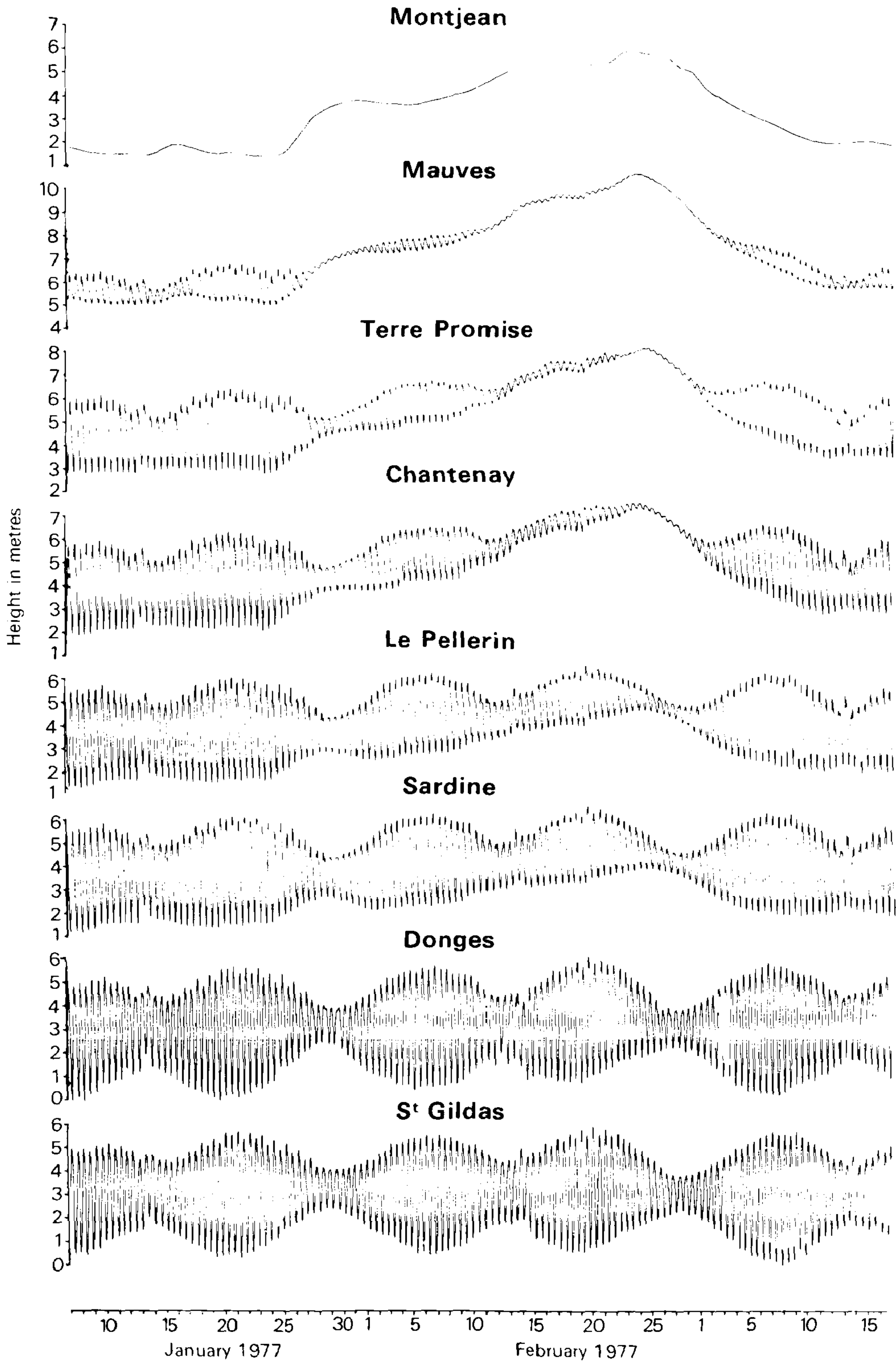

FIG. 17. — The major freshet of 1977.

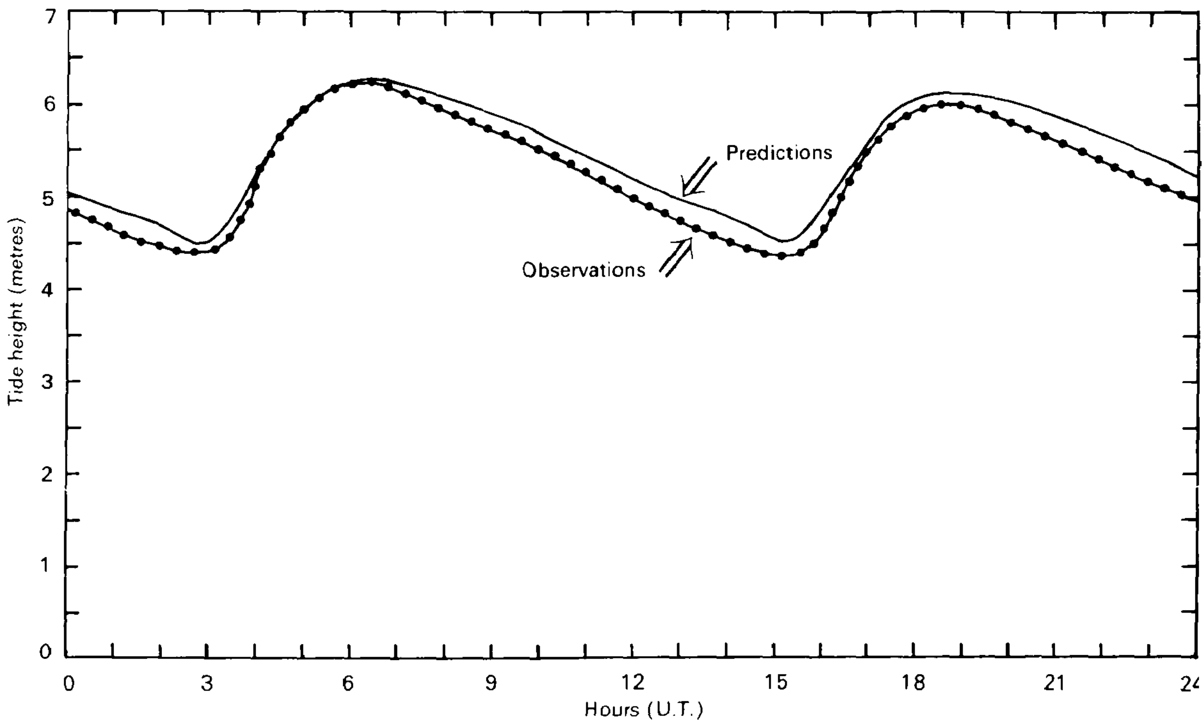

FIG. 18. — Chantenay, 10/12/76, C = 76, S = 4.4 m. The mean level of the predictions is about 20 cm too high; the shape of the curve is well predicted.

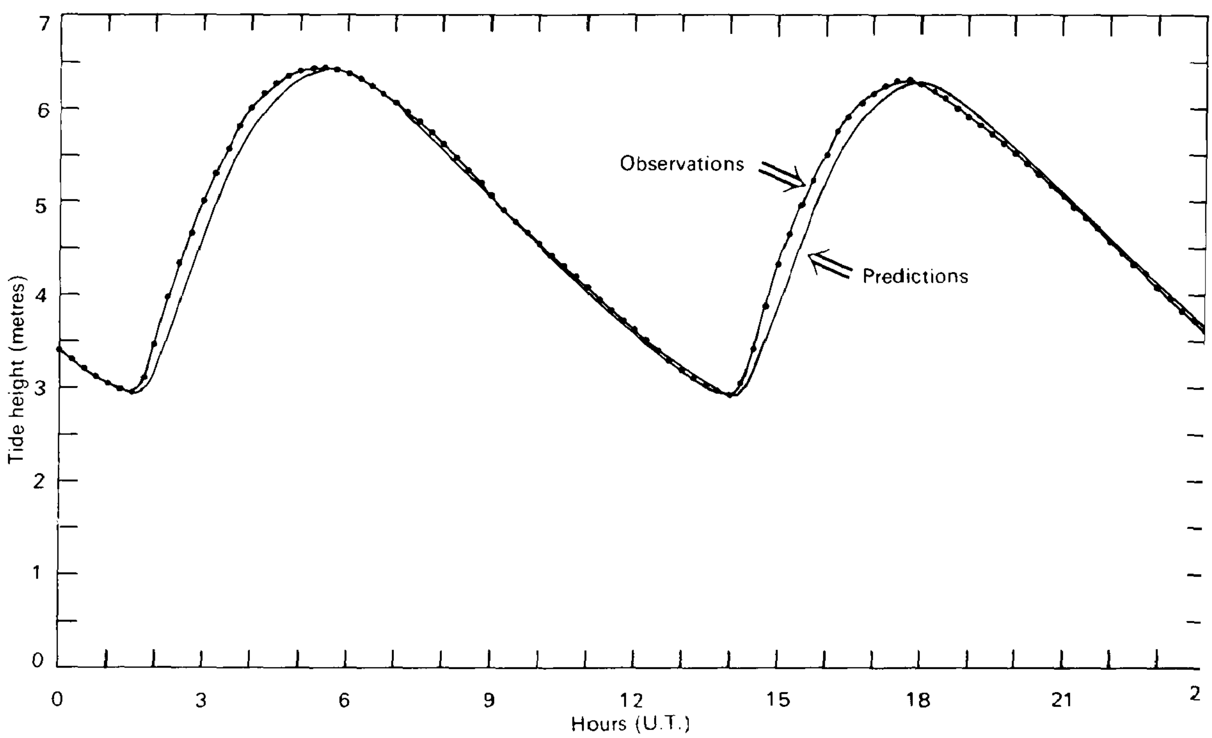

FIG. 19. — Chantenay, 22/12/76, C = 101, S = 2.6 m. The predictions are very close to the observations.

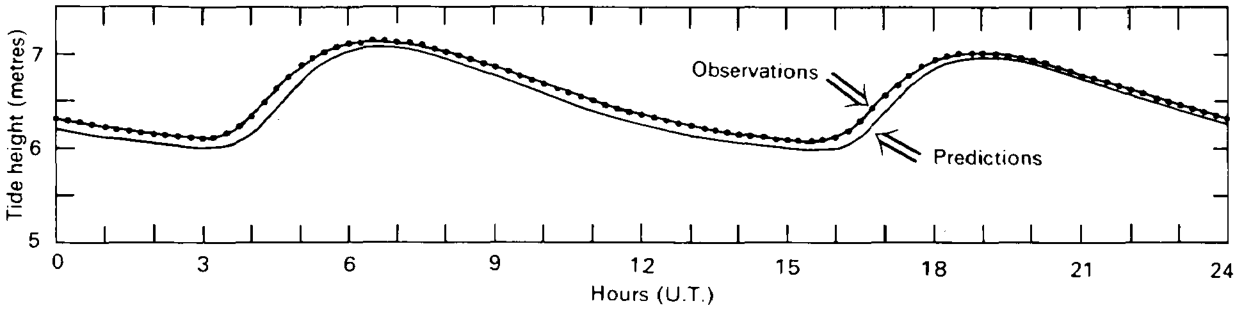

FIG. 20. — Mauves, 22/12/76, C = 101, S = 2.6 m.

4. CONCLUSION

Of the two major problems introduced in Section 1, the first (distortion of the tide-curve) may be dealt with by developing the harmonic method to its fullest extent, but it is better to use a new two-step method, that of species concordance. The second problem (effects of the variation of fluvial discharge) is not easily amenable to treatment by the harmonic method, but is adequately treated by the species concordance method. It is therefore recommended that the species concordance method be adopted in predicting tides in estuaries, especially those with severely distorted tides, and those where there is a great variation in fluvial discharge.

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