A STEP FORWARD IN THE METHODS
OF GETTING A FIX AT SEA

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ABSTRACT

This paper deals with a method of computing a set of astronomical observations to two or more stars without requiring a knowledge of the approximate geographical position of the observer in order to obtain a fix. Programmes for use with the Hewlett-Packard HP 41/CV calculator and in BASIC language are included. The author also describes an extended application of the method to processing observations made by "radio sextants" to provide twenty-four-hour continuous positioning offshore without recourse to radio, satellite, or inertial positioning systems.

It is a fact well known among seamen that methods of getting an astronomical position at sea require the concurrence of dead reckoning. Seamen always know their estimated position, even if in some cases it is a very approximate one.

On the other hand, when a navigator carries out an astronomical observation, measuring the angle of altitude of two celestial bodies of his choice, the circles of altitude intersect at the two extreme points A and B, which represent an ambiguity with regard to the true geometrical location of his observation. However, this ambiguity does not exist in practice because, as we have said, the observer always has a dead reckoning available.

Nevertheless, if we consider position finding from a different point of view, the navigator can determine his fix without the concurrence of dead reckoning.

To do this we start from the navigational triangle, where (see Figures 2 and 3):

- \( P_{nP} \), the line of the poles
- \( ZZ' \), the observer's zenith nadir line
- \( QQ' \), the equator

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HH', the observer's celestial horizon
A, the position of the celestial body, parallactic angle
Z, the azimuth of the celestial body
h, the altitude of the celestial body
\( \varphi \), the observer's latitude
d, the declination of the celestial body
LHA, the local hour angle.

Applying to the spherical triangle APnZ the law of cosines, we obtain:
\[
\cos(90^\circ - h) = \cos(90^\circ - \varphi) \cos(90^\circ - d) + \sin(90^\circ - \varphi) \sin(90^\circ - d) \cos LHA
\]
\[
\sin h_e = \sin \varphi_e \sin d + \cos \varphi_e \cos d \cos LHA_e
\]
(1)

(suffix e indicates the value is estimated).

Fig. 1

Fig. 2

Fig. 3
The equation (1) gives us a line of position on observing the altitude of a heavenly body "h", and we can obtain from the formula the computed or estimated altitude and "h2". If we then compare these two altitudes, we will get an increment of altitude "Δh", which with its azimuth determines the corresponding line of position. Its intersection with another line of position, from the observation of another celestial body, will provide the fix required.

Up to this point, we are still dependent upon dead reckoning.

Now if we establish a system of equations such as:

\[
\sin h_1 = \sin \varphi \sin d_1 + \cos \varphi \cos d_1 \cos \text{LHA}_1 \text{ for celestial body A}_1
\]

\[
\sin h_2 = \sin \varphi \sin d_2 + \cos \varphi \cos d_2 \cos \text{LHA}_2 \text{ for celestial body A}_2
\]

which converts to:

\[
\sin h_1 \sin \varphi \sin d_2 + \sin h_1 \cos \varphi \cos d_2 \cos \text{LHA}_2 = \sin h_2 \sin \varphi \sin d_1 + \sin h_2 \cos \varphi \cos d_1 \cos \text{LHA}_1
\]

and, dividing by \(\cos \varphi\):

\[
\sin h_1 \tan \varphi \sin d_2 + \sin h_1 \cos d_2 \cos \text{LHA}_2 = \sin h_2 \tan \varphi \sin d_1 + \sin h_2 \cos d_1 \cos \text{LHA}_1
\]

and, taking out \(\tan \varphi\) as common factor:

\[
\tan \varphi = \frac{\sin h_1 \sin d_2 - \sin h_2 \sin d_1}{\sin h_1 \sin d_2 - \sin h_2 \sin d_1}
\]

We still have the latitude as a function of the longitude. If we apply the same formula to a third celestial body of altitude "h3", declination "d3" and local hour angle "LHA3" and we go on with the same transformations we will have:

\[
\tan \varphi = \frac{\sin h_3 \cos d_3 \cos \text{LHA}_3 - \sin h_1 \cos d_3 \cos \text{LHA}_3}{\sin h_1 \sin d_3 - \sin h_3 \sin d_3}
\]

If we equalize formulas (2) and (3) and go on with the same conversions we will get:

\[
\frac{\sin h_2 \cos d_3 \cos \text{LHA}_3 - \sin h_1 \cos d_3 \cos \text{LHA}_3}{\sin h_1 \sin d_3 - \sin h_3 \sin d_3}
\]

and this expression expands to:

\[
\sin h_2 \cos d_3 \cos \text{LHA}_3 \sin h_1 \sin d_3 - \sin h_2 \cos d_3 \cos \text{LHA}_3 \sin h_3 \sin d_3
\]

\[
- \sin h_1 \cos d_3 \cos \text{LHA}_3 \sin h_1 \sin d_3 + \sin h_1 \cos d_3 \cos \text{LHA}_3 \sin h_3 \sin d_3
\]

\[
= \sin h_3 \cos d_1 \cos \text{LHA}_1 \sin h_1 \sin d_2 - \sin h_3 \cos d_1 \cos \text{LHA}_1 \sin h_2 \sin d_1
\]

\[
- \sin h_1 \cos d_3 \cos \text{LHA}_3 \sin h_1 \sin d_2 + \sin h_1 \cos d_3 \cos \text{LHA}_3 \sin h_2 \sin d_1
\]

or:

\[
\sin h_2 \cos d_3 \cos \text{LHA}_3 \sin h_1 \sin d_3 - \sin h_2 \cos d_3 \cos \text{LHA}_3 \sin h_3 \sin d_3
\]

\[
- \sin h_1 \cos d_3 \cos \text{LHA}_3 \sin h_1 \sin d_3 + \sin h_1 \cos d_3 \cos \text{LHA}_3 \sin h_3 \sin d_3
\]

\[
= \sin h_3 \cos d_1 \cos \text{LHA}_1 \sin h_1 \sin d_2 - \sin h_3 \cos d_1 \cos \text{LHA}_1 \sin h_2 \sin d_1
\]

\[
- \sin h_1 \cos d_3 \cos \text{LHA}_3 \sin h_1 \sin d_2 + \sin h_1 \cos d_3 \cos \text{LHA}_3 \sin h_2 \sin d_1
\]
and, taking out $\cos LHA_1$, $\cos LHA_2$ and $\cos LHA_3$ as common factors:

\[
\cos LHA_1 \left( \sin h_2 \cos d_1 \sin h_1 \sin d_1 - \sin h_2 \cos d_1 \sin h_1 \sin d_1 \right) \\
+ \cos LHA_2 \left( \sin h_1 \cos d_2 \sin h_3 \sin d_1 - \sin h_1 \cos d_2 \sin h_3 \sin d_1 \right) \\
= \cos LHA_3 \left( \sin h_1 \cos d_3 \sin h_2 \sin d_1 - \sin h_1 \cos d_3 \sin h_2 \sin d_1 \right)
\]

then, cancelling like terms and simplifying we have:

\[
\cos LHA_1 \cos d_1 \sin h_1 \left( \sin h_2 \sin d_3 - \sin h_3 \sin d_2 \right) \\
+ \cos LHA_2 \cos d_2 \sin h_1 \left( \sin h_3 \sin d_1 - \sin h_1 \sin d_3 \right) \\
= \cos LHA_3 \cos d_3 \sin h_1 \left( \sin h_2 \sin d_1 - \sin h_1 \sin d_2 \right)
\]

To facilitate the introduction of these formulae in a computer and/or programmable calculator, we enter the following variables:

\[
A = \cos d_1 \sin h_1 \left( \sin h_2 \sin d_3 - \sin h_3 \sin d_2 \right) \\
B = \cos d_2 \sin h_1 \left( \sin h_3 \sin d_1 - \sin h_1 \sin d_3 \right) \\
C = \cos d_3 \sin h_1 \left( \sin h_2 \sin d_1 - \sin h_1 \sin d_2 \right)
\]

and subsequently:

\[
A \cos LHA_1 + B \cos LHA_2 = C \cos LHA_3
\]

where:

\[
LHA_1 = GHA_1 + L \\
LHA_2 = GHA_2 + L \\
LHA_3 = GHA_3 + L
\]

$GHA_1$, $GHA_2$ and $GHA_3$ are the hour angles of the celestial bodies at Greenwich, and “$L$” the longitude, which at first we consider as positive for east longitudes, and the same formula with its sign solution will indicate for us the west longitudes.

If we follow the process of conversions and make the corresponding substitutions we obtain:

\[
A \cos (GHA_1 + L) + B \cos (GHA_2 + L) = C \cos (GHA_3 + L)
\]

and, expanding the cosines:

\[
A (\cos GHA_1 \cos L - \sin GHA_1 \sin L) \\
+ B (\cos GHA_2 \cos L - \sin GHA_2 \sin L) \\
= C (\cos GHA_3 \cos L - \sin GHA_3 \sin L)
\]

\[
A \cos GHA_1 \cos L - A \sin GHA_1 \sin L \\
+ B \cos GHA_2 \cos L - B \sin GHA_2 \sin L \\
= C \cos GHA_3 \cos L - C \sin GHA_3 \sin L
\]

then, taking out $\cos L$ and $\sin L$ as common factors:

\[
\cos L (A \cos GHA_1 + B \cos GHA_2 - C \cos GHA_3) \\
= \sin L (A \sin GHA_1 + B \sin GHA_2 - C \sin GHA_3)
\]

and, dividing by $\cos L$:

\[
A \cos GHA_1 + B \cos GHA_2 - C \cos GHA_3 \\
= \tan L (A \sin GHA_1 + B \sin GHA_2 - C \sin GHA_3)
\]

or:

\[
\tan L = \frac{A \cos GHA_1 + B \cos GHA_2 - C \cos GHA_3}{A \sin GHA_1 + B \sin GHA_2 - C \sin GHA_3}
\]
In this way we obtain the longitude and thus the latitude, using known data: the hour angles at Greenwich $GHA_1$, $GHA_2$ and $GHA_3$; the declinations $d_1$, $d_2$ and $d_3$, (all data obtainable from the Nautical Almanac using the universal time (U.T.) of observation) and the three altitudes observed by the navigator ($h_1$, $h_2$ and $h_3$).

We have seen that for calculating the observer's geometrical location by applying the attained formulae, it is necessary that the observation of those three celestial bodies be taken simultaneously, that is to say, at the same universal time U.T. This is impossible for only one observer and would require the collaboration of three observers. On the other hand observations are not so precise that they allow us to attain a common geometrical location for the three corresponding circles of altitude, particularly since the navigator observing three celestial bodies will usually give more weighting to one line of position than to others, depending on conditions of the horizon, the collimation, meteorological states, etc.

The latitude can be obtained by any of the following formulae:

$$\tan \text{lat} = \frac{\sin h_1 \cos d_1 \cos LHA_1 - \sin h_2 \cos d_2 \cos LHA_2}{\sin h_1 \sin d_2 - \sin h_2 \sin d_1}$$

for stars $A_1$ and $A_2$

or else:

$$\tan \text{lat} = \frac{\sin h_2 \cos d_1 \cos LHA_1 - \sin h_1 \cos d_2 \cos LHA_2}{\sin d_2 \cos d_1 \cos LHA_1 - \sin d_1 \cos d_2 \cos LHA_2}$$

which can be deduced easily from the previous formulae, with:

$$\tan L = \frac{A \cos GHA_1 + B \cos GHA_2 - C \cos GHA_3}{A \sin GHA_1 + B \sin GHA_2 - C \sin GHA_3}$$

where:

$$A = \cos d_1 (\sin h_3 \sin d_2 - \sin h_2 \sin d_1)$$

$$B = \cos d_2 (\sin h_1 \sin d_3 - \sin h_3 \sin d_1)$$

$$C = \cos d_3 (\sin h_1 \sin d_2 - 2 \sin h_2 \sin d_1)$$

$$\tan \text{lat} = \frac{\sin h_3 \cos d_1 \cos LHA_1 - \sin h_1 \cos d_3 \cos LHA_3}{\sin h_1 \sin d_3 - \sin h_3 \sin d_1}$$

for stars $A_1$ and $A_3$

or using the above sinus formula:

$$\tan \text{lat} = \frac{\sin h_2 \cos d_3 \cos LHA_3 - \sin h_3 \cos d_2 \cos LHA_2}{\sin h_3 \sin d_2 - \sin h_2 \sin d_3}$$

for stars $A_2$ and $A_3$.

In any case, if we were in the situation where the observations were taken by a single observer, he could use the obtained position from the above formulae as an estimated one and subsequently rework his position as a normal running fix, or else as follows:

Let us suppose on board a ship sailing course $AB$ (see Fig. 4), the celestial body $A_1$ is observed to obtain the line of position $CD$ or $EF$ or even $GH$; this line of position only indicates which are the geometrical points of the position. A short interval of time later, another celestial body $A_2$ is observed, getting the line of position $IJ$ or $KL$ or even $MN$.

These lines of position do not necessarily give geometrical points such as "O" or "P" on the course, for example, but, as is obvious, this could be on any of the
lines of position in the figure, in agreement with the astronomical observations taken.

Now the problem which poses itself is how to find out which geometrical point corresponds to the fix we are trying to get.

Let us suppose that as a consequence of the observation taken, the lines of position that we have to work on are GH for celestial body A 1 and IJ for A 2, outside the course AB, for example.

We then transfer the line of position IJ to the position of the first one GH. Observing the triangle $A'TB'$, right angle in T (see Fig. 5), we obtain:

$$\Delta h = D \cos (C - Z_2)$$

where:
- $\Delta h$ increment of altitude
- $D$ distance navigated
- $C$ track followed by the ship
- $Z_2$ azimuth of star A 2.

We see that the increment of altitude "$\Delta h$" is the correction to be applied to the height of star A 2, so as to run it from position 2 to 1.

$$\Delta h = D \cos (180^\circ - (C - Z_2)) = - D \cos (C - Z_2)$$

and in general $\Delta h = D \cos (C - Z)$. 

![Diagram](image)
This formula gives us the increment of altitude and its corresponding sign, and it is a general one which can be verified for the four quadrants, for both azimuth and course, whenever course and azimuth are taken in a clockwise direction, i.e. from the north to the east, as is usual in nautical astronomy.

We can follow the same reasoning for a third celestial body and work out the fix from the last observed position.

To illustrate with an example:

A navigator sails course S 10 W at a speed of 30 knots, on the 4th July 1984 and observes the following stars:

- Denebola: U.T. = 20 h 55 m 49 s, \( d_1 = 14^\circ 39.7' \), above the celestial horizon.
- Spica: U.T. = 20 h 57 m 45 s, \( h_2 = 53^\circ 24.1' \)
- Sibik: U.T. = 20 h 59 m 50 s, \( h_3 = 27^\circ 00.0' \)

What is his fix?

The first calculation we undertake is that of obtaining the fix using the heights just as they are and correcting them later on for transference of the lines of position of Denebola and Spica to that of Sabik.

**Denebola**

- U.T. = 20 h 55 m 49 s
- \( d_1 = 14^\circ 39.7' \)
- Aries H.A. = 223°00.4'
- Correction = 13°59.5'
- Correc. Aries H.A. = 236°59.9'
- SHA = 182°56.3'
- GHA\(_i\) = 419°56.2'

**Spica**

- U.T. = 20 h 57 m 45 s
- \( h_2 = 53^\circ 24.1' \)
- GHA\(_2\) = 396°23.6'

**Sabik**

- U.T. = 20 h 59 m 50 s
- \( h_3 = 27^\circ 00.0' \)
- GHA\(_3\) = 340°38.1'

*Fig. 6. — Flow Chart.*
In the programmable calculator HP 41/CV in which previously we have loaded the program by means of the corresponding cards, we enter the following data:

\[
\begin{align*}
    h_1 &= 60.052 \\
    h_2 &= 53.241 \\
    h_3 &= 27.000 \\
    d_1 &= 14.397 \\
    d_2 &= -11.048 \\
    d_3 &= -15.424 \\
    \text{GHA}_1 &= 419.562 \\
    \text{GHA}_2 &= 396.236 \\
    \text{GHA}_3 &= 340.381
\end{align*}
\]

Once these data have been entered, following the program, we get:

**HP 41/CV Print-out**

```
XEQ "LATLON"
ALTURA ASTROS ?
a1 ENTER +
a2 ENTER +
a3 R/S
   60.052 ENTER +
   53.241 ENTER +
   27.000 RUN

DECLINACION ASTROS ?
d1 ENTER +
d2 ENTER +
d3 R/S
   14.397 ENTER +
   -11.048 ENTER +
   -15.424 RUN

HORARIO ASTROS ?
H*G1 ENTER +
H*G2 ENTER +
H*G3 R/S
   419.562 ENTER +
   396.236 ENTER +
   340.381 RUN

HEMISFERIO NORTE O SUR ?
N S R/S
N RUN

LAT. 24° 59.0' N

LON 29° 59.7' W
```

This is a first approximation of the fix we want to obtain: with this E.P. (estimated position) we can calculate the azimuths of the stars Denebola and Spica.

**Denebola**

\[
\begin{align*}
    \text{GHA}_1 &= 419°56.2' \\
    \text{Lon} &= 29°59.7' - \text{Lon} = 29°59.7' - \\
    \text{LHA} &= 389°56.5' \\
\end{align*}
\]

We enter in HP 41/CV calculator:

```
HORARIO ASTROS ?
H*G1 ENTER +
H*G2 ENTER +
H*G3 R/S
   419.562 ENTER +
   396.236 ENTER +
   340.381 RUN

HEMISFERIO NORTE O SUR ?
N S R/S
N RUN

LAT. 24° 59.0' N

LON 29° 59.7' W
```

```
24.9833 STO 00
14.6617 STO 01
389.9417 STO 02
XEQ "ALT"
104.5 ***
STO 00
360.0 ENTER +
RCL 00
   255.5 ***
```

For Denebola:

\[
\begin{align*}
    \text{Lat} &= 24°59' = 24.9833° \\
    d_1 &= 14°39.7' = 14.6617° \\
    \text{LHA} &= 389°56.5' = 389.9417°
\end{align*}
\]

getting azimuth 255.5°.
For Spica:

Lat. = 24°59' = 24.9833°

d₂ = -11°04.8' = -11.0800°

LHA = 366°23.9' = 366.3983°

Now we can correct the heights of Denebola and Spica using the formula Δh = D cos (C - Z), to get the fix at U.T. = 20 h 59 m 50 s.

Denebola

Course = 190°

Azimuth = 255.5°

C - Z = 065.5°

U.T. = 20 h 59 m 50 s

U.T. = 20 h 55 m 49 s

Δh = +0.8'

h = 60°05.2'

h = 0°56.0'

Δh = 0.8' +

Correc. h₁ = 60°06.0'

h₂ = 53°24.1'

Δh = 1.0' +

Correc. h₂ = 53°25.1'

Spica

Course = 190°

Azimuth = 190.6°

C - Z = 000.6°

U.T. = 20 h 59 m 50 s

U.T. = 20 h 57 m 45 s

Δh = +1'

h = 53°24.1'

h = 0°56.0'

Δh = 1.0' +

Correc. h₂ = 53°25.1'

We enter the data once again in the HP 41/CV calculator:

h₁ = 60.060

d₁ = 14.397

GHA₁ = 419.562

h₂ = 53.251

d₂ = -11.048

GHA₂ = 396.236

h₃ = 27.000

d₃ = -15.424

GHA₃ = 340.381

thereby getting the desired fix:

Lat. 24°58.0’N

Lon. 30°00.4’W

To check this position we work it out as a normal running fix, starting from the approximate position Lat. 24°59.0’N Lon. 29°59.7’W and getting:

Lat. 24°58.2’N

Lon. 30°00.2’W
Using the attached BASIC program will lead us to the same result.

The primary advantage (among others) of this method is its use with the radio-sextant.

It is possible at present to get a fix without using the classic sextant (ignoring radio-positioning systems and satellites). To obtain the angle of altitude of a heavenly body, it is possible to match it by means of radio-sextants, since heavenly bodies emit radiations on their own frequencies showing a visible spectrum or radio wave.

This system is used to calibrate the position of inertial systems of navigation; if we have three receivers available to measure simultaneously the corresponding radiation of groups of three stars, it would always be possible to have available a position of high precision (even for research vessels), whatever meteorological conditions are present, throughout the 24 hours of a day, since these radiations are also received in the day time. In this way it would be feasible to obtain a fix independent from onshore radioelectric stations or satellites.

These formulae can also be applied to land observations, since the attainment of an astronomical point of a geodetic triangular net would not be restricted to the observation of circumpolar stars, that is to say, stars having the same altitude or else the same local hour angle at both sides of the meridian. This system has the advantage that the observer has very easy access to his means of observations, since he can choose the most suitable stars to give the very best intersections.

This paper is intended at present for yachtsmen; however, when electronic development is completed, the principles involved could be a useful tool for navigators, surveyors and also for military applications.
**HP 41/CV Calculator Program**

```
01 *LBL "LATLON" 67 "H*G3 R/S" 132
02 "ALTURA" 57 "H*G3 R/S" 132
10 ACA 75 "XEQ "BES" 132
11 CF 12 76 RCL 06 141 RCL 09
12 PRBUF 77 RCL 08 142 ★
13 "a1 ENTER" 78 "XEQ "BES" 143 RCL 13
14 PRA 79 STO 09 144 RCL 12
15 "a2 ENTER" 80 RCL 02 145 ★
16 PRA 81 XEQ "BES" 146 -
17 "a3 R/S" 82 STO 02 147 RCL 13
18 PRA 83 RCL 03 148 RCL 05
19 TONE 9 84 XEQ "BES" 149 COS
20 STOP 86 RCL 04 150 ★
21 TONE 9 88 STO 04 151★
22 PRA 90 STO 04 152★
23 RDN 91 STO 05 153★
25 RDN 90 STO "XEQ "BES" 155 RCL 06
26 STO 91 STO 05 156 COS
27 "DECLINACION" 92 RCL 07 157★
28 ACA 93 XEQ "BES" 158 RCL 18
29 CLA 94 STO 07 159 RCL 07
30 ARC 95 RCL 00 160 COS
31 ACA 96 XEQ "BES" 161★
32 CLA 97 STO 00 162★
33 ARC 98 RCL 01 163 RCL 20
34 SF 12 99 XEQ "BES" 164 RCL 08
35 ACA 100 STO 01 165 COS
36 SF 12 101 SIN 166★
37 PRBUF 102 STO 01 167★
38 "d1 ENTER" 103 RCL 05 168 RCL 15
39 PRA 104 SIN 169 RCL 06
40 "d2 ENTER" 105 STO 10 170 SIN
41 PRA 106★
42 "d3 R/S" 107 RCL 02 171★
43 PRA 108 SIN 172 RCL 18
44 TONE 109 STO 11 173 RCL 07
45 STOP 110 RCL 04 174 SIN
46 ADV 111 SIN 175★
47 STO 05 112 STO 12 176★
48 RDN 113★
49 RDN 114★
50 RDN 115 RCL 05 179 SIN
51 STO 03 116 SIN 180★
52 "CORARIO" 117 STO 13 182 / 53 ACA 118 RCL 03 183★
54 CLA 119 COS 184 STO 21
55 ARC 120 STO 14 185 RCL 06
56 ACA 121★
57 CLA 122★
58 ARC 123 STO 15 186★
59 SF 12 124 RCL 11 187 COS
60 ACA 125 RCL 03 188 STO 22
61 CF 12 126 SIN 189 RCL 14
62 PRBUF 127 STO 16 190★
```

HP 41/CV Calculator Program

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IMPROVED METHOD OF GETTING A FIX AT SEA
This program gives you the azimuths and altitudes as well, to run the fix as a normal one.

To get azimuths you subtract the value provided by the calculator from 360° if \( \sin \text{LHA} > 0 \) or else take azimuth just as it is in the clockwise direction. See pages 128 (bottom), 129 (top).

Latlon Latitude, longitude
a1, a2, a3 stars’ altitudes (heights)
d1, d2, d3 stars’ declinations.

H\(^{*}\)G 1, H\(^{*}\)G 2, H\(^{*}\)G 3 Greenwich hour angles?

Northern or southern hemisphere?
**BASIC PROGRAM**

10 REM Fix without dead reckoning
20 REM
30 REM
40 DIM A(3), D(3), H(3)
50 FOR I = 1 TO 3
60 PRINT "STAR HEIGHT?" A(I),
70 INPUT A(I)
80 PRINT A(I)
90 F = A(I)
100 GOSUB 870
110 A(I) = F
120 PRINT "STAR DECLINATION?" D(I),
130 INPUT D(I)
140 PRINT D(I)
150 F = D(I)
160 GOSUB 870
170 D(I) = F
180 PRINT "GREENWICH HOUR ANGLE?" H(I),
190 INPUT H(I)
200 PRINT H(I)
210 F = H(I)
220 GOSUB 870
230 H(I) = F
240 PRINT
250 NEXT I
260 REM
270 REM ********************
280 REM * Calculation *
290 REM ********************
300 REM
310 X = SIN(A(1)) * COS(D(1)) * (SIN(A(2)) * SIN(D(3)) - SIN(A(3)) * SIN(D(2)))
320 Y = SIN(A(1)) * COS(D(2)) * (SIN(A(3)) * SIN(D(1)) - SIN(A(1)) * SIN(D(3)))
330 Z = SIN(A(1)) * COS(D(3)) * (SIN(A(2)) * SIN(D(1)) - SIN(A(1)) * SIN(D(2)))
340 Lon = (180 / PI) * ATN((X * COS(H(1)) + Y * COS(H(2)) - Z * CCS(H(3))) / (X * SIN(H(1)) + Y * SIN(H(2)) - Z * SIN(H(3))))
350 FOR I = 1 TO 3
360 H(I) = H(I) + Lon * PI / 180
370 NEXT I
380 Lat = (180 / PI) * ATN((SIN(A(2)) * COS(D(1)) * COS(H(1)) - SIN(A(1)) * COS(D(2)) * COS(H(2))) / (SIN(A(1)) * SIN(D(2)) - SIN(A(2)) * SIN(D(1))))
390 REM
400 REM ********************
410 REM * Hemisphere *
420 REM ********************
430 REM
440 PRINT "NORTH OR SOUTH HEMISPHERE (N/S) ? ",
450 INPUT J$
460 PRINT J$
470 IF J$ = "N" THEN GOTO 510
480 IF J$ = "S" THEN GOTO 620
490 GOTO 440
500 REM
510 REM ******************** NORTHERN HEMISPHERE ******
520 IF Lat < 0 THEN GOTO 540
530 GOTO 640
540 IF Lon < 0 THEN GOTO 560
550 GOTO 580
560 Lon = Lon + 180
570 GOTO 590
580 Lon = Lon - 180
590 Lat = ABS(Lat)
600 GOTO 660
610 REM
620 REM ******************** SOUTHERN HEMISPHERE ******
630 IF Lat > 0 THEN GOTO 540
640 REM
Note

The author of this work has prepared a second bilingual edition of his book "Nautical Astronomy — Fix without dead reckoning", where the errors that can arise with this system and their elimination are treated.

BIBLIOGRAPHY