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# TIDE PREDICTIONS USING SATELLITE CONSTITUENTS

by B.D. ZETLER(*), E.E. LONG(**) and L.F. KU(***)

## ABSTRACT

Conventional harmonic tide predictions for the last century have used $f$ factors to modify the amplitudes of lunar constituents and $u$'s to correct the constituent equilibrium phases ($V_0$) as a means of approximating for a given period (one year or less) the effect of the 18.61 year cycle of the revolution of the moon's node. Historically, there was little choice; friction in geared mechanical tide-predicting machines imposed finite limits on the number of constituents used.

DOODSON (1921) clearly identified and evaluated satellite constituents; his study was updated using the latest astronomical constants by CARTWRIGHT and TAYLER (1971) and by CARTWRIGHT and EDDEN (1973). Nevertheless, satellite constituents, now readily usable on modern computers, have not been used for tide predictions. As a result, predictions have really been quasi-harmonic, requiring modifying amplitudes and phases periodically, at present every year for U.S. predictions, every two months for Canadian, and every 30 days for U.K. predictions. With satellite constituents, nineteen years of hourly tide predictions for Seattle (1921-1939) were computed from initial settings for 1 January 1921.

It was not to be expected that the accuracy of harmonic tide predictions would be improved significantly by the new procedure; comparisons of annual residual variances for predictions by U.S. and Canadian procedures indicate that any improvements are small. Nevertheless, this new method removes the need for rather contrived (however clever) procedures, in particular that of constituents modifying $M_1$ and $L_2$ by cycles per 8.85 years (revolution of lunar perigee) in the $f$ and $u$ corrections for these constituents.

(*) Scripps Institution of Oceanography, University of California, San Diego, La Jolla, CA 92093, USA.

(**) National Ocean Service, NOAA, Rockville, MD 20852, USA.

(***) Canadian Hydrographic Service, Ottawa, Ontario, K1A 0E6, Canada.

## HISTORICAL BACKGROUND

For about a century, tide predictions have been prepared in hydrographic offices around the world using the following formula :

$$h(t) = H_0 + \sum_{i=1}^{N} f_i(t) H_i \cos(2\pi\sigma_i t + V_i + u_i(t) - k_i)$$

in which :

$h$ = height of tide at any time $t$
$H_0$ = mean height of water level above datum used for prediction.
$\sigma_i$, $H_i$ and $k_i$ = the frequency, the amplitude and the epoch of the $i^{th}$ main constituent
$V_i$ = equilibrium argument of the $i^{th}$ main constituent at $t = 0$
$f_i(t)$ and $u_i(t)$ = the amplitude modulation and the phase modulation of the $i^{th}$ main constituent.

The amplitude modulation and the phase modulation are due to the revolution of the moon's node (period of 18.61 years) and the revolution of the lunar perigee every 8.85 years and, in the following discussion, are represented by a set of periodic components called the satellite constituents. The amplitude modulations are ± 19 %, ± 4 % for the $O_1$ and $M_2$ constituents and the corresponding modulations in phase are ± 5° and ± 2°, respectively. Because of the slow and small changes in the modulation, the values of $f_i(t)$ and $u_i(t)$ are computed at certain intervals only :

(1) In the United States, the National Ocean Service computes $f$ and $u$ for the *middle* of the year.
(2) The Canadian Hydrographic Service computes $f$ and $u$ for every two months for the *beginning* of the 2-month period.
(3) The British Institute of Oceanographic Sciences computes $f$ and $u$ for the *middle* of each 30-day period, having changed from doing this every four months about ten years ago.

In principle, the more often the change is made, the more accurate are the predictions. The disadvantage in changing often is the use of more computer time. Although this method of tide prediction has been called harmonic, actually the need to modify $f$ and $u$ periodically makes it really only quasi-harmonic. The method proposed in this paper, the inclusion of all significant constituents ($N$ main constituents plus $N'$ satellite constituents) with the simplified formula (omitting $f$ and $u$),

$$h(t) = H_0 + \sum_{i=1}^{(N+N')} H_i \cos(2\pi\sigma_i t + V_i - k_i)$$

surely was obvious to DOODSON (1921) for he identified and evaluated these added constituents. However, in those days, and indeed until the early 1960's, all predictions were prepared on mechanical tide prediction machines; friction in the gears imposed a severe limit on the number of constituents that could be used. Furthermore, the increased speed and core memory in present-day computers makes it possible to analyze for these added constituents directly if a 19-year series of observations is available, an unthinkable task a generation ago. AMIN (1976) computed the 19-year analysis using the method used in CARTWRIGHT and TAYLER (1971). When a 19-year analysis is impossible or impractical, it is possible to infer the amplitudes and phases of the added satellite constituents.

For this particular study, Seattle was first chosen because 19 consecutive one-year analyses were available and therefore it was hoped that an optimum prediction could be obtained using vector averages of the annual harmonic constants. However, this plan was discarded because it is desirable to show a more general application to NOS tide predictions; instead, whatever constants are presently used on the Form 444 for the station chosen would be used in this study. In selecting a station, the following criteria were considered important : a moderate-sized range, a significant diurnal component, relatively small shallow-water tides, an available 19-year series of observations in computer-compatible format, and finally a station for which NOS does not amplify analyzed amplitudes to force a fit to observed ranges. Oddly enough, these criteria led us back to Seattle.

## DISCUSSION

DOODSON's (1921) study listed all possible frequencies in the tidal potential whose amplitudes were at least $10^{-4}$ times the $M_2$ amplitude. He showed that each of these was the sum or difference of integral products of six fundamental frequencies as follows :

$\sigma_A^{-1}$ = 1 lunar day (period of Earth's rotation relative to Sun)
$\sigma_B^{-1}$ = 1 month (period of Moon's orbital motion)
$\sigma_C^{-1}$ = 1 year (period of Sun's orbital motion)
$\sigma_D^{-1}$ ≈ 8.85 years (period of lunar perigee)
$\sigma_E^{-1}$ ≈ 18.61 years (period of regression of lunar nodes)
$\sigma_F^{-1}$ ≈ 20,900 years (period of solar perigee).

The frequency of tidal periodic components is expressed as :

$$\sigma_K = S_A\sigma_A + S_B\sigma_B \ldots S_F\sigma_F, \quad S_x = 0, \pm 1, \pm 2, \ldots \quad x = A \text{ to } F$$

He used an identification code for each frequency using a six digit number which, to avoid negatives, used a base of 5 for the last five digits. Thus the frequency of $M_2$ is 255555, indicating the use of lunar days as opposed to SCHUREMAN's Table 2 (1941) in which the argument ($V$) is identified in solar days. CARTWRIGHT and TAYLER (1971) updated Doodson's table using computers and modern astronomical constants but changed Doodson's notations by not adding 5's; thus $M_2$ is 200000. Since this study uses the Cartwright and Tayler tables, corrected by CARTWRIGHT and EDDEN (1973), Cartwright's system of frequency notation is used here. $V$'s have been computed using Cartwright's tables but 180° has been added for all odd species to correspond to his normalization, followed by the addition of the angular correction listed in Schureman's Table 2.

We identify the satellite constituents by adding ($\pm S_E$) to the main constituent notations to indicate a change of $\pm S_E$ in the fifth digit ($\pm S_E$ cycles per 18.61

years). Thus $M_2$ is listed as 200000 but $M_2$ (− 1) is 2000-10. $M_1$ and $L_2$ are special cases because they also have nearby frequencies differing by $\pm S_D$ in the fourth integer ($\pm S_D$ cycles per 8.85 years). For these two constituents, a double notation of ($\pm S_D$, $\pm S_E$) is used. Thus, where $L_2$ is 210-100, $L_2$ (+2, +1) is 210110.

If a 19-year analysis is possible and feasible, then by all means this should be done to obtain the harmonic constants for all required constituents including the satellite constituents. However, there is a noise continuum in all tidal records (MUNK and BULLARD, 1963; GROVES and ZETLER, 1964; MUNK *et al.*, 1965); the phases of analyzed constituents whose amplitudes are about equal to or less than the continuum at nearby frequencies are unreliable (random) and these analyzed constants should not be used for prediction purposes. Although Shureman would not have known the word "continuum", he recognized the existence of a continuum and, in an informal office memorandum, rejected all analyzed amplitudes less than 0.03 foot (about 1 cm). Although his limitation is not frequency-dependent as opposed to the continuum which is high at low frequencies and then decreases monotonically with higher frequency except for tidal cusps near large tidal lines, his limitation was a reasonable compromise for all constituents of one or more cycles per day. Because the level of the continuum is lowered about 75 % by increasing the length of the record analyzed (from one year to 19 years), a lower rejection limit of perhaps 0.25 to 0.5 cm is reasonable.

If a 19-year analysis is not available, then harmonic constants for satellite constituents must be inferred. Amplitudes are inferred by multiplying the ratio of the coefficients in CARTWRIGHT and EDDEN (1973) (satellite constituent/main constituent) by the analyzed amplitude for that main constituent. Phases for satellite constituents are assumed to be equal to the analyzed main constituents, but a 180° correction is added when the coefficients are opposite in sign.

The generation of a shallow water satellite constituent is assumed to be identical to the generation of the corresponding shallow water constituent. The phase of the satellite shallow water constituent is therefore equal to that of the corresponding main constituent and its amplitude is $R'H$, where $H$ is the amplitude of the shallow water constituent and $R'$ is a ratio determined by the parental constituent(s). For example, $M_4$ is generated by the double interaction of $M_2$ and its amplitude modulation is

$$f_{M_4} = (1 + R_{M_2} \cos 2\pi\sigma t)^2$$
$$\simeq 1 + 2R_{M_2} \cos 2\pi\sigma t$$

where $R_{M_2}$ is the ratio $M_2(-1)/M_2$ and $2\pi\sigma = 1$ cycle/18.61 years. Therefore the amplitude ratio of $M_4(-1)$ is $2R_{M_2}$.

We have ignored theoretical nodal modulation for three constituents, $S_a$, $S_{sa}$ and $S_1$ because experience has shown these are more important as meteorological tides. In a personal communication, Cartwright has agreed with this treatment as well as the use of 001000 and 11-1000 for $S_a$ and $S_1$ respectively, rather than 00100-1 and 11-1001 as they are listed in Cartwright and Edden. The latter changes conform to accepted international practice.

SCHUREMAN (1941, p. 43) describes an error, a factor of $\sqrt{2.307}$, in the node factors for $M_1$. He then mistakenly states that predicted tides are not affected "since the node factors and reduction factors are reciprocal and compensating". However,

## TABLE 1

**Seattle Harmonic Constants**

| Constituent | | | Amplitude in cm | | | g (kappa Prime) in ° | | |
|---|---|---|---|---|---|---|---|---|
| Name | Cartwright Number | Speed in °/h | NOS analysis | added inferred | 19-yr. analysis | NOS analysis | added inferred | 19-yr. analysis |
| Sa | 001000 | 0.04106864 | 7.38 | | 6.93 | 289.1 | | 289.2 |
| Ssa | 002000 | 0.08213728 | 2.71 | | 2.99 | 214.0 | | 218.0 |
| $2Q_1(-1)$ | 1-302-10 | 12.85207974 | | 0.22 | 0.18 | | 130.7 | 158.3 |
| $2Q_1$ | 1-30200 | 12.85428615 | 1.19 | | 0.83 | 130.7 | | 144.7 |
| $Q_1(-1)$ | 1-201-10 | 13.39645444 | | 1.42 | 1.49 | | 143.5 | 138.6 |
| $Q_1$ | 1-20100 | 13.39866086 | 7.53 | | 7.42 | 143.5 | | 141.8 |
| $\rho_1(-1)$ | 1-22-1-10 | 13.46930806 | | 0.33 | 0.32 | | 138.3 | 136.0 |
| $\rho_1$ | 1-22-100 | 13.47151447 | 1.74 | | 1.49 | 138.3 | | 140.2 |
| $O_1(-1)$ | 1-100-10 | 13.94082915 | | 8.64 | 8.85 | | 143.9 | 141.6 |
| $O_1$ | 1-10000 | 13.94303556 | 45.81 | | 45.64 | 143.9 | | 143.6 |
| $M_1(-2, -1)$ | 100-1-10 | 14.48520386 | | 0.26 | 0.48 | | 172.0 | 131.3 |
| $M_1(-2, 0)$ | 100-100 | 14.48741027 | | 1.40 | 2.13 | | 172.0 | 135.0 |
| $M_1(-1)$ | 1001-10 | 14.49448752 | | 0.11 | 0.10 | | 172.0 | 1.0 |
| $M_1$ | 100100 | 14.49669393 | 3.90 | | 3.53 | 172.0 | | 174.4 |
| $M_1(+1)$ | 100110 | 14.49890034 | | 0.78 | 0.91 | | 172.0 | 159.4 |
| $P_1(-1)$ | 11-20-10 | 14.95672495 | | 0.28 | 0.70 | | 154.9 | 132.7 |
| $P_1$ | 11-2000 | 14.95893136 | 25.24 | | 25.36 | 154.9 | | 155.0 |
| $S_1$ | 11-1000 | 15.00000000 | 1.77 | | 1.94 | 262.1 | | 235.7 |
| $K_1(-1)$ | 1100-10 | 15.03886223 | | 1.65 | 1.41 | | 157.0 | 172.1 |
| $K_1$ | 110000 | 15.04106864 | 83.06 | | 82.74 | 157.0 | | 156.9 |
| $K_1(+1)$ | 110010 | 15.04327505 | | 11.26 | 11.38 | | 157.0 | 155.0 |
| $J_1(-1)$ | 120-1-10 | 15.58323693 | | 0.11 | 0.04 | | 163.6 | 95.2 |
| $J_1$ | 120-100 | 15.58544334 | 3.63 | | 4.04 | 163.6 | | 189.6 |
| $J_1(+1)$ | 120-110 | 15.58764976 | | 0.72 | 0.99 | | 163.6 | 174.7 |
| $OO_1$ | 130000 | 16.13910171 | 1.98 | | 2.98 | 170.1 | | 202.0 |
| $OO_1(+1)$ | 130010 | 16.14130812 | | 1.27 | 1.86 | | 170.1 | 196.3 |
| $OO_1(+2)$ | 130020 | 16.14351454 | | 0.27 | 0.47 | | 170.1 | 180.8 |
| $2N_2(-1)$ | 2-202-10 | 27.89314838 | | 0.11 | 0.02 | | 85.9 | 287.2 |
| $2N_2$ | 2-20200 | 27.89535479 | 2.83 | | 2.35 | 85.9 | | 86.4 |
| $\mu_2(-1)$ | 2-220-10 | 27.96600199 | | 0.12 | 0.28 | | 7.0 | 340.6 |
| $\mu_2$ | 2-22000 | 27.96820840 | 3.20 | | 3.24 | 7.0 | | 12.1 |
| $N_2(-1)$ | 2-101-10 | 28.43752308 | | 0.79 | 0.74 | | 112.7 | 108.9 |
| $N_2$ | 2-10100 | 28.43972949 | 21.24 | | 21.28 | 112.7 | | 113.1 |
| $\nu_2(-1)$ | 2-12-1-10 | 28.51037670 | | 0.13 | 0.07 | | 132.2 | 29.9 |
| $\nu_2$ | 2-12-100 | 28.51258311 | 3.60 | | 4.38 | 132.2 | | 127.0 |
| $M_2(-1)$ | 2000-10 | 28.98189779 | | 3.99 | 3.96 | | 139.6 | 141.0 |
| $M_2$ | 200000 | 28.98410420 | 106.95 | | 106.86 | 139.6 | | 139.3 |
| $\lambda_2(-1)$ | 21-21-10 | 29.45341888 | | 0.08 | 0.24 | | 164.6 | 182.9 |
| $\lambda_2$ | 21-2100 | 29.45562529 | 1.71 | | 2.13 | 164.6 | | 171.5 |
| $L_2(-1)$ | 210-1-10 | 29.52627250 | | 0.18 | 0.48 | | 185.6 | 181.2 |
| $L_2$ | 210-100 | 29.52847891 | 4.97 | | 4.76 | 185.6 | | 180.6 |
| $L_2(+2, 0)$ | 210100 | 29.53776257 | | 1.24 | 0.27 | | 185.6 | 196.7 |
| $L_2(+2, +1)$ | 210110 | 29.53996898 | | 0.55 | 0.31 | | 185.6 | 52.0 |
| $T_2$ | 22-3001 | 29.95893342 | 1.52 | | 1.57 | 158.2 | | 136.9 |
| $S_2$ | 22-2000 | 30.00000000 | 25.82 | | 25.79 | 157.9 | | 157.5 |
| $K_2(-1)$ | 2200-10 | 30.07993086 | | 0.09 | 0.11 | | 155.9 | 94.4 |
| $K_2$ | 220000 | 30.08213728 | 7.22 | | 7.50 | 155.9 | | 156.4 |
| $K_2(+1)$ | 220010 | 30.08434369 | | 2.15 | 2.42 | | 155.9 | 158.0 |
| $K_2(+2)$ | 220020 | 30.08655010 | | 0.23 | 0.26 | | 155.9 | 157.2 |
| $M_4(-1)$ | 4000-10 | 57.96600199 | | 0.16 | 0.13 | | 91.2 | 99.3 |
| $M_4$ | 400000 | 57.96820840 | 2.10 | | 2.14 | 91.2 | | 98.2 |
| $MS_4(-1)$ | 42-20-10 | 58.98189779 | | 0.04 | 0.04 | | 119.0 | 186.9 |
| $MS_4$ | 42-2000 | 58.98410420 | 1.01 | | 1.11 | 119.0 | | 121.5 |
| $M_6(-1)$ | 6000-10 | 86.95010619 | | 0.10 | 0.14 | | 326.3 | 332.5 |
| $M_6$ | 600000 | 86.95231260 | 0.91 | | 0.90 | 326.3 | | 333.5 |

this is true only when analyzing and predicting for the identical period. We recommend going back to the analysis and correcting the reduction factor before going on to use the corrected $M_1$ amplitude and the $M_1$ node constituents in a new prediction.

Table 1 lists the Cartwright number and hourly speeds for 55 constituents, 30 of which are satellite constituents. The analyzed and inferred amplitudes and phases are listed as well as comparable values from 18.61 year analyses (1921-1939) and (1902-1920). The 18.61 year analyses were computed using Fourier calculations for individual frequencies; relatively small sideband effects of nearby tidal lines have not been removed. Table 1 permits comparisons of inferred constants with those obtained in an 18.61-year analysis. For the major constituents, the fit is quite good for both amplitudes and phases. Matching may become poorer, however, for small amplitude constituents and it would be well to retain a rejection limit for analyzed constituents of 0.25 to 0.5 cm. Discrepancies in amplitude and/or phase between analyzed and inferred constituents may also be due to duplicating of some frequencies from different sources. For example, $L_2$ and the shallow water $2MN_2$ have the same argument, 210-100. If $2MN_2$ is relatively large, the inferred $L_2$ satellite constituents will be inaccurate. Similarly, the satellite constituent $M_1$ $(-2, 0)$ has the same argument, 100-100, as the diurnal interaction tide of $M_2$ and $M_1$. Analyses for 19 years or more for these satellite constituents may give useful constants for prediction purposes without separating out the individual contributions from various sources just as analyzed $S_2$ constants are used routinely without separating the gravitational and radiational contributions (ZETLER, 1971). Although AMIN (1976) does attempt some separations as noted above, there is no obvious way of obtaining comparable values by inference from equilibrium tides.

Predictions for the 19 years (1921-1939) were then prepared from the Form 444 constants plus inferences, and from the constants obtained by 18.61-year analyses. Since no $f$ or $u$ values are involved in these predictions, they were computed in single computer runs starting with initial settings for 1 January 1921. Even with single precision, the cumulative error in phase due to round-off in about 170,000 hours is much less than the round-off error in the initial phase.

## RESULTS

Table 2 shows annual residual variances and 19-year means of these obtained for six different sets of predictions, each subtracted from observed hourly height. In general, the predicted fit improves as the period used for $f$ and $u$ decreases. There is a small anomaly in that Canadian 2-month periods are slightly better than NOS 1-month predictions; this may be due to a different formulation for $f$ and $u$ in Canada than in the U.S. Because we are recommending the satellite constituent approach, this has not been investigated. The variances using 19-year analysis (1921-1939) are significantly better than any other but this may be at least partly due to self-prediction. The variances obtained using the 1902-1920 analysis are lower than any of the $f + u$ results. If an amplitude rejection limit had been used, it seems likely that residual variances obtained from the 1921-1939 analysis would be a little higher and those from the 1902-1920 analysis a little lower, but these calculations have not been made.

TABLE 2

**Seattle, Washington**
**Residual Variances in cm$^2$**

| Year | Annual $f+u$ | Bimonthly $f+u$ | Monthly $f+u$ | Harmonic using inferences | Using 19 yr. analysis |
|---|---|---|---|---|---|
| 1921 | 152.18 | 147.60 | 148.55 | 147.21 | 146.32 |
| 1922 | 164.90 | 160.93 | 162.86 | 161.29 | 156.91 |
| 1923 | 177.82 | 174.88 | 174.66 | 174.81 | 169.36 |
| 1924 | 188.87 | 185.77 | 186.18 | 185.75 | 184.04 |
| 1925 | 171.22 | 168.35 | 170.01 | 168.32 | 165.27 |
| 1926 | 202.90 | 200.86 | 202.44 | 200.63 | 199.56 |
| 1927 | 176.89 | 177.72 | 177.82 | 177.78 | 170.94 |
| 1928 | 175.59 | 174.94 | 174.01 | 174.55 | 165.37 |
| 1929 | 193.15 | 189.41 | 190.17 | 190.37 | 175.68 |
| 1930 | 192.96 | 187.44 | 190.45 | 188.12 | 177.63 |
| 1931 | 202.34 | 198.71 | 201.04 | 199.61 | 190.64 |
| 1932 | 195.28 | 190.81 | 191.57 | 191.51 | 183.11 |
| 1933 | 227.24 | 222.38 | 223.80 | 222.37 | 215.72 |
| 1934 | 163.70 | 158.07 | 161.37 | 157.71 | 152.73 |
| 1935 | 169.18 | 167.45 | 168.62 | 167.35 | 161.09 |
| 1936 | 178.93 | 177.57 | 176.14 | 177.65 | 169.55 |
| 1937 | 190.54 | 186.92 | 186.55 | 188.49 | 183.95 |
| 1938 | 200.86 | 196.49 | 197.60 | 196.11 | 194.91 |
| 1939 | 176.70 | 173.12 | 174.47 | 172.57 | 170.57 |
| Mean | 184.27 | 181.02 | 182.02 | 181.17 | 175.44 |

Although the mean differences are only a few percentage points different, the actual differences in the region of the tidal bands are much larger because the energy in the continuum outside the tidal bands is constant in each set of residuals.

Spectral analyses within the 1 and 2 cpd tidal bands have been calculated for the "1 year $f$ and $u$" and for the "harmonic using inferences" residuals for three years, 1923, 1927, and 1934, representing roughly mean and extreme differences in total variance in cm$^2$.

| Year | (1) 1 yr $f$ and $u$ Residual Variance | | | (2) Harmonic using inferences Residual Variance | | | Difference (1) − (2) Residual Variance | |
|---|---|---|---|---|---|---|---|---|
| | Total | 1 cpd | 2 cpd | Total | 1 cpd | 2 cpd | Total | Tidal |
| 1923 | 177.82 | 19.86 | 6.18 | 174.81 | 16.74 | 6.36 | 3.01 | 2.94 |
| 1927 | 176.89 | 23.53 | 4.72 | 177.78 | 24.22 | 4.94 | −0.89 | −0.91 |
| 1934 | 163.70 | 29.96 | 8.43 | 157.71 | 25.16 | 7.38 | 5.99 | 5.85 |

The above data show clearly that essentially all of the differences in total variance for the annual residuals is in the 1 and 2 cpd tidal bands, as is to be expected. If 1923 is taken as about average, then the improvement in the predictable portion of the spectrum is over 10 % as opposed to less than 2 % using the ratio of total residual variances as the criteria.

## SUMMARY

The National Ocean Service should give consideration to reducing the periods used for $f$ and $u$ modifications in tide predictions. Presumably this would also remove the occasional need to blend predictions at the beginning of a year to more closely fit predictions at the end of the previous year. It also should review Schureman's formula for node factors for $M_1$.

All of the above could be resolved by shifting to a prediction program using satellite constituents, using 19-year analysis when possible and inferred satellite constituents when it is not possible. This approach is not only more accurate but it is also simpler (much less devious); it could resolve a future problem of recalculating tidal forces using modern astronomical constants when SCHUREMAN's manual (1941) is rewritten. The savings in being able to predict many years for an initial setting would greatly exceed the cost of roughly doubling the number of constituents used. Finally, if 19-year analysis and satellite constituent predictions are used, this will resolve problems due to the node tables being inaccurate for shallow-water stations and due to ambiguities in node factors when there are multiple sources for individual frequencies.

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